Can Light Travel Faster than Light?

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Recently Wang, Kuzmich and Dogariu have reported a work (*Nature*, Vol 406, p. 277, 20 July 2000, hereafter referred to as [1]) in which they generate light pulses apparently moving at speeds higher than the speed of light in vacuum. They also claim to have observed the peak of a light pulse which appeared to leave the vapour cell before entering it. While the authors of the paper have taken care to point out that these observations are not in conflict with the special theory of relativity and causality, newspaper reports of this work have often given the misleading impression that these observations suggest a need to revise the foundations of the special theory of relativity and the principle of causality.

Here we will try to analyze the effects reported by Wang and others and take the opportunity to clarify for ourselves whether the experiments reported in [1] constitute a violation of the basic notions of causality and special relativity. Such a violation would of course require a revision of all of physics.

The special theory of relativity sets an upper limit on the speed of information transfer. Do the observations reported in [1] violate this principle? Let's review a bit of textbook material in an attempt to resolve this question.

In a non-dispersive medium the velocity V_p, of propagation of the phase of a monochromatic wave is independent of the frequency ν of oscillations. In such cases the velocity of *energy transmission* is identical to the velocity V_p, the *phase velocity.* In a dispersive medium, however, the velocity of energy transmission is not easy to define. This is because different frequency components travel at different speeds leading to a distortion of a pulse shape while it travels through the medium. This makes it hard to define the average velocity. This is where the concept of *group velocity* comes into play. Let's study the propagation of a group of waves (a signal), consisting of a narrow band of wavelengths.

Let's consider the wave motion due to the superposition of two monochromatic waves of angular frequencies $\omega_0 \pm \Delta\omega$ and equal amplitudes A. The wave of angular frequency $\omega_0 - \Delta\omega$ has an amplitude y_-:

$$y_- = A\cos[(\omega_0 - \Delta\omega)t - (k_0 - \Delta k)x] \tag{1}$$

and the wave of angular frequency $\omega_0 + \Delta\omega$ has an amplitude y_+:

$$y_+ = A\cos[(\omega_0 + \Delta\omega)t - (k_0 + \Delta k)x] \tag{2}$$

The resultant wave has an amplitude y:

$$y = y_- + y_+ =$$

$$A[\cos[(\omega_0 - \Delta\omega)t - (k_0 - \Delta k)x] + \cos[(\omega_0 + \Delta\omega)t - (k_0 + \Delta k)x]] = 2A\cos(\omega_0 t - k_0 x)\cos(\Delta\omega t - \Delta k x). \quad (3)$$

This is a modulated wave with an average angular frequency ω_0 in the carrier wave $\cos(\omega_0 t - k_0 x)$ and a slowly varying amplitude modulation $2A\cos(\Delta\omega t - \Delta k x)$. The *phase velocity* of the carrier wave is $V_p = \omega_0/k_0$. The velocity of propagation of the modulation is $\frac{\Delta\omega}{\Delta k}$, which in the limit of the two frequencies becoming nearly equal is given by the group velocity $V_g = \frac{d\omega}{dk}$. Now, the angular frequency $\omega(k)$ is related to the wave number k via

$$\omega(k) = \frac{ck}{n(k)} \quad (4)$$

where c is the speed of light in vacuum and $n(k)$ is the refractive index of the medium of wave number k. The *phase velocity* V_p is given by

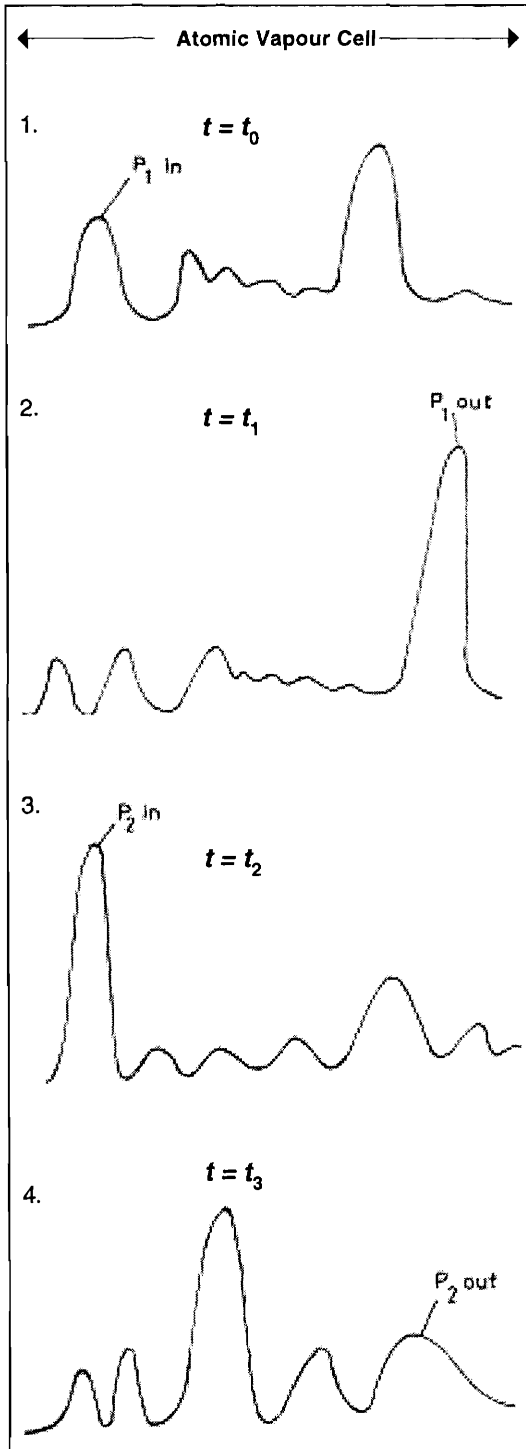
$$V_p = \frac{\omega(k)}{k} = \frac{c}{n(k)}, \quad (5)$$

which expresses the well-known fact that the speed of light is inversely proportional to the refractive index. The *group velocity* V_g is given by

$$V_g = \frac{d\omega(k)}{dk} = \frac{c}{n(\omega) + \omega\frac{dn(\omega)}{d\omega}}. \quad (6)$$

There is no difficulty in defining V_g as long as the medium is purely dispersive (i.e. $\omega = \omega(k)$ is real). But in the region of *anomalous dispersion* where there is *absorption* or *gain* (as in [1]) the wave number k becomes complex and one *cannot attach a clear physical meaning to the group velocity.* (Exercise: Check this statement by introducing complex wave numbers). Thus Wang and others' measurement of a group velocity larger than c, the speed of light in vacuum, is *not in conflict with the special theory of relativity.* This is simply because the group velocity ceases to have the interpretation of the average velocity of a pulse shape in a region of gain assisted anomalous dispersion. The authors of [1] are well aware of the fact that their observations simply verify a well-known fact which is consistent with the special theory of relativity. The novelty in the experiments described in [1] is that they deal with an atomic medium where there is an amplification of light waves rather than an absorption. Absorption usually hampers the measurement of the quantity $\frac{d\omega(k)}{dk}$ in the region of anomalous dispersion.

Let's now move on to the next issue of causality violation. In this work the authors have reported an observation in which they have seen the peak of a light pulse appearing at the exit side of an atomic vapour cell much earlier that it enters it. This observation seems to be in apparent contradiction with the principle of causality. (The causality principle ensures that the cause prece-



Figures 1-4. The wavetrain travels from left to right. 1. The figure shows P_1^{in} entering the vapour cell. 2. The figure shows P_1^{out} exiting the vapour cell. 3. The figure shows P_2^{in} entering the vapour cell. 4. The figure shows P_2^{out} exiting the vapour cell.

-des the effect: the bulb comes on after you turn on the switch). This issue is also easily resolved by making use of the following observation. In a dispersive medium different frequency components move with different velocities. This means that the pulse shape changes and the pulse cannot be characterized by a single velocity. It is therefore meaningless to compare the times of arrival and exit of the peak of the pulse. Let us consider this point a bit more explicitly. Suppose that P_1 and P_2 are two distinct parts of a wave train. Let $P_1^{\rm in}$ be the snapshot of the part P_1 entering the cell at a given time t_0 (*Figure* 1). At a later time t_1 ($t_1 > t_0$) $P_1^{\rm out}$ is the snapshot of the same part P_1 exiting the cell (*Figure* 2). Now a *different* part P_2 enters the cell at a later time $t_2 (t_2 > t_1)$. A snapshot $P_2^{\rm in}$ of the part P_2 of the wavetrain entering the cell is represented in *Figure* 3. $P_2^{\rm out}$ represents the snapshot of P_2 exiting the cell at a time $t_3 > t_2$. Now if we compare *Figure* 2 and *Figure* 3 we notice that $P_1^{\rm out}$ precedes $P_2^{\rm in}$. But this is no cause for alarm since P_1 and P_2 are distinct parts of the wavetrain. We do not see the *same part* of the wavetrain exiting before it enters the cell. To make it even more dramatic let us consider stopping the part P_2 from entering the cell. Even then, we would still see P_1 exiting the cell at $t = t_1$. There is *no* causal connection between P_1 and P_2. The part P_1^{out} exiting the cell at $t = t_1$ is causally related to P_1^{in} entering the cell at $t_0 < t_1$ and *not* to P_2^{in} entering the cell at a later time $t_2 > t_1$.

This obviously means that it is meaningless to compare the times of arrival and exit of a given point, say, the peak of a pulse in such a medium. One is likely to compare different parts of a pulse which move at different rates and arrive at the wrong conclusion that the *same part* of a pulse exits earlier than it enters a cell containing this dispersive medium (in Wang and others' case it is an atomic vapour cell). The observation made in [1] is of a faster moving frequency component appearing at the exit before a slower moving component enters the cell. The only meaningful concept is the *signal velocity* or the true velocity at which information is carried by the light pulse. This is given, for instance, by the velocity at which the front of a step function signal propagates and the authors themselves point out that this velocity has been shown not to exceed c, the speed of light in vacuum. Thus we conclude that there is *no cause for alarm.* The authors of [1] have *not observed* a signal in which the *effect* precedes the *cause.*

(Exercise: Look up Landau and Lifshitz's book (see Suggested Reading) for Kramers–Krōnig's relations. This relation connects the real and imaginary parts of a complex refractive index. The real part introduces a phase of propagation while the imaginary part introduces damping of a light wave propagating through a medium with a complex refractive index. This relation is dictated by the principle of causality which is observed by all known forms of matter.)

Suggested Reading

[1] L D Landau and E M Lifshitz, *Electrodynamics of Continuous Media*, Pergamon Press, 1960.

[2] L Brillouin, *Wave Propagation in Periodic Structures*, First Edition, McGraw Hill Book Company Inc., 1946.

[3] M Born and E Wolf, *Principles of Optics*, Seventh Edition, Cambridge University Press, Cambridge, 1997.

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Ideas come when stepping onto a bus (Poincaré), attending the theatre (Wiener), walking up a mountain (J E Littlewood), sitting at the shore (P S Aleksandrov), or walking in the rain (Littlewood), but only after a long struggle of intensive work.

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