Understanding Mathematics – A Review

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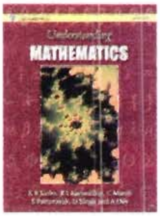

Understanding Mathematics
by K B Sinha, R L Karandikar, C Musili, S Pattanayak, D Singh and A Dey
Universities Press (India) Limited
2000, Rs 120.

It seems to me that people become research mathematicians because of one of the following three broad reasons (allowing for permutations and combinations of all three!):

- ❖ they have discovered the joy of problem solving and are fascinated by solving logical puzzles, and have the ability of coming up with unusual tricks or solutions to problems they encounter,
- ❖ they have come in contact with the depth and beauty of mathematics and have an insight into 'mathematical reality' and would like to explore this reality,
- ❖ they feel that mathematics is the 'language of reality' and therefore would like to learn its grammar and syntax and perhaps do some creative writing themselves.

So any book with the intention of awakening an understanding of mathematics must expose a student to one or more of the above aspects of mathematics. The book under review, I feel, attempts the second and the third. It is addressed to students at the high school and undergraduate level, and the objective as stated by the authors themselves is a hope that after reading this book "some of them will be motivated to take up mathematics as a career". The book is not intended as a text, but rather as supplementary reading.

I would like to begin by discussing the contents of the book and the treatment of various topics. The book is divided into 7 chapters, three of which concentrate on what mathematicians refer to as analysis, two on algebra, one on sets and functions and one on probability and statistics. I read the chapters with the following questions in mind:

- ❖ Is the language accessible to a high school or undergraduate student?
- ❖ Does the chapter give a flavour of the topic to the student and will it arouse their curiosity to learn more about it?
- ❖ At the end of reading the chapter, will the student have a good grasp on the material?
- ❖ Is there a clear goal or a punch line to the chapter?

The first is a chapter on 'sets and functions'. It covers the basic definitions and goes into De Morgan's laws, introduces relations and the notion of a function. I found this to be a well-written chapter with a host of examples, most of which are chosen from the daily life of the student. The coverage is quite extensive and I feel confident that a student would get a good feel for this topic. However the topic might have been further enhanced with historical notes and perhaps an informal discussion on Russell's paradox. I felt that it would have been better to start with concrete examples of a 'relation' and then give a formal definition, rather than the other way around. My experience as a teacher has taught me that students grasp concepts better if they are introduced first concretely and then formalised. It would have been nice if the theorem that equivalence relations lead to partitioning of sets was included, especially since cosets and Lagrange's theorem are discussed in the chapter on groups.

Chapters 2 and 3 cover calculus material from a rigorous standpoint. They introduce the notion of constructing the real line starting from natural numbers, have an elaborate discussion on sequence and series, introduce the notion of limits, continuity, differentiation and integration. Chapter 4 discusses applications of material covered in the earlier chapters and introduces differential equations (first and second order), maxima, minima and the fixed-point theorem. One glaring error in my view is to have put the discussion on maxima and minima and the fixed point theorem after a discussion on differential equations. These topics are far more intuitive and most students find them quite easy to grasp. The fixed-point theorem could have served as the punch line to end Chapter 3. The main strength of these chapters is the variety of examples given to illustrate the various concepts – it is quite an exhaustive collection. I liked the introduction of Peano's axioms, which one does not see in a book at this level and Hilbert's hotel is a nice touch. Chapter 6 deals with probability and statistics and it is a welcome change that statistics is not introduced via concepts like mean, median and mode, but with probability. The concepts of maximum likelihood and hypothesis testing are not really appropriate for students at the plus 2 level unless one is willing to give it a more leisurely and thorough treatment.

Chapter 5 gives a brief introduction to linear algebra, covering transformations, matrices and the algebra and geometry of transformations, eigen values and eigen vectors. This chapter serves as a precursor to Chapter 7, which deals with groups of transformations, and could have been clubbed together with it as a section on algebra. If the purpose of Chapter 5 was to introduce students to this important area in mathematics it could have ended with the theorem that every real symmetric matrix is diagonalizable, perhaps with a discussion that one of the major aims of linear algebra is to classify similar matrices.

Chapter 7 starts with groups and covers rings and fields and has a brief introduction to algebraic geometry. I must confess that the section on groups appealed to me the most. Apart from a confessed bias towards algebra, I found that this section satisfactorily answered the questions raised above. The language is suitable for high school students; it has plenty of pictures and illustrations and after quite a thorough introduction to symmetry groups ends with the lovely Polya enumeration theorem with concrete applications to molecular chemistry. Here, too, one felt that after a leisurely treatment of transformations the section on permutation groups was covered rapidly with a host of results and definitions. In my opinion it would not have been such a great sacrifice to have left out the other sections of this chapter and just given a thorough treatment of groups.

As a teacher at the school level I feel that the language and notation in most parts of the book is unsuitable for high school students. While many chapters start at the level of a high school student they soon jump to that of an undergraduate. Very often there are a lot of new definitions and concepts which go nowhere, and while these may serve to give the reader a wider exposure, my fear is that these would only intimidate a student rather than excite him or her. The book would have been far more effective if all the chapters had been modeled on the section on groups. As mentioned earlier the strength of the book lies in its examples. However, the exercises are rather sporadic and this will undermine its effectiveness. The disparity in style is evident while dealing with historical notes. Some chapters carry them and some do not. I do feel that a good historical perspective goes a long way in motivating students. Many of the topics covered have a rich and lively history. Since one of the objectives of the books is "to explain why definitions and the theorems are the way they appear", perhaps a link with material covered in school could be made. This could be achieved by looking at a topic covered in school and then by a series of questions leading the reader to deeper issues.

The field of mathematics is fortunate to have many excellent books. These include texts, popular books, which serve to inspire students and many which are guides to 'discovering mathematics'. However, many of these are perhaps not affordable to students and even libraries. The book under review being very reasonably priced will be affordable to most students and libraries. I would recommend the book to bright undergraduates and perhaps some high school students who have access to a knowledgeable teacher. Teachers can definitely use the book to learn new material and will have within reach several useful examples.

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There is only one important question:

Is the universe friendly?

Albert Einstein