# Molecular Origami

## Modular Construction of Platonic Solids as Models for Reversible Assemblies

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**Platonic solids, representing the highest order of structural and molecular symmetry have been crafted from single modular elements that carry information relating to lengths and angles.**

The assembly of even the most complicated structure is achieved by Nature using a modular protocol wherein each of the modules holds latent information. Thus, minerals – related to materials, and DNA, polysaccharides and proteins – related to cellular life, are assembled from simple monomers. The information content in the modules would be related to the complexity of structures that can evolve from it and by corollary, those which have the highest symmetry can be constructed from modules having minimum levels of information. In this paper, we demonstrate this, using protocols of origami with the construction of platonic solids.

Origami, the elegant art dating back to 500 AD provides intuitive protocols for the crafting of practically every type of three-dimensional object from two dimensional paper.

Important principles of mathematics are latent in these operations. The unraveling of these would bring origami to the realm of sciences, where it will surely find profound applications [1, 2]. We have illustrated this with the construction of platonic solids, based on mathematical principles [3].

Plato (427-347 BC), philosopher, mathematician and statesman, with commendable insight, limited to five – the number of convex polyhedrons whose faces are congruent, regular polygons forming equal dihedral angles at each edge. If '*m*' number

| Platonic solid | $m$ | $n$ | $E$ | Angle controller [0°] |
|---|---|---|---|---|
| 1. Tetrahedron | 3 | 3 | 6 | 60 |
| 2. Octahedron | 4 | 3 | 12 | 60 |
| 3. Eicosahedron | 5 | 3 | 30 | 60 |
| 4. Cube | 3 | 4 | 12 | 90 |
| 5. Dodecahedron | 3 | 5 | 30 | 109 |

*Chart 1.*

of regular '*n*' sided polygons meet at each vertex, they must satisfy the equation, $1/m + 1/n = ½ + 1/E$, where $E$ represents the total number of edges of the polyhedron. The five fundamental solids that fulfil this requirement, called platonic solids, are tetrahedron, cube, octahedron, dodecahedron and eicosahedron (*Chart* 1).

Platonic solids have played a key role in the development of many facets of human endeavor, ranging from art, architecture, to cosmology to physical and life sciences. Therefore, the assembly of these from a single modular unit in a reversible way would be the obvious choice to illustrate how surfaces can be crafted from units carrying needed information. The modules that assemble to platonic solids need carry only one information, namely, the controller angle!

## The Design Principle

All the designs can be made from thick square sheets of the size 20 cm × 20 cm (readily available A4 size photocopy paper, from which squares of the size 21 cm × 21 cm can be made is most suitable, and all the structures illustrated here are made from this type of paper).

Taking a corner of the A4 sheet to the opposite side, creasing across the diagonal and removing of the protruding strip can make the squares. The procedure involves preparing a number of identical modules by folding paper in a specified manner. Each unit will have a pair of projections or inserts and a complementary pair of receptors. Two units will be joined using the insert of one with the receptor of the other. The angles subtended at the junction will be controlled through simple trigonometric concepts, resulting in good approximations to $60^o$, $90^o$ and $109^o$ needed for constructing the platonic solids. Interestingly the whole process requires neither scissors nor glue! However, it is useful to have a scale to mark the crease while folding and clips to temporarily hold connections till the model is completely built.

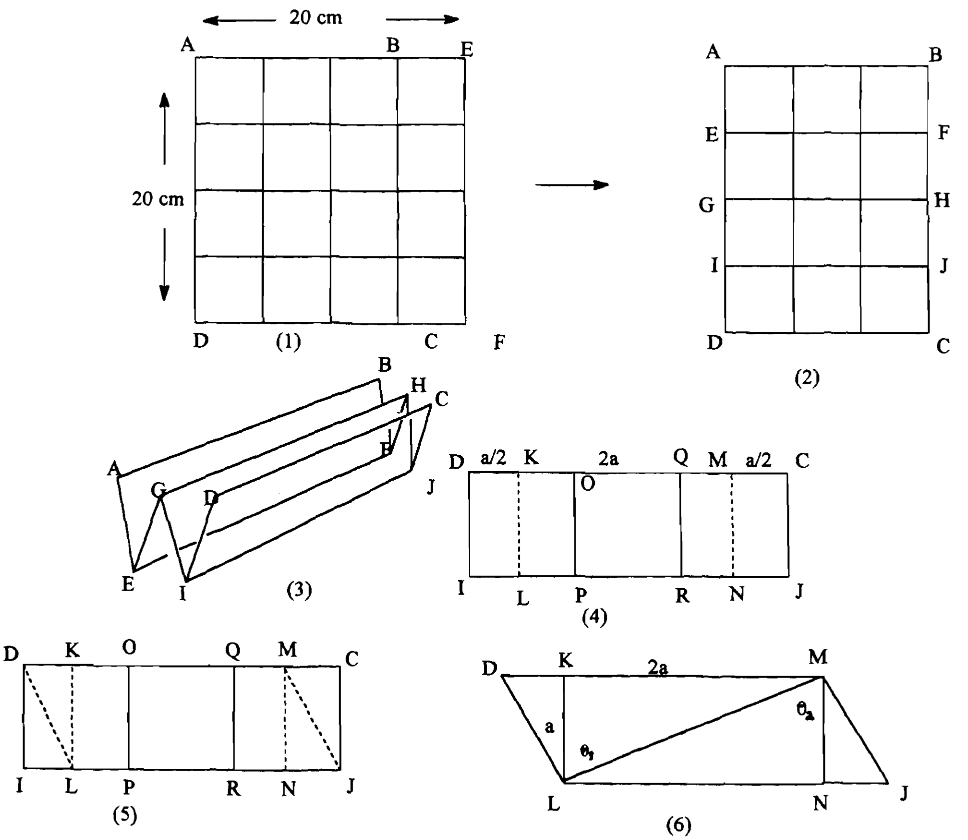
Each module provides, in addition, an edge, corresponding to the eventual nature of the polyhedron. Therefore the number of edges in the platonic solid define how many modules need to be assembled (*Chart* 1). Tetrahedron, octahedron and eicosahedron belong to a sub family in the sense that they represent the congruence of respectively, 3, 4 and 5 equilateral triangles.

## Solids with Triangular Faces

Tetrahedron, octahedron and eicosahedron can all be constructed from an equilateral triangle, constructed from three modules. This is illustrated in *Chart* 2 and *Chart* 3. The triangle so constructed will have, at each corner, an insert and a receptor. These could then be used to add on triangular faces, till the polygon is complete, as shown in *Chart* 4.

## Construction of Module for Tetrahedron, Octahedron and Eicosahedron (*Chart* 3)

Fold the square from A4 sheet of paper (see above) to make equal halves. Fold again in the same manner. Open the sheet and repeat the process in a perpendicular direction. This would result in the $4 \times 4$ grid AEDF (1) (*Chart* 2). Place a scale firmly along BC and tear off strip BCFE, resulting now in the $4 \times 3$-grid ABCD (2). Fold along the horizontal lines, EF, GH, IJ, as shown in 3, to secure strip DCI J (4) that is 4 layers thick. Now, fold the edges, DI, and CJ to, respectively, OP and QR, to make the mid




$\tan \theta_1 = KM/KL = 2a/a = 2 = 2a/a = LN/MN = \tan \theta_2$

$\therefore \theta_1 = \theta_2\ 63.5^\circ \cong 60^\circ$

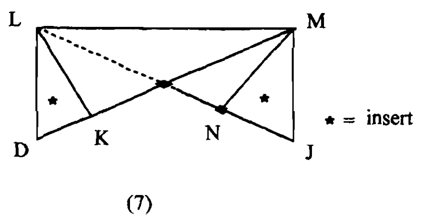



*Chart 2.*

crease lines KL and MN.

If each grid is assumed to be of size '$a \times a$', then in (4), the strips DKIL and CMNJ will be '$a/2 \times a$' and the core '$2a \times a$'. Holding

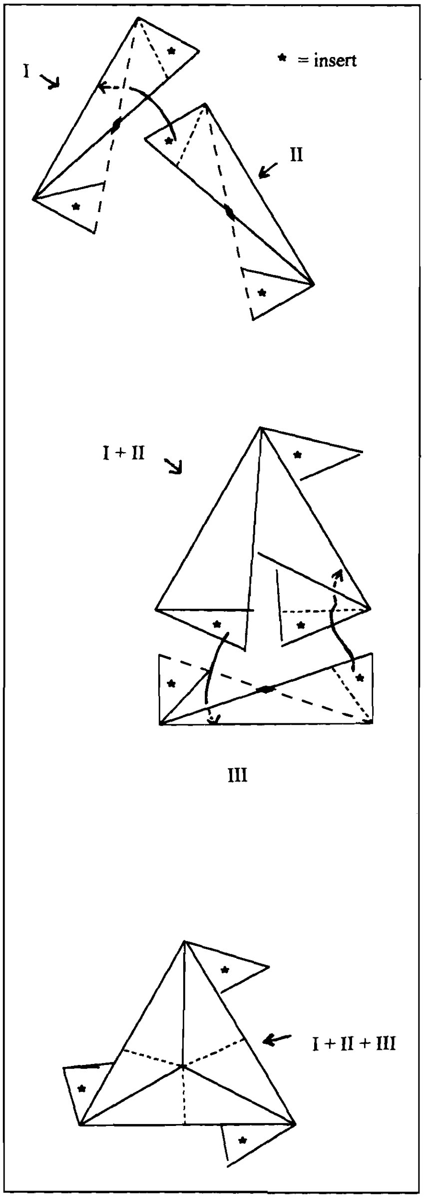



*Chart 3.*

a scale firmly, crease the diagonal DL and MJ (5) and tuck segments DIL and MCJ so that they are not visible (6). The controller angle $\theta$ [KLM=LMN] would be close to the required 60°, as derived in *Chart* 2. To complete the module, holding LM firmly with a scale, bring LDKM forward (7). *Figure* 7 is a typical module. It has two inserts [LKD and MNJ), one in front and the other in the back, and two complementary receptors [DML and JLM]. In this module LM would be the edge ($E$)/side of the polyhedron. As stated previously, tetrahedron would need 6 modules, octahedron 12 and eicosahedron 30. By arranging the folding, three or four at a time, the numbers needed can be made easily.

The assembly of three of the modules to form the equilateral triangle is shown in *Chart* 3. Join module I and module II, by securely tucking the insert of module II behind the edge of I to give composite I+II, with an angle of ~60° with a projecting insert (until well familiar, it is advised to secure each joint with a paper chip, which can be removed after the model is constructed). Tuck in the left insert of 'I+II' to receptor in III and reciprocally, the insert of III to the receptor of 'I+II' to secure the key *triangular motif* [TM], as shown in *Chart* 3. In reality, TM will have a small triangular aperture in center. Each of the three corners has an insert (shown) and a receptor (latent), for further assembly. The step-wise assembly from TM to tetrahedron, octahedron and eicosahedron is shown in *Chart* 4.

***Tetrahedron:*** Make an I+II composite and insert it to the TM, adding another triangle. With a single module connect the corners to make the tetrahedron. Of the three, because of crowding, assembling tetrahedron could be vexing, but with patience this can be done.

***Octahedron:*** Insert the I+II composite to two sides of TM and connect the tips with a single module to give the pyramid base.

Attach an I+II composite to any baseline of the pyramid and connect the tip to the opposite base line to give the octahedron.

***Eicosahedron:*** Construct two pentagonal pyramids, from two TM. Proceed as in the case of the octahedron to attach two I+II composites to the side of TM. Add another I+II composite to any of the new sides. Complete the pyramid by connecting the tips with a single module. The procedure can be clearly seen from *Chart* 4. Connect, in sequence, the corner of one of the pyramids to the base of the other, to create the eicosahedron.

It must be pointed out that the assembly protocol suggested here is one of the many and may not even be the best in the hands of a builder, who will usually develop alternate methodologies more suited to the individual perception.

***Cube:*** The module for cube is a marginal variation of that outlines for platonic solids with triangular faces (*Chart* 1). The modification relates to the change of the controller angle from 60° to 90°!

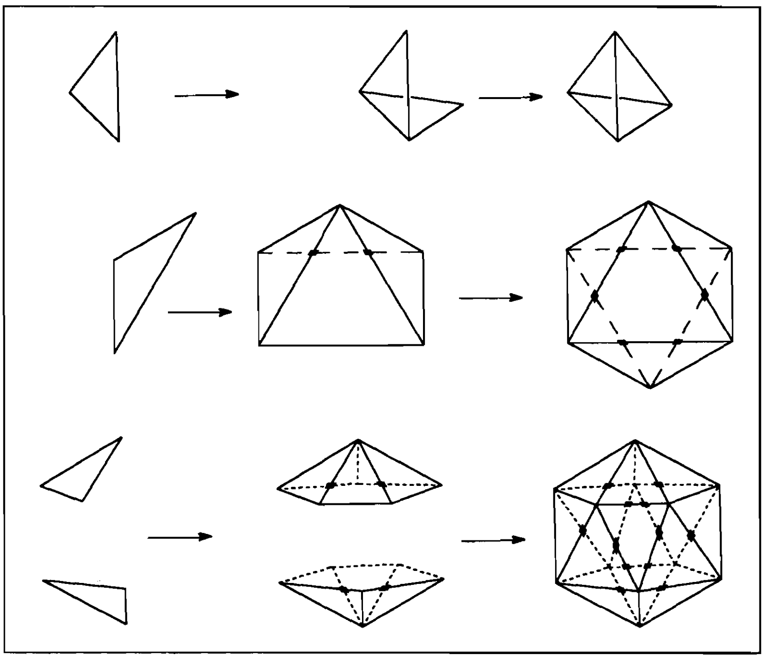


*Chart 4.*

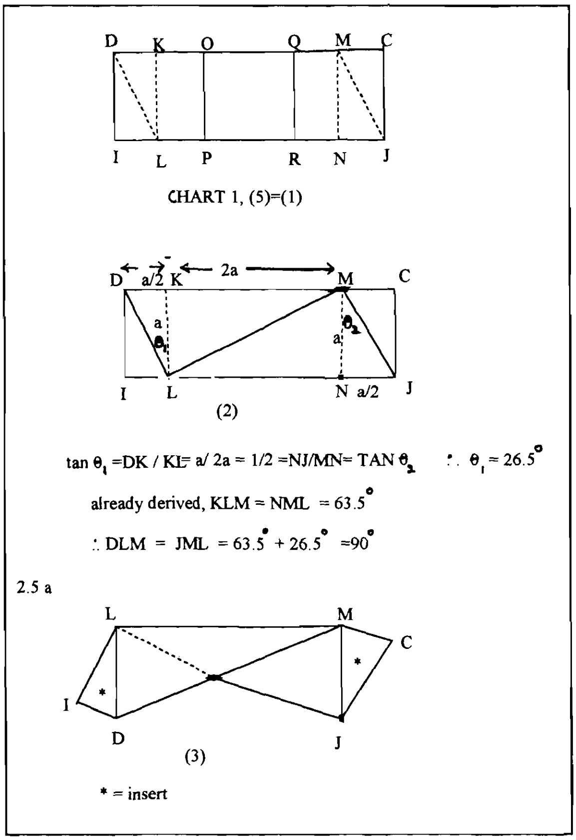



Chart 5.

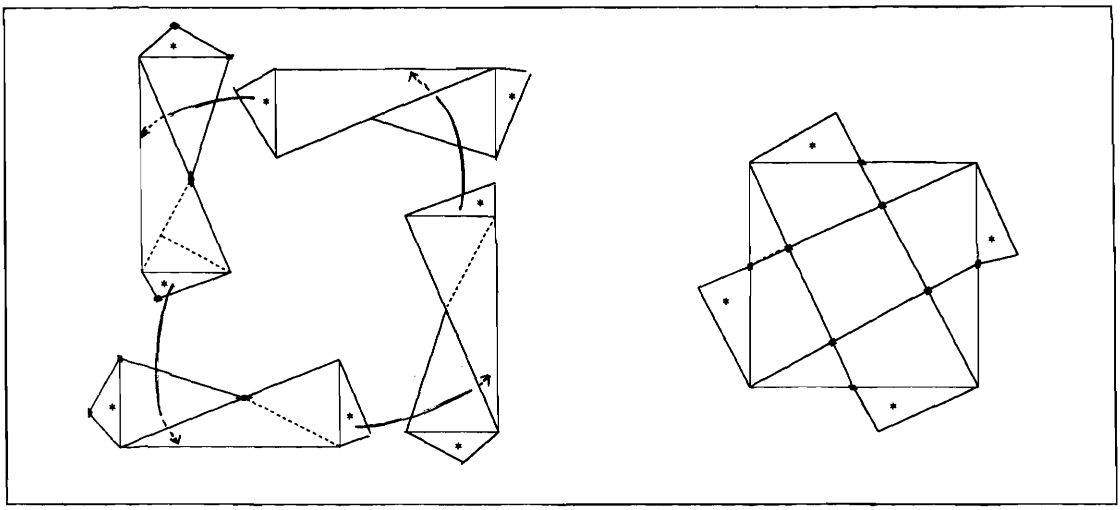
Follow precisely procedure in *Chart* 2 till *Figure* 5, which is again shown as *Figure* 1 in *Chart* 5.

Holding firmly with a scale, crease diagonals DL, ML and MJ (2). Visually it can be seen that the controller angle DLM=JML is the required 90°. This can also be derived mathematically as shown in *Chart* 5. To complete the module, hold LM firmly with a scale, bring the flap MDIL forward. Module 3 has inserts LID and MCJ and as receptors LMD, JLM.

As before, the assembly of 12 units of 3 to the cube starts with building a basic *square module* [SM]. This is clearly shown in *Chart* 6. As could be expected, each corner of the SM will have an insert (seen) and a receptor (latent). The union of two SM with four modules, as shown in *Chart* 7 will result in the formation of a cube.

***Dodecahedron:*** For any 3-D object conceivable, dodecahedron presents the highest symmetry. It also is the easiest platonic

Chart 6.



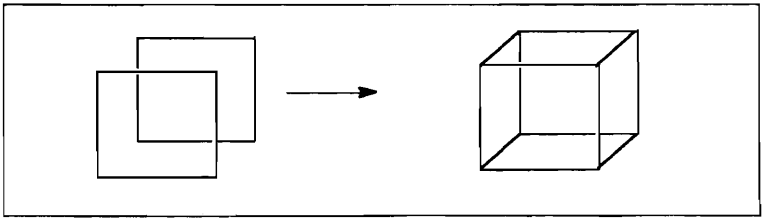


*Chart 7.*

solid to construct and therefore the ideal start for modular assembly exercises!

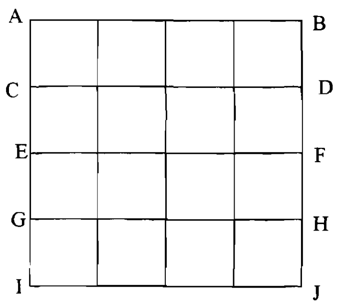
The module is also the easiest to make. Start from the $4 \times 4$ grid (1) in *Chart* 2, which is shown as *Figure* 1 in *Chart* 8. Without any cut, fold it to 2, to give the four layered front strip IJHG (3). Holding firmly with a scale, crease the diagonals, IK, KN and NH (3).

Since IKM and LNH are right-angled triangles with two equal sides, $\theta = 45°$ or $\tan q_1 = 1$, $\theta = 45°$. We have already derived the value of 63.5° for $\theta_2$. Therefore the controller angle IKN=HNK=45° + 63.5° = 108.5°, which is close to the 109° required! The module is completed by bringing forward the flap NKGI by holding firmly on KN with a scale. The module 4 has two inserts [KIG, NHJ] and two receptors [INK, HKN].

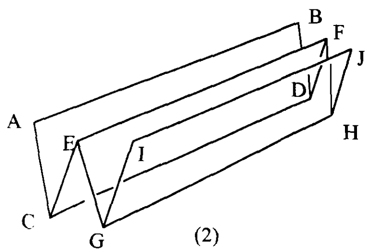
The assembly of five modules of (4) to the *pentagon motif* [PM] is illustrated in *Chart* 9. As with TM and SM, PM will have an insert (seen) and a receptor (latent) at each corner.

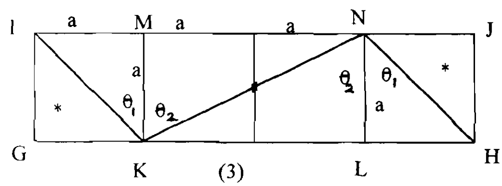
Of the many ways possible, for the assembly of the 30 modules of (4) (*Chart* 8) to dodecahedron, a simple way is to start from a single unit of PM and gradually add pentagons, with (4), in an anti-clockwise manner to reach the closed motif, which is called peristylane or cup! (*Chart* 10). The union of the five apices of peristylane to another unit of PM will generate the beautiful dodecahedron.

In carbon science, dodecahedron represents 20 perfectly, tetrahedrally disposed carbon atoms. Indeed the corresponding hydrocarbon dodecahedrane [$C_{20}H_{20}$] has been made.

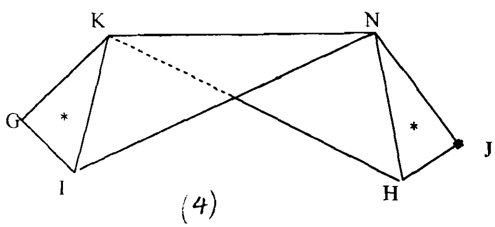



(1) Chart 1= (1)



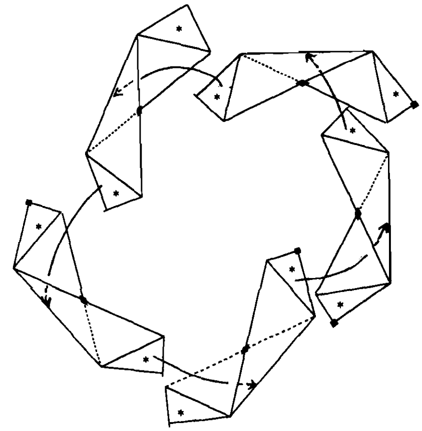




$\theta_1 = 45^\circ$; $\theta_2 = 63.5^\circ$ [ from tan $\theta_2$ = 2a/a=2]; $\therefore$ $\theta_1 + \theta_2 = 108.5^\circ$




* = insert

Chart 8.



* = insert

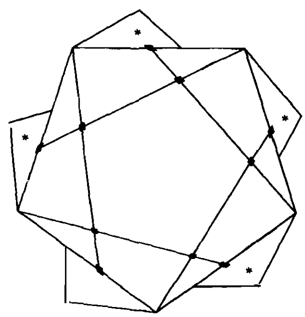


Chart 9.

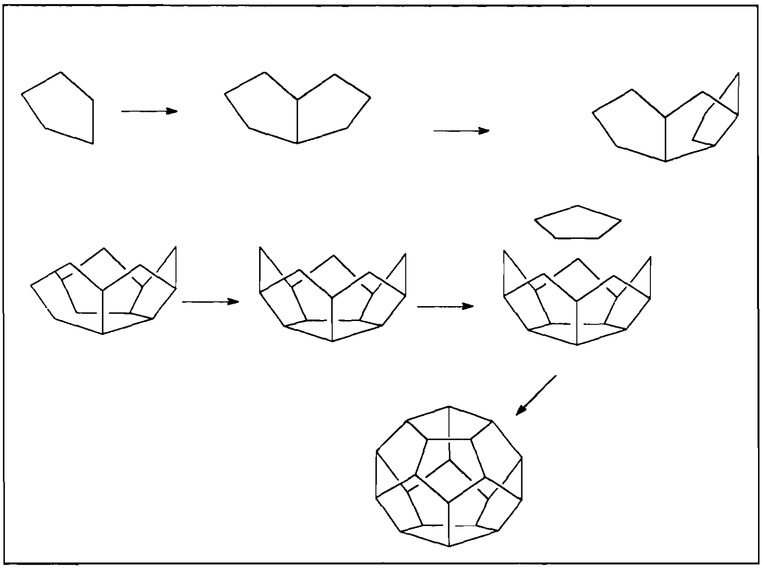


*Chart 10.*

The module (4) (*Chart* 8) used here, can therefore be equally appropriate for construction of saturated carbon compounds, thus providing vast possibilities for modular construction. The basic principles of modular assembly outlined here have potential for further exploration along a myriad of avenues.

## Acknowledgement

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## Suggested Reading

[1] J S Richardson, D C Richardson, The Origami of Proteins in Protein Folding, Eds. L M Gierasch, King, *J American Association for the Advancement of Science*, p.5-17, 1989.

[2] E A Lord, S Ranganathan, U D Kulkarni, Tilings, coverings and quasicrystals, *Curr. Sci.*, 78, 64-72, 2000.

[3] For an intuitive approach to the construction of such solids, see, J Ansel, Modular Folds, Lifestyle Origami, Harper Perenniel, New York, p.79-91, 1992.

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