



This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

B Sury
Indian Statistical Institute, 8th
mile, Mysore Road
Bangalore 560 059, India.

Wrecked Tangles of Mats and Maths of Rectangles

The first two problems require elementary finite group theory as taught in undergraduate courses.

Q 1. If all proper subgroups of a given finite group are abelian, show that the group must be solvable.

Q 2. Let G be a finite group such that for each positive integer n , G has at the most n elements g satisfying $g^n = 1$. Prove that G has to be cyclic.

Q 3. Given two pieces of mat of dimensions 8×8 and 1×6 respectively, how can the 8×8 piece be cut into two pieces so as to make the resultant three pieces fit to form exactly a 10×7 piece?

Q 4. Suppose $f(X), g(X)$ are polynomials with coefficients that can be taken to be complex (or real or rational or anything actually). Assume that $\text{degree}(f) \geq \text{degree}(g) + 2$. If f has no multiple roots, prove that $\sum_{\alpha} \frac{g(\alpha)}{f'(\alpha)} = 0$ where the sum is over the roots of f . *This is for undergraduates – no complex analysis please!*

Q 5. Start with a square of side 1. Right on its top, add a unit square. Next, to the right of this rectangle, add a rectangle of the same height which has unit area. The rectangle at this stage has width $3/2$ and height 2. At the next step, add a rectangle on top with the same width and having unit area. Proceed this way adding rectangles (each time of unit area) alternately to the right and to the top. What is the limit of the ratio of height to the width?