

# Think It Over

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This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

## Solution to The Ten Pirates Problem

The following problem was heard from Prof. Yehuda Vardi of Rutgers State, University, New Jersey, USA.

### Solution given by

S Ravi

Department of Statistics

University of Mysore

Manasagangotri

Mysore 570 006, India.

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**Problem:** Suppose that you are the chieftain of 10 sea pirates and that your group stumbles upon a treasure of 100 gold coins. Further, let the group have a hierarchical structure in the sense that the members of the group have ranks from 1 to 10 with the chieftain having rank 1 and the weakest in the group having rank 10. It is agreed upon by your group that the chieftain will propose a strategy to divide the 100 gold coins among the group members which will be put to vote. If the chieftain succeeds in getting at least 50% of the votes including his own then the coins will be divided as per the strategy and if the chieftain fails to get the required votes then the chieftain will be thrown into the sea and the next person in the hierarchical order will get the chance to propose a strategy which will be put to vote and the same procedure is repeated till somebody gets at least 50% of the votes. Let us also assume that there are no deals between the group members and that the pirates think rationally in the sense that they will think that one gold coin is better than no gold coins. One strategy of the chieftain could be to divide the 100 gold coins equally among the members of the group with each one getting 10 gold coins. The question is what is the best strategy that you, the chieftain, has to propose so as to save your life and which will fetch you the maximum number of gold coins?

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**Solution:** To get an idea about how to solve this problem, let us first consider only two members in the group of pirates, namely, you and one other person having rank 2. Then you may propose the strategy to keep all the 100 coins for yourself and not to give any coins for the other pirate, put this strategy to vote, vote for



Pirates	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	Votes
2	100	0									1
3	99	0	1								2
4	99	0	1	0							2
5	98	0	1	0	1						3
6	98	0	1	0	1	0					3
7	97	0	1	0	1	0	1				4
8	97	0	1	0	1	0	1	0			4
9	96	0	1	0	1	0	1	0	1		5
10	96	0	1	0	1	0	1	0	1	0	5

yourself and thereby gain the required 50% of the votes! If there are three persons, say yourself, B and C, ordered according to heirarchy, then you may propose the strategy of keeping 99 coins to yourself, no coin to the next person below you in the heirarchy, that is B and 1 coin to C. Notice that if you do not get the vote of C then you will be eliminated and if there are only two persons B and C left then C will get no coin and so C is forced to vote for you! Continuing this argument, we get the table given above which will give you a strategy that will fetch you the maximum number of gold coins.

The table above gives the division of coins according to ranking 1 to 10 from left to right where the first column gives the number of pirates in the group, the next ten column give the number of gold coins which each member of the group will get according to ranking and the last column gives the minimum number of votes needed for the strategy.

Note that if there are  $N$  pirates and  $T$  gold coins, then the chieftain's strategy which will fetch the maximum number of votes will be to keep  $(T - N + [N/2] + 1)$  since she/he needs at least  $(N - [N/2])$  votes and to distribute the other to the remaining pirates in the group as follows. Pirate ranked even will get 0 and pirate ranked odd will get 1 gold coin. Here  $T$  is the total number of gold coins and  $[x]$  means the largest integer not exceeding  $x$ .

