Ludwig Prandtl and Boundary Layers in Fluid Flow

How a Small Viscosity can Cause Large Effects

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In 1904, Prandtl proposed the concept of boundary layers that revolutionised the study of fluid mechanics. In this article we present the basic ideas of boundary layers and boundary-layer separation, a phenomenon that distinguishes streamlined from bluff bodies.

A Long-Standing Paradox

It is a matter of common experience that when we stand in a breeze or wade in water we feel a force which is called the drag force. We now know that the drag is caused by fluid friction or viscosity. However, for long it was believed that the viscosity shouldn't enter the picture at all since it was so small in value for both water and air. Assuming no viscosity, the finest mathematical physicists of the 19th century constructed a large body of elegant results which predicted that the drag on a body in steady flow would be zero. This discrepancy between ideal fluid theory or hydrodynamics and common experience was known as 'd'Alembert's paradox' The paradox was only resolved in a revolutionary 1904 paper by L Prandtl who showed that viscous effects, no matter how small the viscosity, can never be neglected. More precisely, it is the Reynolds number Re, a dimensionless measure of the relative importance of inertial to viscous forces in the flow, which is the determining factor. Prandtl postulated that for certain kinds of high Reynolds number or nearly frictionless flows, for example the flow past a streamlined body like an airfoil, the viscous effects would be confined to thin regions called boundary layers. For certain other kinds of high Re

flows, such as the flow past a bluff body like a sphere, viscous effects need not be confined to such thin layers; viscosity then has a more dramatic effect than what its low value might suggest. The key concept of boundary layers has now spread to many other fields; boundary layers often arise in what are known as singular perturbation problems.

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In this article we illustrate the boundary-layer concept by considering flows around three representative bodies, namely a thin plate aligned with the flow, an airfoil and a circular cylinder.

Viscous Stress and the No-slip Condition

It is useful when studying fluid motion to consider the motion of a fluid particle or a small element of fluid. Although forces such as that due to gravity are at times important we ignore them here. For the purposes of this article the only forces that we will consider are those due to pressure and viscosity. These forces can accelerate or decelerate a fluid particle.

For common fluids the viscous force is proportional to viscosity × rate of deformation of fluid element, or viscosity × spatial gradient of velocity.

Another important point relevant to the boundary layer is the no-slip condition at a solid wall: the fluid right next to a solid wall has the same velocity as the wall (*Figure* 1).

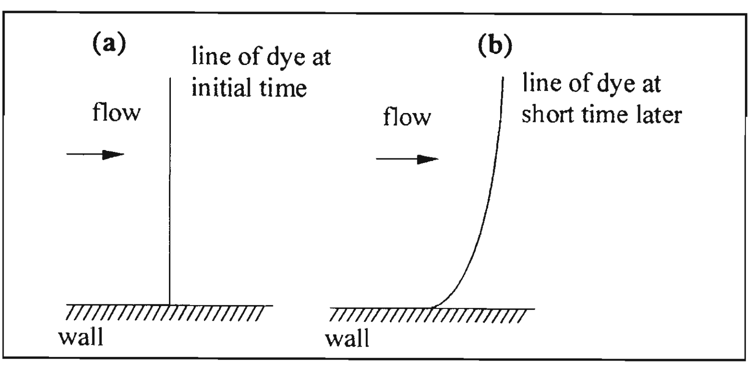


Figure 1. Illustration of the no-slip condition. Fluid is flowing past a stationary solid surface. The figure shows how an initially vertical line of dye is displaced at a later time by the flow. The fluid and the dye next to the wall do not move.

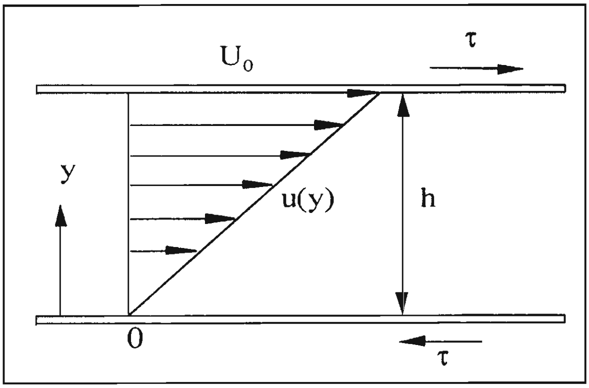


Figure 2. The figure shows the linear velocity variation in a fluid between a stationary bottom plate and a top plate moving with velocity U_0. The force per unit area to be applied on the plates is τ.

On the other hand, in ideal or non-viscous flow the fluid next to a solid surface can 'slip' past it.

To illustrate these two points, consider a fluid between parallel plates, with the lower plate stationary and the upper plate moving with velocity U_0 (*Figure* 2). By the no-slip condition, the fluid next to the lower plate has zero velocity and the fluid next to the upper plate has velocity U_0; if the gap between the plates is small enough, the fluid velocity $u(y)$ varies linearly from zero to U_0, $u(y) = U_0 y/h$.

The viscous stress on either plate is $\mu du/dy = \mu U_0/h$. The larger the shear U_0/h, the larger the force required to move the plate.

Prandtl's Resolution of the Paradox: Flow past a Thin Plate

The key concept of boundary layers has now spread to many other fields; boundary layers often arise in what are known as singular perturbation problems.

Now consider our first example of a thin flat plate placed in a steady uniform flow, with velocity U_∞ and pressure P_∞; the plate is aligned with the flow. If the fluid were ideal, i.e., frictionless or without viscosity, and the plate was of negligible thickness, the flow would be undisturbed (*Figure* 3a). By Bernoulli's equation the pressure on both sides of the plate would be P_∞. The drag on the plate would be zero.

When the fluid has even a small viscosity the no-slip

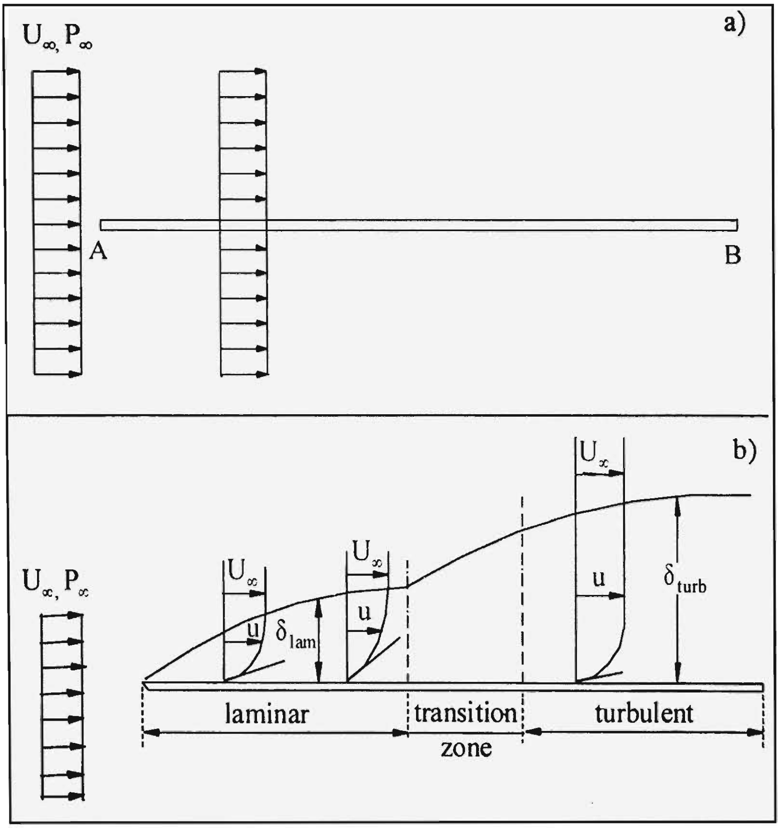


Figure 3. Uniform flow with velocity U_∞ past a thin flat plate. (a) The flow is undisturbed if the fluid is non-viscous. (b) For a fluid with small viscosity thin boundary layers develop on either side of the plate. The boundary layers become turbulent beyond a certain distance from the front edge. The fluid velocity varies sharply from zero at the surface to U_∞ across the boundary layer.

Note: Boundary layer on only the upper surface is shown and the vertical scale is highly exaggerated.

boundary condition changes the situation totally. Imagine a fluid particle which impinges on, say, the top surface. The part of the fluid particle that touches the surface gets 'stuck' to the surface while the rest of the particle keeps moving forward. The particle is highly sheared and the viscous forces are large. Just past the leading edge, only the fluid next to the wall is slowed down. Further down the plate the random motion of the molecules slow down adjacent fluid layers away from the wall – the retarding effect of the wall is spread outward by viscous diffusion of momentum.

For a given length of plate the region over which viscous action is felt is thin if the viscosity is small, that is, when the Reynolds number is large. This thin viscosity affected region is called the boundary layer; outside the boundary layer the flow is practically non-viscous.

Within the boundary layer the velocity gradient normal to the wall is large; the large velocity gradient multiplying a small viscosity gives a non-negligible viscous force.

Since the viscosity is small and the boundary layer is thin, Prandtl drew three important conclusions:

1. The flow outside the boundary layer is practically unaffected and is almost the same as that predicted by ideal flow theory without the boundary layer.

2. There is negligible variation of pressure across the boundary layer, i.e., pressure on the surface $\simeq$ pressure at the edge of the boundary layer.

3. The flow in the thin boundary layer could be dealt with by simplifed boundary-layer equations.

Thus in the case of the flow over a thin plate the velocity just outside the boundary layer is U_∞ and the pressure both in and outside the boundary layer is constant and equal to P_∞. There is a great simplification in what needs to be computed. Instead of having to solve the complicated Navier–Stokes equations, one has to now solve the simpler boundary-layer equations (see *Box* 1). What has happened is that whereas, when $\mu \to 0$, we can treat the major portion of the flow as inviscid, we still need to account for the crucial role of viscosity in bringing the fluid to rest at the boundaries; moreover the inviscid outer flow and the boundary layer must be correctly matched. For the mathematically minded reader a simple model problem which illustrates this procedure is discussed in *Box* 2.

Since the boundary layers are thin the pressure transmits unchanged through the boundary layer.

It is now possible with the simplified procedure to estimate the way the boundary layer grows along the plate. The boundary layer thickness, δ, grows with the downstream distance by viscous diffusion; the larger the viscosity, the faster the diffusion. We observe that fluid is carried downstream by the flow with velocity approx-

Box 1. The Equations Governing Fluid Flow and their Approximate Forms.

Here we summarize for the mathematically minded reader the relevant two-dimensional field equations in cartesian coordinates. In order to simplify matters we only consider the flow over a thin flat plate aligned with the flow. Let ρ and μ be the density and viscosity of an incompressible fluid and let (u, v) be the (x, y) velocity components and p be the pressure. The non-linear Navier–Stokes equations, which govern viscous fluid motion then take the form

$$u_x + v_y = 0 \tag{1a}$$

$$\rho(uu_x + vu_y) = -p_x + \mu(u_{xx} + u_{yy}) \tag{1b}$$

$$\rho(uv_x + vv_y) = -p_y + \mu(v_{xx} + v_{yy}) \tag{1c}$$

where the subscripts indicate partial differentiation. The first of the above equations is the continuity or the conservation of mass equation. The next two are the x and y momentum equations, respectively. For a viscous fluid u and v have to vanish on stationary solid boundaries.

The Euler equations which describe inviscid or frictionless motion can be obtained by just setting $\mu = 0$ in the above equations. Note that the highest derivatives in the second and third equations are lost. Correspondingly, now only the velocity component normal to a stationary solid boundary has to vanish.

Prandtl's boundary layer equations follow from a careful simplification of (1)

$$u_x + v_y = 0 \tag{2a}$$

$$\rho(uu_x + vu_y) = -p_x + \mu u_{yy} \tag{2b}$$

$$p_y = 0. \tag{2c}$$

The continuity equation remains unchanged, the highest x-derivative has been dropped in the x-momentum equation while the y-momentum equation has been considerably simplified to a statement that the pressure is constant across the boundary layer and is determined by the external inviscid flow. Note that the boundary layer equations remain non-linear.

Box 2. A Model Boundary Layer Problem

To illustrate to the boundary layer concept we consider a function $u(y)$ defined on the interval $0 \leq y \leq 1$. Let us assume that $u(y)$ satisfies the simple ordinary differential equation

$$\epsilon\frac{d^2u}{dy^2} + (1+\epsilon)\frac{du}{dy} + u = 0, \quad u(0) = 0, u(1) = 1. \tag{1}$$

If we wish to draw an analogy with the fluid flow equations, we may consider ϵ, u and y to be analogous to the viscosity, streamwise velocity and the direction normal to the plate, respectively. Note that the ϵ in the coefficient of the middle term is there only to simplify the solution. The reader must be warned however that the analogy is only a very crude, qualitative one. In any case, since the differential equation is a linear one with constant coefficients it can be solved exactly by, for example, assuming exponential solutions. The exact solution which satisfies the above boundary conditions is given by

$$u(y) = \frac{e^{-y} - e^{-y/\epsilon}}{e^{-1} - e^{-1/\epsilon}}. \tag{2}$$

Now note that when $\epsilon \to 0$ (i.e. the viscosity vanishes)$-1/\epsilon \to -\infty$ and for $y > 0$ $-y/\epsilon \to -\infty$; as a consequence for every positive y, $u(y) \to e^{1-y}$. This is the *outer solution.* As $\epsilon \to 0$ this solution is a very good approximation to the exact solution over most of the field, $O(e) < y \leq 1$. But it does not satisfy the boundary condition at y=0 since it gives the value e for u instead of 0. This pathology immediately suggests that although the outer solution is valid almost everywhere a boundary layer is required near $y = 0$.

We now outline the boundary layer analysis of (1) as $\epsilon \to 0$. First, to directly get the outer solution we let $\epsilon \to 0$ in (1) and get the equation

$$\frac{du_o}{dy} + u_o = 0. \tag{3}$$

for the outer solution $u_o(y)$. Note that the second derivative term has been dropped as has the small term multiplying the first derivative. Equation (3) can be easily integrated and made to satisfy the boundary condition at $x = 1$. We then get the outer solution: $u_o(y) = e^{1-y}$, just as we had found from the exact solution.

Box 2. continued...

The trickier part, the Prandtl part, is to get the *boundary layer* or *inner solution.* We know from our discussion above that although the outer solution goes to e as $y \to 0$, the correct solution must actually go to *zero* very rapidly in a thin layer of thickness ϵ. This suggests scaling or magnifying the region close to $y = 0$. Let $\eta = y/\epsilon$ be the *inner variable*; note that for fixed y, $\eta \to \infty$ as $\epsilon \to 0$ and so η tends to magnify y. Now we assume that the inner solution $u_i(\eta)$ is a function of the inner variable η alone. Note that now $\frac{d}{dy} = \epsilon^{-1}\frac{d}{d\eta}$ and $\frac{d^2}{dy^2} = \epsilon^{-2}\frac{d^2}{d\eta^2}$. Now if $u_i(\eta)$ is assumed to satisfy (1) with η defined as above, and if the substitutions are made and the leading terms in ϵ alone are collected we find

$$\frac{d^2u_i}{d\eta^2} + \frac{du_i}{d\eta} = 0. \tag{4}$$

This can be immediately integrated twice to yield $u_i(\eta) = A_0 - A_1e^{-\eta}$ where A_0 and A_1 are arbitrary constants. Since the inner solution has to satify the zero boundary condition at $y = 0$ (i.e. $\eta = 0$), $A_1 = A_0$ and so $u_i(\eta) = A_0(1 - e^{-\eta})$. Now how do we connect the inner solution $u_i(\eta)$ to the outer solution $u_o(y)$? This is done by *matching* the two solutions. Since u_o is expected to hold in the outer region (roughly $\epsilon < y \leq 1$) while u_i is expected to hold in the inner region (roughly $0 \leq y < \epsilon$) and since both of them represent u we would expect them to overlap or *match* in some common region of validity. This suggests trying the matching condition

$$\lim_{y\to 0} u_0(y) = \lim_{\eta\to\infty} u_i(\eta). \tag{5}$$

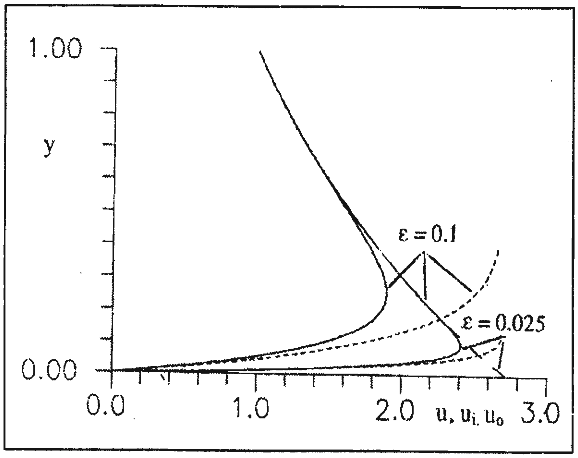


Figure A. Comparison of the exact solution u(y) (——) with the inner and outer solutions $u_i(y)$ (-----) and $u_o(y)$ (- - - -) for two values of ε. Note that as ε becomes smaller the perturbation solution becomes better. Compare with Figure 5.

Box 2. continued...

This condition immediately leads to $A_0 = e$. Thus we find the solution

$$u(y) \approx u_o(y) = e^{1-y}, \quad \epsilon < y \leq 1 \qquad (6a)$$

$$u(y) \approx u_i(\eta) = e(1 - e^{-\eta}) = e(1 - e^{-y/\epsilon}), \quad 0 \leq y < \epsilon \qquad (6b),$$

The two solutions can be combined to give a composite solution approximately valid everywhere but we shall not do that here. We just note that when we compare the approximate solution (6) with the exact solution (2), we find that the *boundary layer analysis* correctly picks up the boundary layer near $y = 0$, while the outer solution picks up the correct solution near $y = 1$ when $\epsilon \to 0$. This is also shown very clearly in *Figure* A.

mately equal to U_∞. How fast the layer diffuses and how fast the fluid is carried downstream together determine the boundary layer thickness. The boundary-layer thickness δ on a flat plate depends on the distance x from the front or leading edge of the plate and on the condition of the flow. If the flow is laminar or smooth, steady and orderly, as it will initially be near the leading edge, $\delta(x)$ is given by

$$\delta(x) = cx/(Re_x)^{1/2} = (cx^{1/2}/U_\infty{}^{1/2})(\mu/\rho)^{1/2}$$

The random motion in a turbulent boundary layer enhances the momentum diffusion rate to many times more than the viscous or molecular diffusion rate obtained in the laminar boundary layer.

where ρ is the fluid density and c is a constant. In this situation, a measure of the relative importance of viscosity is the Reynolds number $Re_x = \rho U_\infty x/\mu$; the smaller the viscosity, or the higher the Reynolds number, the slower is the growth of the boundary layer. Beyond a distance corresponding to Re_x greater than 1×10^6 the boundary layer becomes turbulent. The fluid flow within the boundary layer becomes chaotic; the fluid elements move about randomly in addition to their bulk downstream motion. This random motion enhances the momentum diffusion rate to many times more than the viscous or molecular diffusion rate obtained in the laminar boundary layer. The boundary layer grows more rapidly and nearly linearly with the distance x.

The main effect of viscosity, however small, is to cause drag which is absent in ideal flow. The drag on the plate is entirely due to the tangential stress at the plate surface and is given by $\mu(\partial u/\partial y)_{y=0}$. This tangential stress at the surface is called wall shear stress and is denoted by τ_w. In the laminar boundary layer

$$\tau_w \simeq 0.332 U_\infty^{3/2}\sqrt{\frac{\mu\rho}{x}} \propto \mu U_\infty/\delta.$$

τ_w is maximum near the leading edge and decreases as $x^{-1/2}$ with distance.

On the other hand, when the boundary layer becomes turbulent the random motion tends to make the velocity over more of the boundary layer closer to the free stream speed U_∞; that is the velocity profile is much flatter. As a consequence the decay to the no-slip condition occurs over a smaller distance, leading to a larger wall shear stress compared to the laminar value. In the turbulent boundary layer $\tau_w = C\rho U_\infty^2/2$; C depends only weakly on x and normally lies between 0.002 and 0.004 for a smooth surface.

Flow over an Airfoil

Next, as an example of a streamlined body consider uniform steady flow past an airfoil. *Figure* 4 shows a streamline picture of ideal flow; and the picture, would not look very different if the fluid had a small viscosity. Consider the motion of a fluid particle along a streamline just above the top surface of the airfoil. The fluid particle starting with the velocity U_∞ ahead of the airfoil slows down initially; its speed then increases rapidly beyond U_∞ near the nose, reaches a maximum near about the maximum thickness point, and slowly comes back to about U_∞ near the trailing edge of the airfoil (see *Figure* 4); call this velocity just above the airfoil surface

The lift force on an airfoil, which is mainly caused by pressure difference between the top and bottom surfaces, predicted using ideal flow theory is close to the measured force.

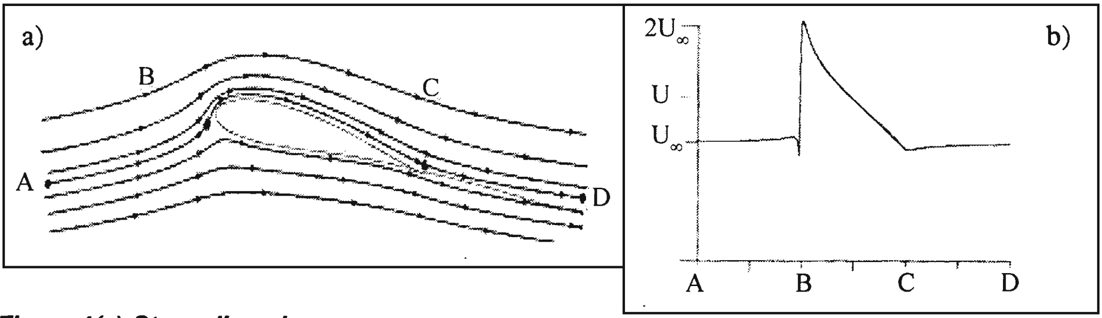


Figure 4(a) Streamlines in non-viscous flow past an airfoil. (b) The speed variation along a streamline that goes just above the airfoil surface.

U_e. By Bernoulli's equation the pressure on the upper surface is, $P_e = P_\infty + 1/2\rho(U_\infty^2 - U_e^2)$; where ρ is the fluid density. Since $U_e > U_\infty$ over most of the upper surface, the pressure there is less than the free stream pressure. Since the opposite happens on the lower surface there is a net upward or lift force on the airfoil.

When the fluid is viscous the velocity on the airfoil, by the no-slip condition, has to vanish. There is one difference between the boundary layer on the flat plate and the boundary layer on the airfoil: in the flat plate case, the velocity on the edge of the boundary layer is a constant U_∞, whereas in the airfoil case the velocity at the edge of the boundary layer is variable and nearly equal to U_e. *Figure* 5 shows the velocity profiles at one station on an airfoil for ideal and boundary layer flow. Since the boundary layers are thin the pressure transmits unchanged through the boundary layer. The

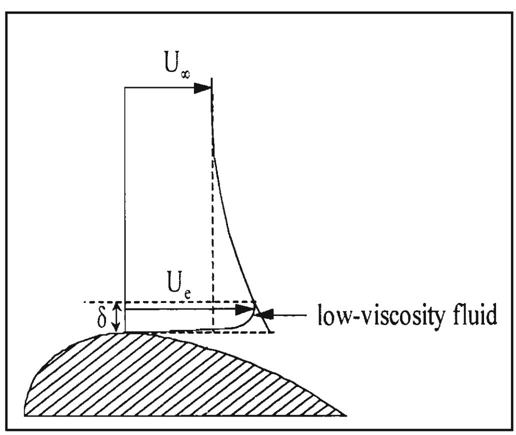


Figure 5. Velocity variation near the surface in (a) a non-viscous fluid and (b) a fluid with low viscosity. The two velocities are different only in the thin boundary layer.

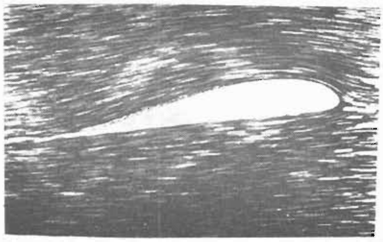


Figure 6. A picture of flow past an airfoil. The flow, which is from right to left, is made visible by tiny particles in the fluid. The boundary layers are too thin to be seen. The drag is mainly caused by tangential viscous stress on the surface.
(Reproduced from G K Batchelor. An Introduction to Fluid Dynamics, Cambridge University Press, 1967.)

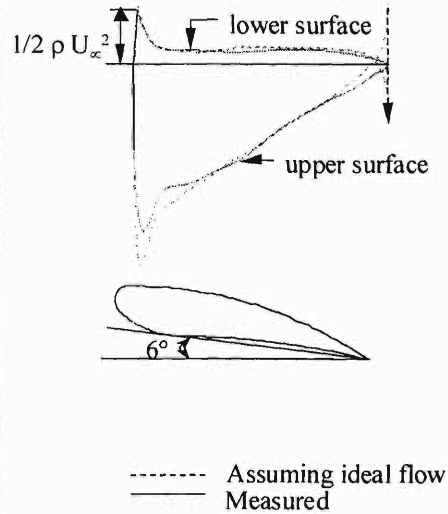


Figure 7. The pressure distributions on the upper and lower surfaces of an airfoil. There is negligible difference between the pressure distribution calculated assuming non-viscous flow and the measured pressure distribution in flow of a low-viscosity fluid. The higher pressure on the lower surface and the lower pressure on the upper surface result in lift.

streamline picture (*Figure* 6) and the surface pressure (*Figure* 7) are very close to those obtained in the ideal flow case. That is why the lift force, which is mainly caused by pressure difference between the top and bottom surfaces, predicted using ideal flow theory is close to the measured force.

On the other hand, the drag is caused by viscous stresses at the wall and cannot be predicted just based on an ideal flow calculation. But a calculation based on boundary layer theory is effective.

Flow around Bluff Bodies: Boundary Layer Separation.

Even a small value of viscosity has a dramatic effect on the flow around a bluff body such as a circular cylinder or a sphere when compared to the ideal, zero viscosity, flow. *Figure* 8 shows the streamline picture of ideal flow over a cylinder and the velocity variation along a streamline close to the surface. For bluff bodies, as in the airfoil case, the velocity increases from the leading edge to the maximum thickness point. For a circular cylinder the velocity increases from the front of the cylinder to

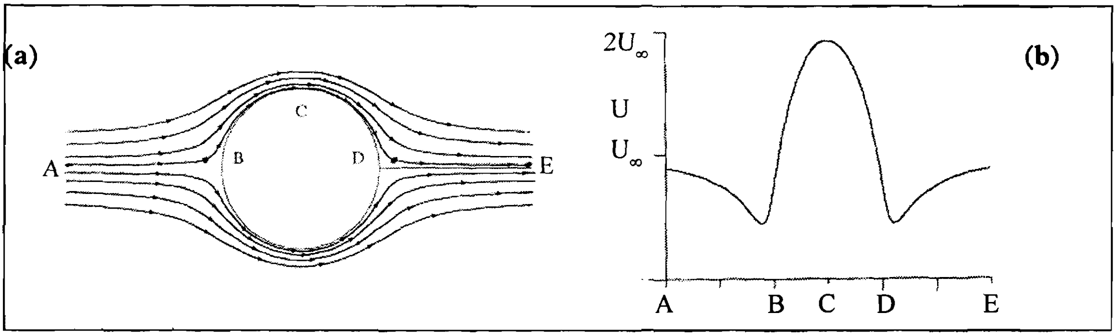


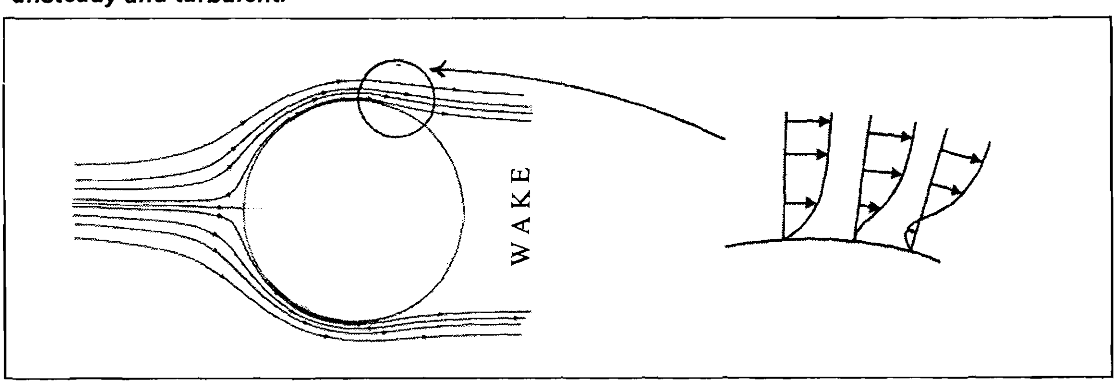
Figure 8. (a) Streamlines in non-viscous flow past a circular cylinder (b) The velocity variation along a streamline that goes just above the cylinder surface. The flow accelerates from point B to point C and decelerates from point C to point D.

a maximum of $\sim 2U_\infty$ at the maximum thickness point and reduces to near zero at the back of the cylinder.

In the case of flow of a fluid with low viscosity, boundary layers form on both the upper and lower surfaces starting from the nose or front stagnation point of the body. The pressure accelerates the boundary layer flow in the front portion; viscous forces retard the flow.

Now we come to the main point. In the rear portion the increasing pressure causes a rapid deceleration of the flow. The slower moving boundary-layer fluid subjected to this rapid deceleration reverses direction(*Figure* 9).

Figure 9. Streamline picture of the flow of a low-viscosity fluid past a circular cylinder. Except for the boundary layers the flow in the front portion of the cylinder is nearly identical to the ideal flow shown in Figure 8. Just after the maximum thickness point the increasing pressure with distance causes the boundary-layer fluid to reverse direction and the boundary layer separates. In the rear portion of the cylinder the flow, called the wake, is completely different from ideal flow; it is unsteady and turbulent.



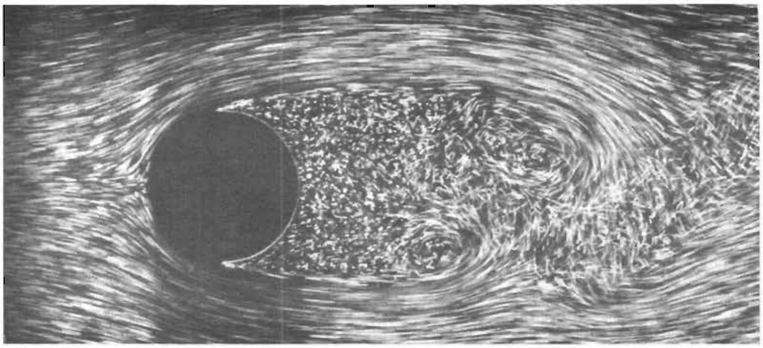


Figure 10. Visualization of the flow over a circular cylinder at a Reynolds number of 2000. The boundary layers form at the nose and separate at about the 90° points. The flow after the separation is turbulent and the fluid in the wake is nearly stagnant. Method of visualization is similar to the one in Figure 6. *(Reproduced from An Album of Fluid Motion, assembled by Milton Van Dyke, The Parabolic Press, 1982).*

Then the boundary layer instead of remaining attached to the body separates from it. Although boundary-layer separation, so characteristic of bluff bodies, is a purely viscous effect in a narrow boundary layer, its consequences are global and far reaching. Consequently, the observed streamline pattern and pressure distribution downstream of separation are totally different from those predicted by ideal flow theory (*Figures* 10 and 11).

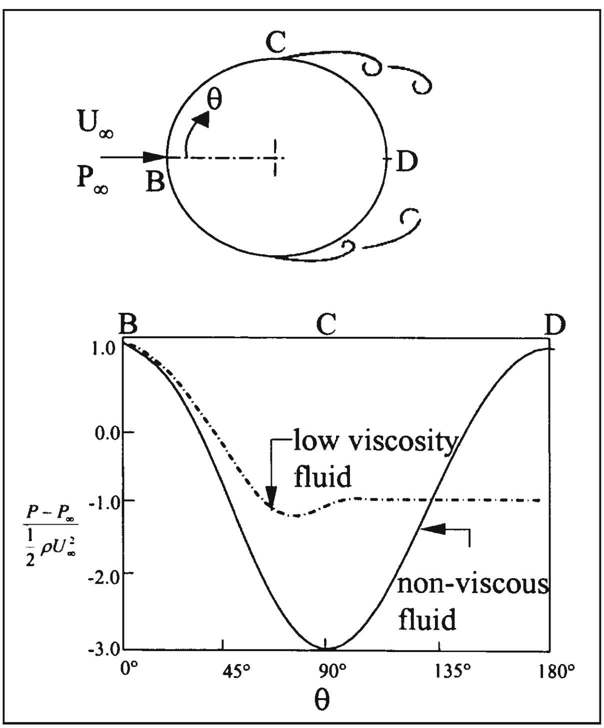


Figure 11. The pressure variations with angle over a circular cylinder in non-viscous flow and in flow of a fluid with small viscosity. In non-viscous flow equal pressures in the front and back results in zero drag. In viscous flow a higher pressure in the front in relation to the pressure in the back gives a drag ~ ½ ρU∞² × frontal area.

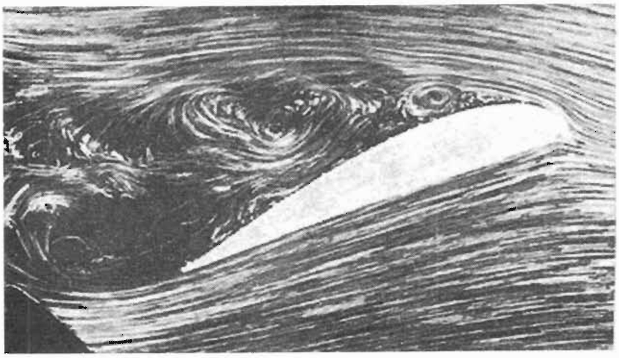


Figure 12. Visualization of flow around an airfoil at a high angle of attack. The boundary layer on the top surface separates resulting in a large drag caused by pressure differential between the lower and upper surfaces. Compare this picture with that of flow around an airfoil at a small angle of attack (Figure 6). Flow is from right to left.

(Reproduced from G K Batchelor, An Introduction to Fluid Dynamics, Cambridge University Press, 1967)

The drag on a bluff body arises mainly from the difference between the higher pressure on the front face and the lower pressure on the back face. The drag force per unit frontal area $\simeq \rho U_\infty^2/2$ is almost independent of viscosity!

Beyond an angle of attack around, say 12^o, an airfoil will stall: the lift drops and the drag sharply increases. Stalling is due to boundary-layer separation (*Figure* 12). An airfoil at a high angle of attack behaves like a bluff body!

Conclusion

In most physical situations the notion that small causes lead to small effects is valid: Prandtl showed, however, that in the case of boundary layers in fluid flow this notion was at times invalid. We have briefly shown using three types of flows how important boundary layers are in practically important flows. But they are also very relevant to many natural flow fields such as those in rivers, in oceans and in the atmosphere, where the scales are very different. Although we are normally unaware of it, boundary layers play an important role in everyday life, for example when we stir our tea.

The slower moving boundary-layer fluid subjected to a rapid deceleration, reverses direction. The boundary layer instead of remaining attached to the body, separates from it.

In order to get a feel for the actual numbers involved, we conclude by estimating some Reynolds numbers and boundary layer characterestics in some everyday situations. We observe that the density and viscosity of water are approximately 1000 kg/m^3 and 1×10^{-3} kg/ms, respectively; the corresponding values for air are approximately 1 kg/m^3 and 0.15×10^{-4} kg/ms. This implies that if U is in km/hr and l is in m, $Re_{\text{air}} \approx 1.85 \times 10^4 Ul$ and $Re_{\text{water}} \approx 2.8 \times 10^5 Ul$. We immediately see that under most normal circumstances encountered in everyday life the Reynolds numbers are likely to be very large; thus boundary layers are likely to be present and the flows are likely to be turbulent.

The boundary layer on a thin 1m × 1m flat plate travelling at 60 kmph ($\simeq$17m/s) in air will remain laminar nearly till the end of the plate, at which point the boundary layer thickness is about 4mm. The drag on the plate $\simeq 0.002 \times 2 \times (1/2\rho U^2) \times 1 = 0.55\text{N} \simeq 0.057\text{kgf}$. The same plate placed head-on will have a drag force $\simeq 1/2(\rho U^2 A) \simeq 140\text{N} \simeq 14\text{kgf}$. A person with a frontal area $\simeq 0.3m^2$ travelling at 60kmph on say a motorcycle will feel a drag force of about 4kgf.

It is close to a hundred years since Ludwig Prandtl introduced the idea of a boundary layer. By doing so he resolved a paradox that had been puzzling scientists for almost two centuries. It took another 50 years to understand mathematically what Prandtl had intuitively, through his own genius, seen to be true. Now his ideas are not only routinely utilized in many flow situations but are being applied to many other branches of the physical and engineering sciences.

Acknowledgements

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Suggested Reading

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[2] D J Tritton, *Physical Fluid Dynamics*, Van Nostrand, 1977

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