Harish-Chandra Research Institute, Allahabad

formerly

Mehta Research Institute of Mathematics and Mathematical Physics

A Brief History of the Institute

The Harish-Chandra Research Institute (known as the Mehta Research Institute of Mathematics and Mathematical Physics until October 2000) came into existence in 1975, with a donation of some land and Rs. 40 lakhs from the B S Mehta Trust in Calcutta. With the aim of converting it into a top-class research Institute in the country, the Department of Atomic Energy took over the reins of the Institute and in January, 1992, H S Mani was appointed as the Director and its charter was changed to include theoretical physics as well as mathematics. In January 1996, the Institute moved to its current location on the banks of the Ganga, just beyond the Sangam, in Jhusi. Thus, although, the Institute celebrated its Silver Jubilee in 2000, in its new incarnation at Jhusi, it is not even five years old.

Within a very short time since its inception, the Harish-Chandra Research Institute has established itself as a leading research Institute in the country in the fields of mathematics and theoretical physics. It is a young and vibrant Institute and is currently in the expansion mode with new faculty with research experience in the latest upcoming fields, joining every year.

Academic Activities at the Institute

The aim of the Institute members is to carry out research and add to the body of international knowledge at the frontiers of their subjects in the fields of mathematics and theoretical physics. What are these subjects and why are the Institute members (as well as scientists elsewhere) interested in studying them? We try to briefly answer this question below for the subjects represented at HRI.

A. Mathematics

A good definition of mathematics is 'the study of mental objects with reproducible properties'. The main subdivisions of mathematics include geometry, algebra and analysis. Geometry is a study of a space M made up of points with various sorts of structures, analysis involves functions and its derivatives, and algebra is a study of a set of things with a law of composition. HRI scientists mainly work on algebra, with particular emphasis on number theory. There are also a few who work on algebraic geometry and analysis (*Box*1).

Box 1.

To give a flavour of the kind of mathematical research being pursued at HRI, we outline a few problems in number theory studied here. Define

$$\Delta(q) = q \prod_{n=1}^{\infty}(1-q^n)^{24} = \sum_{n=1}^{\infty}\tau(n)q^n.$$

It can be seen that the product expansion of $\Delta(q)$ converges for $|q| < 1$, and defines a holomorphic function in this region, and is a special, although very important, example of what is called a modular form. The properties of the coefficients, $\tau(n)$, defined for integers $n \geq 1$, have been studied for a long time. It was conjectured by Srinivasa Ramanujan that

1. $\tau(mn) = \tau(m)\,\tau(n)$, if $(m,n) = 1$.
2. $\tau(p^{n+1}) = \tau(p)\,\tau(p^n) - p^{11}\,\tau(p^{n-1})$ for p a prime, and $n > 1$.
3. $|\tau(p)| \geq 2\,p^{11/2}$ for all primes p.

The first two properties were proved in the early part of the century and now forms a subject in itself. The third part of Ramanujan's conjecture was only proved in the mid-seventies by P Deligne who got the Fields medal for it in 1978. There are many other questions about these coefficients which are still unsolved –

1. Is $\tau(p)$ ever zero for p a prime?
2. For which primes p, is $\tau(p)$ divisible by p?

All of these form part of the theory of modular forms, which has been attracting much attention in recent times especially since it is at the heart of the recent proof of Fermat's last theorem which states that the equation

$$X^n + Y^n = Z^n$$

has no integral solutions for $n > 2$ with $XYZ \neq 0$.

B. Physics

In physics, the general aim is to understand the material world – what everything around us is made of and what causes them to behave as they do. The main areas of research here in HRI are high energy physics or elementary particle physics, string theory, condensed matter physics and astronomy and astrophysics (*Boxes* 2-5).

Quality of life in the campus

HRI is housed in one of the most beautiful campuses in the country. It is situated on the banks of the Ganga, just beyond the confluence of the Ganga and Jamuna (*Sangam*), near the village of Jhusi in the outskirts of Allahabad. The idyllic surroundings with walks through the woods, multi-coloured flowers blossoming in all seasons, and mynahs, peacocks and myriad exotic birds, makes it a naturalist's paradise.

HRI also has an excellent library, a very strong visitor's programme and state of the art computer facilities, with a local network connected to the internet via both VSAT and VSNL links, and with personal high end desk-top computers on every desk. But besides these facilities, HRI also offers an extremely informal academic atmosphere conducive to learning, with perfectly free interactions between the faculty and the students.

Box 2. Elementary Particle Physics

Elementary particle physics seeks to understand phenomena at the sub-atomic level and at the highest energies available today. At this level, particles are governed by three types of interaction – strong, weak and electromagnetic. There exists a well-motivated theory, called the standard model, that explains most phenomena coming under the purview of these interactions, with just one important component missing – a new particle called the Higgs boson. One of the most important experimental goals now of elementary particle physics, is to discover this elusive particle. At the theoretical level, an important component of the standard model, which still continues to defy a complete understanding, despite considerable advancement in related fields, is the problem of confinement, or how quarks are confined within the proton or neutron. In addition, scientists around the world are asking whether there exists something 'beyond the standard model'. The only experimental motivation for this question is the fact that there are certain anomalies in the data for neutrinos coming from the sun, as also from cosmic ray showers. At the theoretical level, a new symmetry among the elementary particles such as supersymmetry, or a new unification of the interactions could also lead to new physics beyond the standard model. All these issues are actively pursued at HRI, which has one of the largest particle physics groups in the country.

Box 3. String Theory

In the usual study of elementary particle physics, the gravitational force (which is responsible for the motion of massive celestial bodies) between particles is extremely tiny and can be ignored as it is in the standard model. But at even higher energies or at tinier distance scales, one finds that the gravitational force does become comparable to the other forces between particles. In this regime, it is found that the laws of gravitation are inconsistent with the rules of quantum theory as we understand them. To reconcile this inconsistency, string theory was invented, which has the basic idea that instead of many elementary particles, one has a single kind of tiny elementary string. As the string oscillates, its normal modes (or its quantum analogs) appear as various elementary particles. In fact, the theory is sufficiently constrained that it also dictates the interactions between the various modes or particles and so it is possible to get all the qualitative features of the particles and their interactions from string theory. However, at present the theory is not understood well enough to get a sufficiently good model that explains all the phenomena in quantitative detail. Nevertheless, string theory has contributed a great deal to the general understanding of quantum dynamics and has consistently come up with new ideas. This and its remarkable internal consistency are the reasons that research in this field continues even in the absence of any experiemental data that can confirm the string paradigm. HRI has the second largest string theory group in the country and has been at the forefront of research in this field. Many new ideas at the frontiers of research were pioneered in this Institute.

Box 4. Condensed Matter Physics

The modern era in theoretical condensed matter physics began in the 1920's with the application of quantum mechanics to solids. The basic idea here is that even though one knows that electrons interact with each other through the Coulomb interaction at atomic length scales, since one has a large number (Avogadro's number) of particles involved in these interactions, new collective phenomena arise, which cannot be trivially predicted from the microscopic theory. This field includes accomplishments ranging from electronic band theory, which explains metals, insulators and semi-conductors, to the theory of superconductivity and the quantum Hall effect. In the second half of the twentieth century, there has also opened up a new branch of condensed matter physics – the so-called soft condensed matter physics – concerned with properties of matter, which are determined by physics arising at length scales much larger than molecular length scales, so that ideas of classical statistical physics suffice. This era has also seen a set of new paradigms introduced in condensed matter physics and spreading to other areas in physics. The study of phase transitions has led to the notions of scaling and universality and also spawned the renormalisation group idea. The concepts of broken symmetry and order parameter have emerged as unifying concepts applicable not only in condensed matter physics, but also in particle physics and cosmology. In HRI, the active areas of research are mainly strongly interacting electron systems, superconductivity and low dimensional electron systems such as quantum wires and dots.

Box 5. Astronomy and Astrophysics

One of the interesting observational facts known about the Universe is that it is expanding. So at earlier times, it must have been hotter and more compact. At sufficiently early stages, it must have been hot enough for the elements hydrogen and helium to be ionised. As the Universe expanded from this hot state, it cooled and at some stage ions and electrons combined to form neutral atoms. Once this happened, there were few electrons left and the mean free path for photons became very large. The spectrum of these photons is thermal with an effective temperature of 2.7K, and these constitute what is known as the Cosmic Microwave Background Radiation (CMBR). It is known that CMBR is remarkably isotropic, with variations in temperature being less than one part in 10^5. This level of isotropy is surprising, particularly as in the standard Big Bang model these photons are coming from regions that are not causally linked. Hence one has to look for mechanisms for making different parts of the Universe look alike, or believe that the initial conditions for our Universe are very special. Inflation is one mechanism that can link different parts of the Universe causally, and hence allow for homogenisation. It is hypothesised that the Universe underwent very rapid expansion at a very early stage, and that everything we see originally came from a causally connected region. This alleviates the problem of an isotropic Universe to some extent, and also predicts that the Universe must be very close to flat. Inflation also provides a straightforward mechanism for generating small perturbations, and these account for the tiny anisotropy that is observed in CMBR. The tiny perturbations in the CMBR that are barely observable grow via gravitational instability to form large structures like galaxies and clusters of galaxies. The study of formation and evolution of these structures is an active area of research. Scientists in HRI mainly work on inflation, numerical simulations on formation and clustering of galaxies, and in making theoretical predictions for the Giant Meter-wave Radio Telescope (GMRT) in popular scenarios of galaxy formation.

On a day-to-day level, life in the campus is extremely comfortable. Faculty, students and post-doctoral fellows live within the campus which has all the facilities needed for normal needs, such as a shop, a bank, a post-office and a medical centre. The campus also has a recreational centre equipped with sports facilities like badminton, table tennis, and (in the near future) swimming. As for other activities, the natural beauty of the terrain, allows plenty of scope for amateur photography and art, besides gardening and bird-watching. The Institute also boasts of a movie club which screens regular commercial and non-commercial movies. Other recreational activities also include occasional music programmes and plays from the city of Allahabad performed at the local community centre housed in the campus.

Recruitment of students

HRI runs a regular graduate programme leading to a PhD degree in both mathematics and physics. Students are called from all over the country, based on an initial screening test called

the JEST, to appear for a written test and interview at the Institute. The selected students are then initiated into course work for about a year, in order to prepare them for the research work for their PhD.

The Institute is currently in an expansion phase and is actively looking for young students and post-doctoral fellows, who would like to pursue research in physics and mathematics. The kind of students that are needed are those who are intensely curious about their subjects, highly motivated and hard-working, and who would like to spend their lives satisfying their curiosity. Since, HRI is a purely theoretical research Institute, the need is for students with some amount of mathematical aptitude as well as the aptitude to innovate and think. HRI students are expected to go for post-doctoral fellowships after they graduate with a PhD and expected to eventually end up in academic positions. Hence, HRI is interested in students who enjoy learning new things and finding new things to learn about and want to spend the rest of their lives doing just that!

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