This section of *Resonance* presents thought-provoking questions, and discusses answers a few months later. Readers are invited to send new questions, solutions to old ones and comments, to 'Think It Over', *Resonance*, Indian Academy of Sciences, Bangalore 560 080. Items illustrating ideas and concepts will generally be chosen.

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The Limit of *n*-fold Sine

Consider the function $f_n(x)$ defined for positive integers n and real numbers x by $f_n(x)=\sin(\sin(\sin(\ldots(\sin x))))$, with n applications of the sine function. Thus $f_2(x)=\sin(\sin x)$, $f_3(x)=\sin(\sin(\sin x))$, and so on.

Let $x \in (0, \pi/2)$ be chosen, and let the sequence

$$f_1(x), f_2(x), f_3(x), \ldots f_{100}(x), \ldots$$

be computed. Naturally the sequence depends on x, but curiously as $n \to \infty$ the dependence seems to diminish. Indeed we find that for $n=4000$,

$$f_n(0.3)=0.027603,\ f_n(0.5)=0.027333,\ f_n(1)=0.027361,$$

while for $n=10000$,

$$f_n(0.3)=0.017289,\ f_n(0.5)=0.017307,\ f_n(1)=0.017314$$

(all values correct to 5 decimal places). In fact it turns out that for fixed x and sufficiently large n, we have $f_n(x) \approx \sqrt{3/n}$ How may this phenomenon be explained?