

Fixed Points

From Russia with Love – A Primer of Fixed Point Theory

A K Vijayakumar



Fixed Points
Yu A Shashkin
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The seventy five pages of this mathematical morsel of a book present a polished little memoir on the elementary theory of fixed points. In tone and treatment, it may perhaps be placed in that distinguished tradition of mathematical writing which produced the elegant exposes on elementary topics belonging to the ‘*Little Mathematics Library*’, brought out by Mir Publishers. Pitched at a slightly more ambitious level, ‘*Fixed Points*’ is an interesting experiment in presenting, from an elementary perspective, material traditionally displayed as applications of homology and homotopy theory. Here, the central technique is through combinatorial theory – especially Sperner’s lemma. Three distinct regions of fixed point theory are considered. The context is always Euclidean space of at most three dimensions. The first is the Brouwer fixed point theorem; the second is the contraction mapping theorem while the third formulates conditions in terms of degree theory for the existence of fixed points. Fourteen crisply crafted chapters carry the

theory forward from a few simple combinatorial notions to the degree of mappings and the Borsuk – Ulam theorem. An idea of the impressiveness of such an agenda can be conveyed by mentioning that continuity makes its appearance as late as the fifth chapter. It is interesting to see how much of the conventional arsenal of algebraic and differential topology – especially covering spaces, homology and homotopy groups – can be skirted by the use of such elementary ideas. But the demonstration comes at an inevitable price. Occasionally, the argument glosses over crucial aspects of the theory. At others, concepts which are extremely subtle – at least for this level of exposition – are presented with a bland “One can prove...” or “It can be shown...” air. To mention just two instances of this kind, Orientation or even the PL approximation of continuous maps, pop out like rabbits from a top hat, without even the conventional courtesy of a diagram or two. But, given the nature of the undertaking, these are perhaps unavoidable shortcomings and in no way compromise the value of such an enterprise. In fact, if the text could be fleshed out with some gaps being filled in and more examples thrown in, a good one – semester course can be run on it at the undergraduate – or even graduate-level. In this sense, the book emphatically achieves its objective. Fixed point theory certainly has enough intuitive geometric content in itself not to require any external justification for its study. But perhaps the text could have profited from an illustration of the power of the techniques developed, by a mention of the

role played by it in, for instance, the Stone – Weierstrass theorem or the existence of solutions of differential equations. But mathematicians are a notoriously eccentric lot when it comes to what they consider ‘applications’. [The great Edmund Landau once claimed that analytic number theory was an applied science, since so many of his students had used it to get their Ph.D. degree. On the other hand, it has been the reviewer’s privilege to listen to a distinguished topologist embed Brouwer’s fixed point theorem into the context of economic theory and then proceed to “derive” it from purely economic considerations!]. The occasional looseness of language in the text (a statement on page 43 claims that a “circle is a retract of an annulus” without caring to specify where the circle is

located in the annulus) can serve as an illuminating exercise for students. The economist C N Parkinson once observed that parliamentary transactions were a dreary business. But in France it must be even more tedious, since the whole thing was done in French. A textbook in topology could be daunting enough, without the reader being required to plough through it in Russian. The Universities Press has put the mathematical community in its debt by bringing out a series of titles translated from Russian, at this level. It has done an excellent job with this book too, which, though perhaps, could have been a trifle more reasonably priced for its size.

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The Medusa and the Snail

Harini Nagendra



The Medusa and the Snail
Lewis Thomas
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1995, pp.175,
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Busy with your regular college syllabus? Then you must feel the need for dipping into something totally different once in a while – and this is a book I would highly recommend. *The Medusa and the Snail* is a collection of

thought-provoking essays on various aspects of the natural sciences, written by Lewis Thomas, that first appeared in the *New England Journal of Medicine*. These essays have been collected and published as a series of books – a review of the first of these, ‘*The Lives of a Cell*’, has already appeared in an earlier issue of *Resonance*.

The Medusa and the Snail is a collection of essays that seem completely different – one could be on cloning, and the next on schizophrenia! But they all have a single common thread running through – each essay explores an aspect of ‘selfness’. It is difficult to give you a sense of what I mean by this,