Numeracy for Everyone

6. Numeracy in Social Sciences – Art and Science of Quantifying Amorphous Concepts, Impressions and Unmeasurables

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Matters in the sphere of politics, economics, sociology, law, etc. affect lives of all. Here too numerical information is often the basis of many decisions, policies and actions. As intelligent citizens, participating actively in social life, we need to understand the logic behind several such matters.

Many students and parents hold the wrong notion that social sciences are totally descriptive and any aptitude for quantification is quite unnecessary to study it. While good amount of work can be done in social sciences without numbers, measurement always strengthens any argument and can be essential in some situations. Here is a simple example. Large sums of money are currently being spent on increasing awareness of AIDS. Funding agencies are keen to know if the work has any impact. So we may record interviews among people vulnerable to AIDS (e.g. truck drivers) before and after the education programs such as awareness camps. The difference if any will be a numerical measure of its effect on society.

The word measurement need not always imply a thermometer or measuring tape, etc. Even if we simply give names and classify, it is a step in the right direction. So sex (male and female), profession (teacher, scientist, lawyer, social worker, etc.) religion (Hindu, Muslim) are cases of naming. Sometimes it is called 'nominal' measurement. If there is some hierarchy, that is yet another step in measurement called ordinal. Let us consider such a case.

Abodes of Gods

If you study religion and rituals of tribals, you may describe

their deities and sacred groves, various sacrifices, etc. None of these will require numeracy skills. But consider the following proposition: higher the degree of elaborateness in a cult or worship spot, greater is the degree of destruction in the surrounding forest. If true this will mean that more primitive tribals will live in better surroundings. But how does one quantify either?

A deity in the open consisting of a rock covered with red lead can be regarded as the most primitive. A roof, walls and floor make the temple more elaborate. If the deity is beautifully carved, elaborateness is greater. So we have three ordered categories.

Now order the surrounding forest by the tree density i.e., number of trees per unit area and decide on suitable intervals. Then there can be categories such as – very dense, moderately dense and disturbed. So, a contingency table can now be designed as shown in *Table* 1.

Here we can actually measure association between the two. If all cases fall in cells a, b, c and other cells are empty then the proposition is strongly supported. If, on the other hand, all cases fall in cells d, b, e the conclusion is reversed in that elaborate temples seem to harbour dense vegetation around. In practice there may be non-zero counts in all/most of the 9 cells and a suitable measure such as chi-square may have to be calculated to quantify intermediate level of association. We hope the point is clear.

Gallup Polls

Ours is a democratic constitution. Periodic elections are the means by which we, the people, exert our supreme right to

Status of Surrounding Forest	Elaborateness Level of Deity		
	Low	Medium	High
Disturbed	e		c
Moderately dense		b	
Dense	a		d

Table 1. Association between deity elaborateness and forest status.

[1] Part 1. Why Quantification?, *Resonance*, Vol.4, No.5, 19-30, 1999.
Part 2. Dice of Life, *Resonance*, Vol.4, No.9,14–23, 1999.
Part 3. Just for Ecologists, *Resonance*, Vol.5, No.5, 15-25, 2000.
Part 4. Numeracy in Research Planning, *Resonance*, Vol.5, No.11, 14-25 , 2000.
Part 5. A Circular Path of Conjectures, Observations, Interpretation and New Conjectures about Diseases and the Causes there of, *Resonance*, Vol.6, No.3, 8-18, 2001.

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choose our rulers. Every general election is almost an upheaval. Thousands of candidates seek the blessing of the electorate. Therefore any means of assessing inclination of voters is of great interest. Consequently, sample surveys of voters evoke tremendous excitement. Such surveys seem to have started in early nineteenth century in United States. For a general description of sample surveys see Part 4 of this series[1].

Literary Digest, a prestigious magazine in USA conducted in the 1930's a mail questionnaire survey of about 2.4 million voters and estimated that Alfred E Landon will win over Franklin D Roosevelt in the presidential race. One advantage to the public (and disadvantage to the pollsters) in pre-election polls is that the day of reckoning is right there. In this instance, actual result was the opposite of the prediction. Roosevelt won handsomely. A post facto analysis of what went wrong in the survey was inevitable. The sample size was absolutely massive and the survey could not be faulted on that score. However, selection of voters turned out to be biased. The survey used lists of car and telephone owners to send the query forms. These were all individuals in the average and above average income groups. In this section of society, the Republican candidate was popular. But Democrats had support in lower income groups which were more populous.

This signal failure of electoral poll emphasised the importance of proper selection of a truly representative sample. This was not the only election survey at that time. Several others were carried out too. One of them was designed by George Gallup. It ensured better representation of the electorate and did forecast Democratic victory. Ever since, the name of George Gallup has become associated with election surveys. In fact often they are called Gallup polls. Since the thirties, methodology of such surveys has been improved considerably, and their credibility today is high. In the latest American Presidential elections, pollsters had predicted a close fight between George Bush and Al Gore and indeed it turned out to be so.

Before	After		
	Democrat	Republican	Total
Democrat	50	20	70
Republican	5	25	30
Total	55	45	100

Table 2. Preference before and after debate.

Apart from anticipating the winner there can be other aspects of elections which need a quantitative study. Of late, television debates have become an important part of American Presidential elections. It is of interest to see if such debates make voters change their mind and in which direction. Suppose in a hypothetical study 100 voters expressed their preference before and after a debate and the result is given in *Table* 2.

Here 25 out of 100 voters have changed their minds. 20 people switched from Democrat to a Republican candidate while 5 switched the other way round. Should the number be regarded as essentially equivalent or have Republicans benefited more? Without going into technicalities we will say that the difference squared divided by the number who switched i.e. $(20–5)^2/25$ is a good measure of relative effect of debate. If this quantity is large we may regard the debate to have helped Republicans significantly. In technical jargon this is called McNemar test after the psychologist who invented it.

Surveys are useful not only to study elections but in fact to study virtually any aspect of society. Kenneth Boulding, a famous social scientist termed the survey method 'telescope of the social sciences'. This is a very apt simile because surveys help researchers scan social skies. In Arabian stories, Harun Al Rashid, the Khalipha of Baghadad had to walk the streets of the city in disguise at night to understand ground reality. This subterfuge is no longer needed. Sample surveys can obtain information about almost any feature of social life. Many newspapers, including the *Times of India*, conduct frequent opinion polls about current issues and publish results. These include surveys

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on internet. *Times of India* puts out a question every day and publishes the reactions received. A word of caution is in order. The responses come over internet. So at best they represent those Indians who have access to internet. That is a tiny fraction. Secondly these are responses from volunteers. Often volunteers are not representative of the whole group. Of course, to be fair to *Times of India* it must be admitted that they make no bones of these limitations. All schools and colleges can undertake the exercise of conducting such surveys locally on questions of local and broader interest. National Sample Survey Organization (NSSO), a professional group set up by the Government of India conducts periodic 'rounds' of expenditure surveys. These reveal the felt needs of people in urban and rural areas. Results of such surveys constitute major inputs into the national planning processes.

Measuring the Unmeasurable

Economics is the science and art of using limited resources to satisfy our varied needs. We bring home a salary, spend the money on food, housing, clothes, education, entertainment, health and what not. Many people get a basic salary plus a dearness allowance (or cost of living allowance/COLA). What is the cost of living?

Economics is of course concerned with prices, demand and supply. When price falls demand rises (case of negative correlation). Cost is another name of price. So far so good. We understand what is the cost of milk or bread. But what is the cost of living as a whole? Clearly, we need an overall figure measuring this concept. Here we encounter a peculiar situation. The entity to be measured does not really have physical existence. It exists only in our heads. It is an amorphous concept. In this sense we are trying to measure the unmeasurable. Cost of an item can easily be measured through market inquiry. But to measure (overall) dearness level, we have to operationalise this concept first before measurement is possible. Such an operationalised measure is consumer price index. This takes us

into the art of index construction. An index is constructed when direct measurement is not possible. Human height can be measured in cm and fruit yield can be measured in kg. And no index is necessary. On the other hand, intelligence, health, poverty, etc. cannot be measured so simply. Here a formal calculation procedure is spelt out and once agreed upon, it is followed by all. This general agreement is the strength behind any particular index. Consider the case of measuring intelligence.

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We can say that Mr. X is more intelligent in mathematics than Mr. Y since the latter got fewer marks than X. But that is only one aspect of intelligence. There are others. Verbal skill, recognising geometric patterns, judging weights, having a ready wit and so on. But how do we combine such measures? Psychologists have answered this question by means of a formula we call the IQ or intelligence quotient. Once such an index is devised, authors have to help others to understand it. In case of IQ we know that average humans have IQ of around 100, geniuses have values say above 140 while value below 70 probably suggests an imbecile. Secondly, in certain cases, when people know that the individual is a very bright person, the index value should come out quite high. Such agreement increases credibility of an index. Once people see that where they are sure, the index agrees with them, they feel comfortable about using that index in situations which are ambivalent.

Coming back to dearness level, construction of a 'consumer price index' involves 3 steps. First, prepare a list of items that a typical consumer buys. Importance of each item is assessed and a weight is given accordingly. Next we calculate a weighted sum of prices.

Lastly the figure is expressed as a multiple of the corresponding value for a base or reference year. Index value for base year is 100. The corresponding value for current year is usually much higher. Since the second world war, by and large, there is continuous rise in the index. There are very few items in case of

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which prices are on the decline. Computers are among such articles of purchase. But otherwise, the overall index is found to creep up throughout inexorably. This rise is termed as inflation. Generally we express inflation rate as % increase in consumer price index. In developed countries, rate of inflation is very low, say a couple of percent per year. In India it is higher. This year it is about 6%. When inflation rate enters double digits, all Indians feel concerned (or such is the assessment of some analysts). The population tends to become more resentful of the rulers. But double digit inflation in India is nothing compared to the so called runaway inflation sometimes experienced by countries in Latin America. In this situation, with rate of inflation one (or even two) order of magnitude higher, everyone rushes to convert money into goods. Suppose you have 100 rupees, it is enough to buy a shirt. But tomorrow, the same shirt will cost 200 rupees. Then it is best not to keep your money idle but quickly exchange it for goods. Another economic nightmare is grinding poverty.

Recently Amartya Sen made all Indians proud by winning the Nobel Prize in Economics. Among other things he is known for his research on measurement of poverty. Sen's index of poverty (or minor variations thereof) is the standard method of poverty measurement at least in research literature. So, how do we measure poverty? Here a crucial term is poverty line.

This is the level of income needed to satisfy a person's basic needs. Make a list of all items that are essential (e.g. food items, clothing, medicine, etc.). Decide the quantity of each item needed. Obtain information on price of each item. Hence find the amount of money needed to satisfy basic needs. Anyone with income below this line is regarded as poor.

At a conceptual level, this sounds very reasonable. But making a list of essential commodities is not a trivial task. Some people consider meat/fish essential at least occasionally. Others abhor meat. South Indians cannot survive without rice and north Indians need to have wheat. Alcohol is an essential commodity

for many tribal groups. It is part of all religious rituals. Some people consider tobacco and alcohol as essential.

Even if items are agreed upon, their quantities are not easy to decide. We may then refer to nutrition experts who may know how much food a man needs. You may find it surprising, but estimates of nutritional requirement are constantly revised, often downwards. So how reliable can estimated poverty line be at any moment? Thus it is very difficult to come up with a really good evaluation of poverty line. Suppose we do have one reasonable value to be used. Still measurement of poverty does not follow. There can be various ways of using poverty line. 'Head count' is simply the number (or proportion) of people whose income falls below poverty line. This index is not too bad, except for one hitch. Among people who have incomes below poverty line, some incomes are just below while others are far below. This gap is ignored by 'head count'. Also there is the angle of 'relative deprivation'. If everyone shares the same suffering, there is no relative deprivation and poverty hurts less in such a case. Inequality makes the impact of poverty more severe. Amartya Sen showed how all these considerations can be given due weight in constructing a 'good' poverty index. This index P is defined as $P = HI + (1-I)*G$. Here H is head count or proportion of people with income below poverty line. I is the income gap or average deficit for poor people defined as $I = (z - \bar{x})/z$. z is the poverty line. $\bar{x}$ is the average income among the poor. G is the Gini coefficient of inequality among poor given by

$$G = \frac{\sum_{i=1}^{q} \sum_{j=1}^{q} |x_i - x_j|}{2q^2 \bar{x}}$$

If there is no inequality among poor, G is zero (can you see why?) and Sen's index is simply HI. The question of avoiding use of a seemingly arbitrary poverty line has foxed many an expert. Here is one possible alternative.

'Head count' is simply the number (or proportion) of people whose income falls below poverty line.

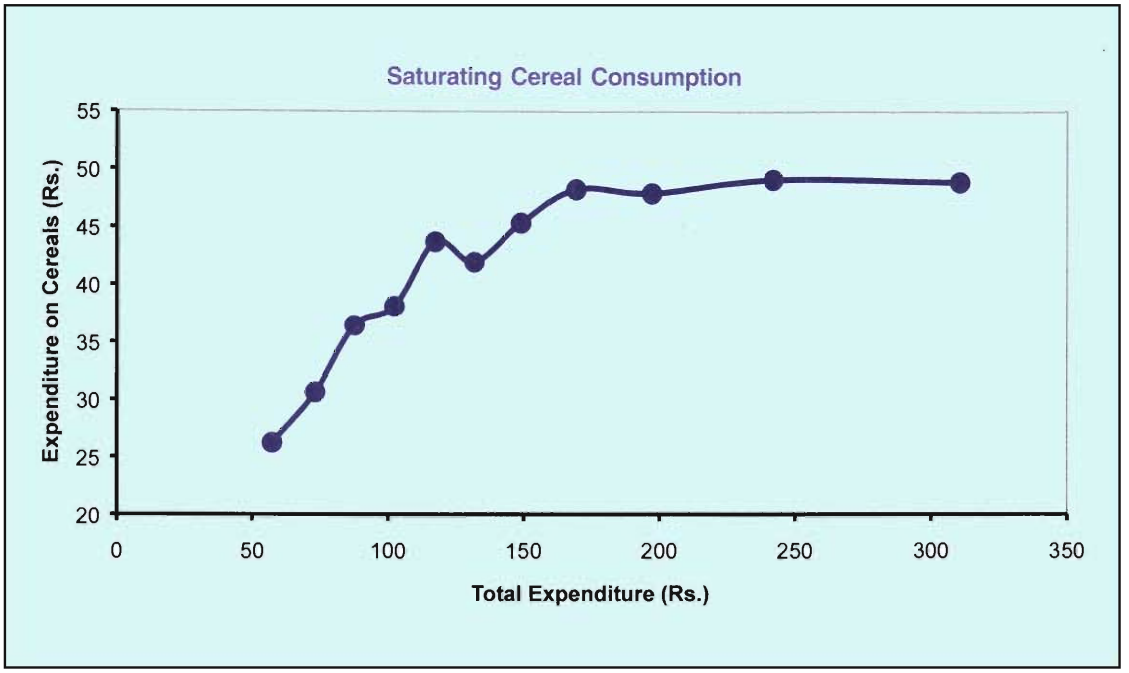

Figure 1. NSS 45th round rural (1989-90) per capita per month expenditure data. Total expenditure is used as a surrogate for total income.

If we plot income level on X-axis and money spent on cereals (rice/wheat) on Y-axis, the data seem to follow a rising but saturating curve. See *Figure* 1. This is a well-known phenomenon. As you spend more on cereals, you get satiated and any further income is spent elsewhere. The level to which this curve approaches can be regarded as requirement. We can say that people reveal requirement through their expenditure pattern. Poverty (in cereal consumption) can be assessed by comparing actual consumption with this requirement.

Index construction is only one area in economics. But quantitative inputs are needed in many. To give a couple of examples, consider forecasting share prices or tax collection or export earning. If you read a good newspaper or magazine, you will invariably find an economics section. We leave it as an exercise for you to read such sections and identify matters involving numeracy.

Let us now turn to a different area in social science, viz. law and judiciary.

Uncertainties in Law

Estimating residual life is a statistical problem.

"Oh come off it! What has statistics to do with law? Law may be an ass. But it is firm and rigid. Nothing uncertain about it." Or so you may think. But the truth of the matter is that in many criminal cases there is not enough evidence for the judge to be absolutely sure of guilt (or innocence) of the accused. So any judgment is prone to error. Errors can be of two types. An innocent person may be wrongly pronounced guilty. Or a guilty person may be acquitted. It is virtually impossible to avoid both errors. Then the tendency is to avoid more serious error. Attitude in India (inherited from the British) is that it is more important to ensure that no innocent person gets punished. So a judge tries to see if guilt is established beyond reasonable doubt. You see, doubt and uncertainty are part of jurisprudence.

But statistics can come handy in the legal field in many ways. We will briefly describe a few cases documented in literature.

The first case concerns a workers' union in an aluminum smelting company in Canada. The workers during a strike, disconnected electric power supply to the aluminum plant. Now aluminum smelting is a continuous process in which the ore is heated to very high temperature in crucibles. These crucibles became cold and cracked after power outage. The company sued the union for damages, demanding that the union should pay for purchase of new crucibles. Union lawyers argued that the crucibles had been in use for a long time and needed replacement anyway. So the matter boiled down to how much longer the crucibles would have served had it not been for the union mischief. In other words what was the expected residual life of the crucibles. Any reasonable compensation should be commensurate with this duration.

Estimating residual life is a statistical problem. If there was a fixed period beyond which a crucible becomes dysfunctional, the matter would have been very simple. But it is not so.

Techniques of 'survival analysis' have to be deployed to complete the calculation. Details of that would take us way beyond the scope of this writing.

The second case is a dispute between a municipal corporation of a major city in USA and a company contracted to collect coins from parking meters. These are metal devices with clocks, mounted on pipes and hoisted near each car parking slot in public places such as roads. Users of the parking slot insert coins in meters and are eligible to use the slot for time proportional to the amount of money inserted. The contracting company was to open meters periodically and collect coins from thousands of such meters and to deliver the money to the city authorities. It was suspected that considerable pilferage was involved. So, as a first step, authorities decided to estimate the total amount of money deposited in parking meters. They used the capture-recapture technique [2]. They secretly marked some coins and inserted them in different meters. Only some of these coins came back in the bags representing the day's collection. It became clear that a considerable portion of the collected money was siphoned off. (Let us say D dollars.) Hence employees of the company were followed on a day, their activities secretly videotaped and then a case was filed. The company pleaded guilty to pilferage on the dates for which videotaped evidence was available. Municipal corporation demanded that the company should pay back D dollars per day for the entire period of contract. A counter argument made was that the collections were much lower in previous periods. This necessitated estimation of true collection in previous periods. Such estimation requires fairly sophisticated statistical methods.

[2] Part 3. Just for Ecologists, *Resonance*, Vol.5, No.5, 15-25, 2000.

We hope examples given here convince you of the usefulness of numeracy in social sphere.

Exercises

1. Plot changes in share prices of some companies in the field of information technology (e.g. Wipro, Infosys) over the last 8 weeks.

2. Seek opinions of your classmates about a TV serial. Then seek opinions of about equal number of people of your parents' age group about the same program. Is the percentage of people who like the program same in both groups?

3. Following data are on the last 12 parliament elections in India.

Election	1	2	3	4	5	6
(T) total voters (crores)	17	19	22	25	27	32
(P) % votes cast	39	39	45	39	45	40

Election	7	8	9	10	11	12
(T) total voters (crores)	36	40	50	51	59	61
(P) % votes cast	43	36	38	39	43	38

Plot graphs of T and P overtime and comment on them. Try to fit simple linear regression and check which slope is steeper.

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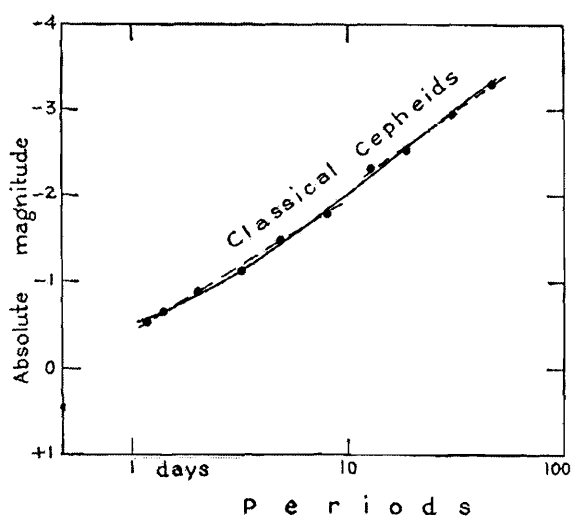

The figure shows the relation between the period of a cepheid variable star and its actual brightness, as discovered by Henrietta Leavitt. Astronomers use the 'magnitude' scale to describe the brightness of a star – the brighter a star is, the lesser its magnitude is. Absolute magnitude denotes the actual brightness of a star in this scale.
(Source: The Story of Variable Stars by Leon Campbell and Luigi Jacchia)