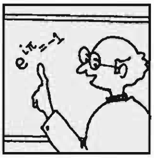

In this section of* Resonance, *we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

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International Physics Olympiad '98
Eddy Curents, Flux Linkages

This article is in continuation of my earlier article where I presented the statement of the experimental problem of the International Physics Olympiad'98 (IPhO). In the present article, I shall give the solution to the problem, along with relevant theoretical discussions and experimental observations whenever necessary. Although I wasn't able to do all the parts of this problem accurately in the actual competition, I have tried my best to keep this article error free.

Aim: To study magnetic shielding with eddy currents and the response of two coils mounted on a closed ferrite core to an external alternating (sinusoidal) voltage applied to one of them.

Part 1: Magnetic Shielding by Eddy Currents

Time dependent magnetic fields induce surface and volume currents in conductors. These are known as 'eddy currents'. The eddy currents induced in turn produce counteracting magnetic fields. This phenomenon is commonly known as 'magnetic shielding'. In superconductors the induced eddy currents maintain a magnetisation $\mathbf{M} = -\mathbf{B}_{ext}$ in the interior and this magnetisation is exactly opposite to the applied magnetic

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field $\mathbf{B}_{ext}$. Hence in superconductors the eddy currents will expel the magnetic field completely from the interior of the conductor. The magnetic state of a superconductor can be described as 'ideal diamagnetism'. Normal metals are not as effective in shielding magnetic fields as superconductors because of finite conductivity. To describe the shielding effect of aluminium foils we proposed the following phenomenological model:

$$\mathbf{B} = \mathbf{B}_0 \exp(-\alpha\, \mathbf{d}),$$

where $\mathbf{B}$ is the magnetic flux density beneath the foils. $\mathbf{B}_0$ is the magnetic flux density at the same place in the absence of the foil. α is the attenuation coefficient and $\mathbf{d}$ is the foil thickness.

The justification of this model comes from electromagnetic theory. In conducting materials, the free-current-density vector $\mathbf{J}_{free}$ and the applied electric field $\mathbf{E}$ are related by Ohm's law :

$$\mathbf{J}_{free} = \sigma \mathbf{E}, \tag{1}$$

where σ is the conductivity of the material.

We all know that Maxwell compiled the results obtained by Ampere, Faraday and others and gave equations for general time varying electric and magnetic fields. Ampere's law with the well-known correction of Maxwell is given by :

$$\nabla \times \mathbf{H} = \mathbf{J}_{free} + \partial \mathbf{D}/\partial t, \tag{2}$$

where $\mathbf{H}$ is the magnetic field intensity and $\mathbf{D}$ is the electric flux density. These are related to the magnetic flux density $\mathbf{B}$ and the electric field strength $\mathbf{E}$ by: $\mathbf{B} = \mu\mathbf{H}$, and $\mathbf{D} = \varepsilon\,\mathbf{E}$, respectively, where μ and ε are the magnetic permeability and the electric permittivity respectively, of the medium. Substituting for $\mathbf{H}$ and $\mathbf{B}$ and using Ohm's law in Maxwell's equations for Ampere's law one gets :

$$\nabla \times \mathbf{B} = \mu \mathbf{J}_{free} + \mu\varepsilon\,(\partial \mathbf{E}/\partial t) \tag{3}$$

or,

$$\nabla \times \mathbf{B} = \mu\sigma\mathbf{E} + \mu\varepsilon\,(\partial \mathbf{E}/\partial t). \tag{4}$$

Maxwell's equation describing Faraday's law is given by:

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t. \tag{5}$$

Taking curl on both sides of these equations and simplifying we obtain:

$$\nabla^2 \mathbf{E} = \mu\varepsilon (\partial^2 \mathbf{E}/\partial t^2) + \mu\sigma (\partial \mathbf{E}/\partial t) \tag{6}$$
$$\nabla^2 \mathbf{B} = \mu\varepsilon (\partial^2 \mathbf{B}/\partial t^2) + \mu\sigma (\partial \mathbf{B}/\partial t). \tag{7}$$

In one dimensional spatial coordinate, these equations admit solutions of the form:[1]

$$\mathbf{E}(x,t) = \mathbf{E}_0 e^{j(kx-\omega t)} \text{ and } \mathbf{B}(x,t) = \mathbf{B}_0 e^{j(kx-\omega t)} \tag{8}$$

[1] For detailed solution to these equations the reader may refer to any standard textbook on classical electromagnetic theory, e.g. *Introduction to Electrodynamics* by David J Griffith.

where ω is the frequency of the sinusoidal electric and magnetic fields. On substituting these in the above equation we obtain:

$$k = \beta + j\alpha$$

where

$$\beta, \alpha = \omega \sqrt{\frac{\varepsilon\mu}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2} \pm 1 \right)^{\frac{1}{2}} \tag{9}$$

The imaginary part of k is responsible for the attenuation of the wave i.e.

$$\mathbf{E}(x,t) = \mathbf{E}_0 e^{(-\alpha x)} e^{j(\beta x-\omega t)} \text{ and } \mathbf{B}(x,t) = \mathbf{B}_0 e^{(-\alpha x)} e^{j(\beta x-\omega t)}. \tag{10}$$

So the amplitude of the magnetic flux density decreases with depth inside the foil as $\mathbf{B}_0 e^{(-\alpha x)}$. Here α is known as the *attenuation coefficient*. In this experiment, we were asked to use one of the U-shaped ferrite cores to generate a time-varying (sinusoidal) magnetic field and place the aluminum foils of known thickness in the field. The magnetic field ratio $\mathbf{B}/\mathbf{B}_0$ was to be determined by placing another 'pickup' coil beneath the foil and by measuring the induced emf which is in turn proportional to the time derivative of the field. Sufficient data was to be collected for frequencies in the range of 6-18 kHz and the attenuation coefficient α was to be determined for the aluminum foils (25-175 μm). We were also asked to plot α versus frequency and write an approximate mathematical expression describing the function $\alpha(f)$ from the graph (*Figure* 1).

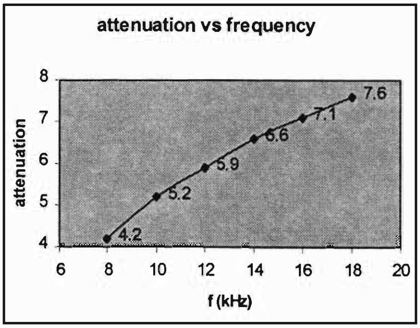


Figure 1.

From the aforementioned theory (see (9)) we can clearly see that for good conductors in relatively low frequency ranges, for which $\sigma >> \varepsilon\omega$, α varies as $f^{1/2}$ (this can be seen from the following graph which has been plotted for d = 75 μm).

In this section we saw that the field intensity decreases exponentially with depth inside conducting materials. This electromagnetic shielding finds a host of important practical applications. One such example is the famous 'Faraday Cage' which is a metal encasement used to shield sensitive electronics equipment from stray electromagnetic fields.

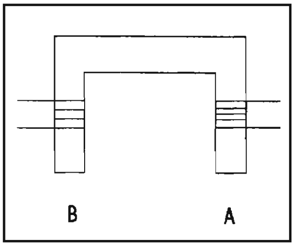
Part 2: Magnetic Flux Linkage

In this section, the response of two coils on a closed ferrite core to an external alternating voltage (V_g) from a sinusoidal signal generator was to be studied.

Theory

In the following theoretical discussion and for the subsequent experiment (*Figure* 2), it is assumed that the ohmic resistance in the two coils, the hysteresis losses and the eddy current losses have insignificant influence on the measured currents and voltages. These are reasonable assumptions. However, because of these simplifications in the theoretical treatment, some minor deviations occurred between the measured and the calculated values.

Figure 2.



Single Coil

Let us consider a core with a single coil carrying a current I. The magnetic flux φ that the current generates in the ferrite core is proportional to the number of turns in the coil (N) and the current in the coil (I). It was shown in Part 1 that :

$$\varphi = \mu_0 \mu_r gNI = \mu gNI = cNI, \qquad (11)$$

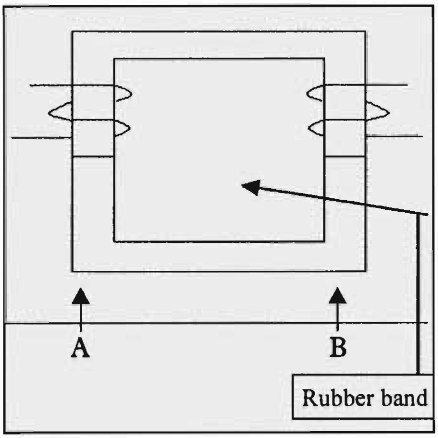


Figure 3.

where g is a geometrical factor characteristic of the ferrite core (the magnetic circuit).

The self induced voltage is given by Faraday's law:

$$\varepsilon(t) = -N(d\varphi/dt) = -cN^2(dI/dt).$$

Therefore the self inductance L of the coil is given by $L = cN^2$.

If $I(t) = I_0 \sin \omega t$, then $\varepsilon(t) = -\omega cN^2 \cos \omega t$.

Therefore in terms of the rms values of voltage and current,

$$\varepsilon = \omega c N^2 I \tag{12}$$

Two Coils

Now we take the two U-shaped ferrite cores given (one with the two coils A and B and the other with no coil on it) and fasten them together with a rubber band as shown in *Figure* 3.

It was proposed in the question that a part of the flux φ_A due to the current in coil A, i.e. $k\varphi_A$ goes through coil B and vice versa. k is called the coupling factor.

Now suppose that in general the potential difference across coil A is ε_A and across coil B is ε_B. Let the current through coil A be I_A and coil B be I_B. Then:

$$\varepsilon_A = -N_A \; d/dt[cN_A I_A - kcN_B I_B].$$

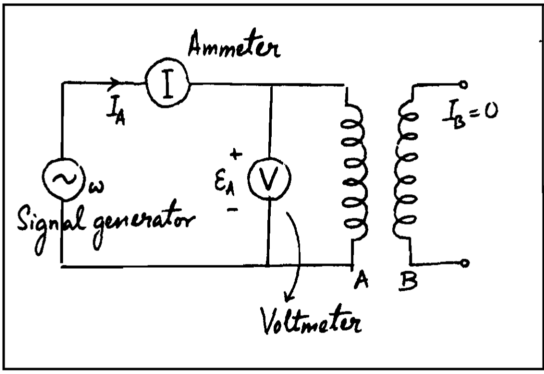
In terms of rms values of voltages and currents, we have

$$\varepsilon_A = -\omega c N_A{}^2 I_A - \omega kcN_A N_B I_B, \text{ and similarly} \tag{13}$$

$$\varepsilon_B = \omega c N_B{}^2 I_B - \omega kcN_A N_B I_A \tag{14}$$

Experiment

The procedural part of this section of the experiment is divided into six subdivisions. Note that the internal resistance of the given AC source has been denoted by R and the frequency



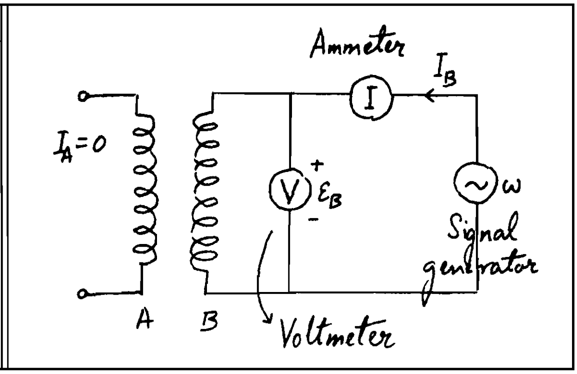


Figure 4 (left).
Figure 5 (right).

denoted by f. The various sections of this part are given.

Throughout the following experiment we have used an AC voltage source supplying 7.5 V (rms) at f = 10 kHz. ($\omega = 2\pi f$ = 62.83 krad/s).The number of turns in the two coils are: N_A = 150 turns and N_B = 100 turns.

1) We were asked to derive algebraic expressions for the self inductances L_A and L_B and the coupling factor k in terms of measured and given quantities.

Calculation of L_A

We can see from (13) and (14) that if we set $I_B = 0$, then as $L_A = cN_A^2$, $R << \omega L_A$, $L_A = \varepsilon_A/\omega I_A$. The circuit to measure L_A therefore will be as in *Figure* 4. The experimental value obtained for L_A was $L_A = \varepsilon_A/\omega I_A = 34 \pm 3$ mH.

Calculation of L_B

We can see from (13) and (14) that if we set $I_A = 0$, then as $L_B = cN_B^2$, $R << \omega L_B$, $L_B = \varepsilon_B/\omega I_B$. The circuit to measure L_B therefore will be as in *Figure* 5. The experimental values obtained for L_B was $L_B = \varepsilon_B/\omega I_B = 15 \pm 1$ mH.

Calculation of k

For calculating the value of k short circuit coil B, i.e. set $\varepsilon_B = 0$. Then we obtain from (14): $k = N_B I_B / N_A I_A$. The circuit for calculating k would therefore be as in *Figure* 6. The experimental

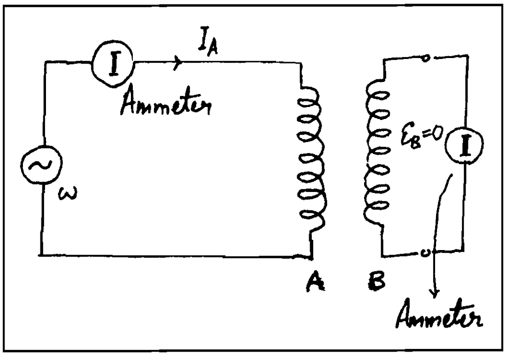


Figure 6.

value obtained for k was
$k = N_B I_B / N_A I_A = 0.95 \pm 0.09.$

2) In this part we were to calculate the current in the coil A (I_P) when the coil B is short circuited. But this is what we did in the previous part.

Putting the condition $N_B I_B = kN_A I_A$ (and that $\varepsilon_B = 0$) in (13) we obtain on simplification:

$$I_P = I_A = \varepsilon_A / \omega L_A (1-k^2). \tag{15}$$

Experimental value obtained: $\varepsilon_A = 7.2\text{V}$, and $I_P = 35 \pm 3$ mA.

Value obtained on substituting earlier calculated values of the parameters is $I_P = 34.56$ mA.

3) The coils A and B can be connected in series in two different ways such that the two flux contributions are either added to or subtracted from each other.

3.1. The self inductance of the serially connected coils (in the case when the two flux contributions add) L_{A+B} was to be calculated by deriving a suitable expression in terms of other parameters and the calculated value was to be compared with the experimentally obtained value.

When the two flux contributions add, we have:

$$\varepsilon_A = \omega c N_A^2 I + \omega k c N_A N_B I \text{, and}$$

$$\varepsilon_B = \omega c N_B^2 I + \omega k c N_A N_B I$$

where I is the current flowing in the serially connected coil.

$$L_{A+B} = (\varepsilon_A + \varepsilon_B)/\omega I = c(N_A^2 + 2kN_A N_B + N_B^2)$$

or, $$L_{A+B} = L_A + L_B + 2k\,(L_A L_B)^{1/2} \tag{16}$$

So here we set up the circuit as shown in *Figure* 7. From measurements made on this circuit, the quantity $(\varepsilon_A + \varepsilon_B)/\omega I$ was found to be 99 ± 8 mH.

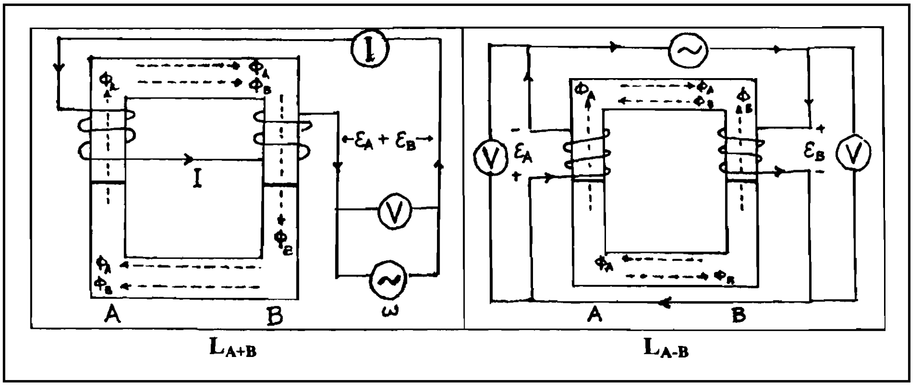


Figure 7.

$L_A + L_B + 2k\,(L_A L_B)^{1/2} = 92$ mH (using values of L_A, L_B and k determined earlier).

3.2. When the two coils are connected serially such that their flux contributions tend to oppose each other, the ratio of the potential differences across coils A and B is to be measured. Also, we were to find a mathematical expression for the ratio of the voltages and compare the value obtained from the expression with the experimental values.

Experimental values were :

Supplied voltage $\varepsilon = 24.0$ V
$\varepsilon_A = 15.5 \pm 0.6$ V, $\varepsilon_B = 8.4 \pm 0.5$ V
$\varepsilon_A/\varepsilon_A = 1.86 \pm 0.12$.

Mathematical expression for the ratio will be:

$$\frac{\varepsilon_A}{\varepsilon_B} = \frac{\omega c N_A^2 I - \omega k c N_A N_B I}{\omega c N_B^2 I - \omega k c N_A N_B I} = \frac{N_A(N_A - kN_B)}{N_B(N_B - kN_A)} \qquad (17)$$

This gives : $\varepsilon_A/\varepsilon_B = [N_A(N_A - kN_B)]/[N_B(N_B - kN_A)] = 1.94$, using value of k found earlier.

(We find that this is near the measured value of the ratio.)

4) In this part we were to verify that the self inductance of a coil is proportional to the square of the number of turns on it.

This can be done as follows: $L_A/N_A^2 = 1.51 \times 10^{-6}$ H (using values of L_A and N_A calculated earlier).

Similarly we find that $L_B/N_B^2 = 1.50 \times 10^{-6}$ H.

So we find that the ratio of self inductance to the square of the number of turns on the coils is almost a constant. This indicates that self inductance of a coil is proportional to the square of the number of its turns.

5) In this part, it was to be verified that in the preceeding discussion in Part 2 neglecting the ohmic resistance (R_A) of the primary coil was justifiable.

So let's write down the exact equations describing the coil voltages in terms of the respective rms phasors, when $\varepsilon_B = 0$. [2]

$$\varepsilon_A = I_A(R_A + j\omega L_A) - jM\omega I_B,$$
$$0 = I_B(R_B + j\omega L_B) - jM\omega I_A,$$

where M is the mutual inductance of the two coils.

$$\Rightarrow I_B = jM\omega I_A / (R_B + j\omega L_B).$$

Simplifying further we obtain:

$$I_A = \frac{\varepsilon_A}{R_A + j\omega L_A + \dfrac{M^2\omega^2}{R_B + j\omega L_B}}. \tag{18}$$

If R_B is quite small as compared to $|j\omega L_B|$ and by the well-known result $M^2 = k^2 L_A L_B$, the equation for I_A becomes:

$$I_A = \frac{\varepsilon_A}{R_A + j\omega L_A\left(1 - k^2\right)} \tag{19}$$

Now if $\omega L_A(1 - k^2)$ is much greater than R_A, we can neglect the primary resistance R_A and we get back our earlier expression for I_A, i.e. (15)

From experimental data, we find that

$R_A = 2.4$ ohms,

$(\omega L_A)_{(min)}(1 - k^2) \cong 100$ ohms.

[2] Here I have used the same symbols for voltage and current for the phasor quantities. For reference on phasors the reader is requested to refer to *Electrical Engineering Fundamentals* by Vincent Del Toro.

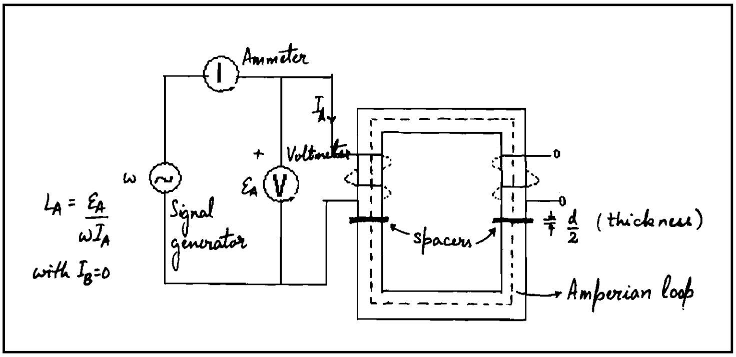


Figure 8.

This justifies our assumption.

6) This is the final part of this experiment (*Figure* 8). Here two thin spacers had been put in between the two U-shaped ferrite cores. The problem was to find an expression for and the actual value of the relative magnetic permeability (μ_r) of the ferrite core using Ampere's law and the fact that the normal component of the flux density **B** remains continuous across the ferrite paper interface. (Note : The last statement follows from the Maxwell's equation $\nabla.\mathbf{B}=0$ which is an implication of the fact that magnetic monopoles do not exist in nature.)

The solution to this problem goes as follows:

Set up the circuit as shown above. We see that $I_B = 0$. Ampere's law says that in a closed loop,

$$\oint \frac{1}{\mu} \mathbf{B} dl = I_{\text{total}} . \tag{20}$$

[Note: Here we have omitted the displacement current term (ε $(\partial\varphi_E/\partial t)$) of the Ampere–Maxwell's equation, φ_E being the flux of the electric field through the loop of integration. The reason for this is that typically at these frequencies for good conductors the displacement current is negligible. It has significance only at very high frequencies and when there is appreciable capacitance in the circuit.]

Suppose that the total length of the dotted line shown above is l. Then we have,

$$\mathbf{B}l = \mu N_A I_A , \tag{21}$$

[3, 4] The theoretical deductions carried out in the last part of the problem requires the knowledge of basic definitions and methods of solving magnetic circuits (for example the definition of 'reluctance' and that reluctance can be added algebraically for serially connected cores. The reader may refer for the same to *'Basic Electrical Engineering'* by Nagrath.

where **B** is the flux density inside the coil.

$$\text{Also } \mathbf{B} = \varphi/\text{A}\ , \tag{22}$$

where φ is the magnetic flux inside the core and A is the area of cross section.

Magnetic reluctance of the closed loop $R_M = l/\mu_A$.[3] (23)

Also we know that $L_A I_A = N_A \varphi$. (24)

Putting together all the above equations we have

$$L_A = N_A{}^2/R_M \tag{25}$$

Now consider the reluctance of the magnetic circuit with no spacers:

$$R_M(\text{no spacers}) = l/\mu\text{A} = R_1\,(\text{say}),$$

And with the spacers :

$R_M(\text{with spacers}) = (l-d)/\mu\text{A} + d/\mu_0\text{A} = R_2\,(\text{say})$.[4]

where d = 86 μm is twice the given thickness of the thin papers (thickness = 43 μm).

Let us call L_A(no spacers) = L_1 , and L_A(with spacers)= L_2
From (25), we get :

$$R_1/R_2 = L_2/L_1 \tag{26}$$

With a little manipulation with the above equations we get:

$$L_1 = L_2\left(1 - \frac{d}{l} + \frac{d}{l}\mu_r\right)$$

As $d << 1$, we may neglect the second term inside the bracket in the right hand side

The final expression for μ_r is:

$$\mu_r = \frac{l}{d}\left(\frac{L_1}{L_2} - 1\right). \tag{27}$$

When the whole procedure described above was carried out, the value of μ_r for the ferrite core came out to be $\cong 2300$

When μ_r is known, the geometrical factor of the coil g (see the question paper) can be calculated from the relation

$$g = c/\mu_0 \mu_r. \tag{28}$$

So, we have seen in the above experiment an example of a simple magnetic circuit, i.e. a 'circuit' whose 'working force' is magnetic flux. The characterization of magnetic circuits by magnetic reluctance, which is something analogous to electrical resistance in d.c. electric circuits, is a very useful method to analyze these circuits. This method of analysis works very effectively for more complicated magnetic circuits as well. The magnetic flux linkage that we studied in this experiment, finds the most extensive use in transformers, which is a device used to 'step up' or 'step down' ac voltages. Transformers are nowadays almost a part of our lives as they find a large variety of applications.

The experiments that are asked at the IPhOs are always innovative and require a considerable amount of theoretical analysis and planning before starting. Error estimation is an indispensable part of scientific experimentation. Every experimentally determined quantity must be reported to the appropriate number of significant figures alongwith the estimated error. I did not do the error estimation properly in the actual competition as a result of which I lost quite a few marks. So I would like to suggest the future IPhO trainees to practice doing quick and rough error estimation during all the experiments done at the training camp itself.

On the whole, participating in the IPhO was a nice experience for all of us. In India as far as training for the experimental competition is concerned, we have excellent laboratories specially aimed towards training students for the IPhO at the HBCSE, Mumbai (the institution where the training camp is held). This year's (1999) Indian team performed excellently at the IPhO '99 (held at Padova, Italy) winning four silvers and a bronze medal.

So, with this initial experience and further practice, I sincerely hope that the future Indian teams will perform increasingly well at the coming IPhOs.

Acknowledgements

I thank Dr. Vijay Singh (Professor, Department of Physics, IIT Kanpur) for helping me in editing the piece and arranging for experimental data for the actual experiment. He is actively involved in the IPhO programme and was the delegation leader for this year's (1999) Indian team. I am grateful to S S Prabhu (Retd. Professor, Department of Electrical Engineering, IIT Kanpur) for helping me with some of the theoretical parts of this experiment.

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When are the Roots of a Cubic Real?

The condition for a cubic equation with real coefficients to have only real roots is derived analytically.

Theorem: The equation

$$x^3 + ax^2 + bx + c = 0$$

with real coefficients a, b, c will have only real roots if, and only if,

$$(2a^3 - 9ab + 27c)^2 \leq 4(a^2 - 3b)^3$$

Proof. As a cubic equation has either two extreme values or no extremum, we have the following cases:

Case (i): The equation will have only one real root if it has no extremum.

Case (ii): If both the extrema lie above or below the real axis, then the equation will have only one real root.