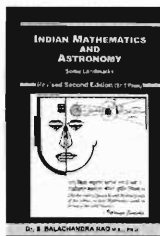


## Indian Mathematics and Astronomy: Some Landmarks

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*Indian Mathematics and Astronomy:*

*Some Landmarks\*\**

By S Balachandra Rao

Jhana Deep Publications, Bangalore,  
1994, Pages, VIII + 234, Price Rs. 75/-  
(in India).

This book is 'mainly addressed to the student community and general readers, with a view to providing an introduction to the development of mathematics and astronomy in India', as is announced in the Preface. The present reviewer congratulates Prof. Rao on his successfully achieving this aim. The scarcity of books of this kind seems to have deprived the young Indian generation of the chance of being informed of their own scientific heritage. In this very compact book of about 200 pages the author has packed almost all the essential aspects of the history of Indian mathematics and astronomy from the Vedic period to Ramanujan.

In the introduction entitled as 'Chapter 0' the author gives an overview of ancient Indian mathematics. Most topics in this chapter are taken up more in detail in the following chapters. Here are two comments of the reviewer on the topics which are found only in this chapter. The author claims that 'the

zero-symbol, namely a dot ( $\cdot$ ), was used' in Pingals's *Chandaḥsūtra*, but this statement is not based on textual evidence. At least we should distinguish zero as a concept from zero as a mathematical symbol. The author follows the old view of dating the *Bakhshālī Manuscript* in the 3rd century A.D., but the reader is advised to refer to the most recent and convincing result of Prof. T Hayashi in his *Bakhshālī Manuscript*, edited with an English translation and commentary, Groningen, 1995.

The main part of this book is divided into 11 chapters. The author's attitude is quite fair being free from nationalistic bias, which have been often embarrassing to the present reviewer. The reader can get considerably fair evaluation of the Indian contribution to mathematics. But there still remain some views which might misguide the reader. Let us review them following the order of the chapters.

Chapter 1 outlines the mathematics of the *Śulvasūtras*. The interesting topics of square-roots and squaring the circle are dealt with. Commenting on the correctness of the value of  $\sqrt{2}$  the author proposes to 'rename' the 'Pythagoras theorem as *Śulva Theorem*'. It seems that the author did not remember the fact that the similar or even better values were attested in the Old Babylonian cuneiform texts belonging to the time long before the *Śulvasūtras*.

Chapter 2 summarizes the *Vedāṅgajyo-*

\*Reproduced from *Ganita Bharati* (edited by R C Gupta), Vol. 19, pp. 128-132, 1997. \*\* A second and revised edition has been brought out in 2000.

*tiṣa*. According to the author, Lagadha's *Vedāṅgajyotiṣa* is assigned to 'the period of 12th to 14th century B.C.', based on the position of the winter solstice which was  $23^{\circ}20'$  apart from that in the day of Varāhamihira. In discussing such matters one should be careful of the fact that the origin of a myth and origin of the texts based on the myth are quite different. Any myth could go back to the time immemorial, as some people like to say, but texts cannot. It is true, for instance, that Kṛttikā (Pleiades) was near the vernal equinox in 2300 B.C., and thus it is not wrong to think that the origin of such an idea goes back to the Indus Valley Civilization, but this does not mean that already at that time the ecliptic coordinate system was well defined.

In Chapter 3 the author focuses his attention to Āryabhaṭa I, summarizes the contents of the *Āryabhaṭīyā*, and discusses his date and place of activity, and his innovative contributions in mathematics and astronomy. It is true that Āryabhaṭa used the word *āsanna* ('approximately') for his excellent value  $\pi = 62832/20000$  but there is no evidence to show that  $\pi$  was regarded as irrational by Āryabhaṭa himself.

Chapter 4 is devoted to Bhāskara I, who is the author of the oldest extant commentary on the *Āryabhaṭīyā*. As Prof. Rao points out, it is interesting to note that Bhāskara I went into oblivion in north India, and that his works survived only in the south, especially in

Kerala where Āryabhaṭa was highly esteemed and where his school found a remarkable revival in Mādhava school. As the starting point of this long and extraordinary tradition, the author's stress on Bhāskara I's significant position is truly appreciated.

Chapter 5 describes the works of Varāhamihira. The author follows the old tradition that Varāhamihira was a 'Magadha-dvija' and that he migrated from Magadha to Ujjainī where he was active as a court astrologer. But there is a view that '*magadha*' was a result of misinterpretation of the original form '*maga*', which stands for the Zoroastrian priesthood of Iranian origin. There are several evidences to support this view, especially in the *Pañcasiddhāntikā*, where Iranian deities were associated with the days in a month. The rest of Prof. Rao's accounts on Varāhamihira's achievement, including his indebtedness to Greek astrology, is well founded and guides the reader to the right direction.

Chapter 6 dwells on Brahmagupta, especially on his controversial character. His critical views on the established authorities, which are often very severe, are neatly summarized in this chapter. The author also quotes the words of Al-Bīrunī who was one of the earliest ancient scholars who could offer a fair evaluation of Brahmagupta's contribution. The reader is introduced to the famous Brahmagupta's theorem on cyclic quadrilateral and his ingenious solution of the indeterminate equation of the sec-

ond degree (*vargaprakṛti*). Brahmagupta's most outstanding contribution, the second order interpolation, is also explained in a clear manner.

Chapter 7 deals with Mahāvīra, one of the most renowned Jain mathematicians belonging to the 9th century. The history of Jain mathematics is one of the most neglected chapters of Indian mathematics and therefore further study is strongly expected. I hope this chapter stimulates the young scholars' interest in this unexplored field. The author regards that the 'āyatavṛtta' (lit. oblong circle), treated in the *Ganitasārasaṅgraha*, is identical to 'ellipse', but in fact, they are different in the strict sense of modern mathematics.

Chapter 8 makes a good introduction to the achievements of Bhāskara II, whose popularity outstrips all other Indian mathematicians. The author is correct when he says that Bhāskara II was the first Indian astronomer who took into account the effect of evection in lunar equation, but to say that 'Bhāskara's discovery preceded this in the west (by Tycho Brahe) by nearly four centuries' is wrong, since Ptolemy in the second century A.D. already knew the second anomaly' of the moon which was caused by evection. Among several innovations ascribed to Bhāskara II, the *cakravāla* method of solving the indeterminate equation of the second degree and the concept of differentials are skillfully explained in this chapter. The author attracts the reader by quoting with Eng-

lish translation the charming examples of mathematics given in the *Līlāvati*.

Chapter 9 is devoted to Gaṇeśadaivajña who is one of the best commentators of Bhāskara II's works. A quotation from Gaṇeśa's father Keśava is quite interesting in that he criticizes the traditional *Siddhāntas* and stresses the importance of observation and modification of parameters. Prof. Rao explains Gaṇeśa's interesting way of giving the date of composition of the *Vṛndāvana-ṭīkā*. The resulting date is 'Māghaśukla-pratipad of Śaka 1500'. Prof. Rao regards this date as falling in A.D. 1578, but this is wrong. According to the present reviewer's computer program based on the *Sūryasiddhānta*, the date corresponds to January 27 (Tuesday), A.D. 1579. Such kind of error (in this case the author simply added 78 to 1500 without taking into account the month) is frequently committed when Indian date is converted into the Western date or vice versa.

Gaṇeśadaivajña's *Grahalāghava* gave the most significant influence on the Pañcāṅga makers. It is interesting to know that even today many popular calendars are based on this work.

Chapter 10 forms the most important part of the entire book, giving considerably detailed accounts on the outstanding activities of the astronomers in Kerala. The author rightly criticizes the ignorant view of some famous historians of mathematics that there were no im-



portant mathematicians after Bhāskara II, although the high achievements of south Indian astronomer-mathematician had been brought to light by C M Whish as early as in A.D. 1835.

The author introduces the reader to the works of Parameśvara, Dāmodara, Nīlakaṇṭha, Jyeṣṭhadeva, Śāṅkara, Nārāyaṇa and Acyuta. He then turns his eyes to the very beginning of this great tradition, named the *parahita* system of Haridatta, and the first Kerala astronomer Govindasvāmin. It is fascinating to know that a stream of such a high standard of learning survived until the time of Prince Śāṅkara Varmā who wrote *Sadratnamālā* in A.D. 1819, only 16 years to go before Whish's 'rediscovery' of the tradition.

The reader might be impressed by learning that Newton–Gauss interpolation formula was anticipated by Govindasvāmin Taylor expansion and Gregory's series by Mādhava, etc. Prof. Rao's exposition is based on the pioneering works by Whish and C T Rajagopal and the recent works by K V Sarma, T A Sarasvati Amma, R C Gupta, A K Bag, and Kuppanna Sastri. Especially K V Sarma's editions and translations of the Sanskrit texts and his personal communication with the author constituted the main source of information. It is a pity that sometimes most interesting quotations are given without reference to the original Sanskrit source.

At the end of this well written chap-

ter the author adds a brief section 'Heliocentric model in Kerala astronomy', which is based on the recent paper of Ramasubrahmanian *et al.* It is true that Nīlakaṇṭha tried to construct a new geometrical model in which the effects of two epicycles, i.e. *mandavṛtta* and *sīghravṛtta* be represented in a unified form, but the result cannot be called heliocentric' in a proper terminology. If the present reviewer's interpretation is correct, Nīlakaṇṭha's model was such that the *sīghravṛtta* was no more located on the deferent, but it is an eccentric circle around the earth, and that the manda epicycle rotates on the *sīghravṛtta*. Thus, in modern interpretation, the *sīghra* epicycle represents the solar orbit and the manda epicycle explains the anomaly due to the combined eccentricities of the sun and the planet. Thus we can say that Nīlakaṇṭha's model was closer to Tycho Brahe's than to Copernicus' In this model the earth was always considered as stationed at the centre of the universe. Of course, if we interchange the solar orbit with the orbit of the earth, we can get the heliocentric model. But this was not Nīlakaṇṭha's idea.

The last chapter of this book is dedicated to Ramanujan. The author gives interesting, though elementary, examples which would stimulate younger readers. These examples show that the mathematics genius was also the product of the great tradition of the south Indian mathematics.

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