

Reliability – How to Quantify and Improve?

Improving the Health of Products

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The basic concepts relating to reliability, including the statistical distributions employed are explained in this article. The approach of physicists, material scientists and engineers to improve reliability are emphasized. The qualitative measures would greatly help in improving quality and life of the products we use.

Societies, particularly the industrialised ones depend largely on proper functioning of several equipment – mechanical, electrical and electronic – for industries, farms and even homes. The reliability of a water pump or electric motor driving the pump or a tractor is of great concern to a farmer. Industries pay much attention to reliability of production equipment as equipment shutdown would reduce production and profits. With increasing growth of service sectors such as hospitals and hotels, reliability of medical equipment, elevators and power systems assume great importance. Housewives are interested in the reliability of kitchen equipment, refrigerators, washing machines and even toys. Reliability which is an important quality attribute is discussed widely.

How do we quantitatively assess the reliability? What is the mathematical formalism to describe ‘life’ and ‘failure pattern’ of equipment? How can we improve reliability? These questions are answered briefly in this article.

In common parlance, reliability is assessed by two quantities: (1) the life of an equipment expressed in years or months since the date of purchase and (2) how often the equipment/component fails and requires repair or replacement in a year. Both these measures are useful for qualitative studies but are inadequate for rigorous scientific analysis.

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Life of an Equipment

Most often we tend to express reliability of an equipment or device with its calendar or chronological life. For instance, we may state that a home refrigerator has lasted for twelve years. Assuming that it has been continuously used, we can subtract the periods of switching off due to power shutdown (say 10%) and reckon the life period as 10.8 years. [A more rigorous approach would be to count the 'on' periods only and the number of on-off cycles of the compressor units.]

For most quantitative analysis, however, we need to use the operating hours of the equipment, rather than calendar period. For instance, the life of an electric motor should be expressed in hours of operation rather than in terms of calendar months. It is commonly accepted that the life of an automobile engine should be expressed in terms of kilometers (or miles) run by the automobile.

Failure Rate

Many equipments fail at various times and if repairable are repaired and put to use. [This can be compared to the frequent visits of a person to a hospital or clinic; this frequency is not easily correlated with the life span of the person!] The number of failures in a given period, say in 1000 hours or million hours of operation is a measure of reliability. The failure rate may be high in the early life of an equipment due to manufacturing or assembly faults and due to improper usage by customers. Again the failure rate increases with time, due to gradual weakening or wear of equipment (due to several causes or failure modes such as corrosion, metal fatigue, wear, non-lubrication and so on).

Bathtub Curve

The failure-rate study of electronic equipment exhibits an interesting pattern. When (instantaneous) failure rate or hazard rate is plotted against time, we observe a curve that resembles a

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bathtub. The curve has three distinct regions – first one, with decreasing failure rate is called the ‘infant stage’; the middle one, with constant failure rate is called the ‘adult or maturity stage’ and the third one, with increasing failure rate is called the ‘wear-out stage’ or ‘old age’ (*Figure 1*). The analogy to human health is apparent. The first stage corresponds to the frequent illnesses suffered by babies in the first few years of life. The adult stage is characterised by healthy life period, with fewer illnesses and a normal productive life. The third stage is the field of gerontologists with increasing illnesses of bodily organs.

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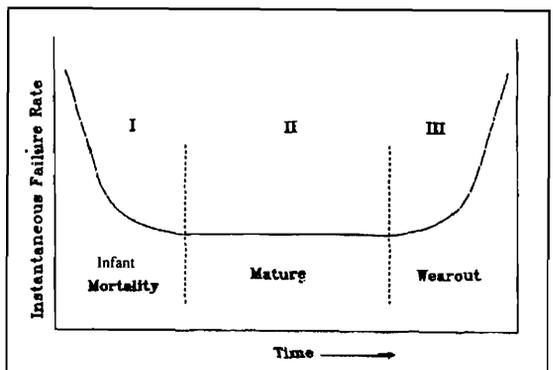
The middle region of constant failure rate is important to engineers. This rate is related to ‘random’ or ‘accidental’ failures and is easily modelled with Poisson distribution which is often used for analysing road or train accidents. It is also expressed in terms of ‘exponential distribution’ which is identical to Poisson distribution as we shall see later. [This stage is often treated as homogeneous Poisson process by statisticians.]

The third stage, with increasing failure rate, indicates the end of useful life. Even though the equipment may be usable with frequent repairs or replacement of parts, the maintenance cost is likely to be heavy at this stage. For durable equipment (not throw away or planned obsolescence items) the engineer would like to extend the second stage as much as possible.

Figure 1. Bathtub curve – failure rate vs time.

The Mathematics of Reliability

The life of equipment and the failure rates are statistical quantities. Thus, if we test ten electric motors or light bulbs for their life period, the results would vary. The histogram of these statistical quantities may re-



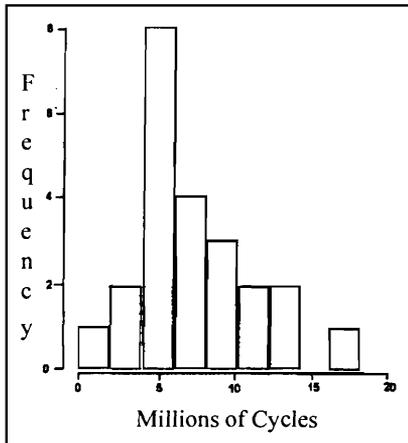


Figure 2. Histogram of the ball bearing failure data.

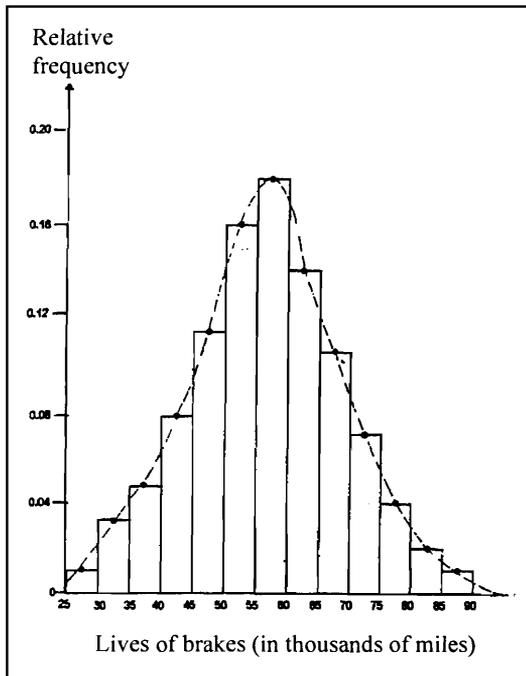
veal the pattern – particularly whether they are symmetrically distributed or not. A typical example for the life of ball bearings is shown in *Figure 2*. The histogram is not symmetrically distributed.

In most cases, the life of equipment (say the life of brake systems or automotive tyres expressed in miles or kms) would be symmetrically distributed. The life period is then modelled with normal or Gaussian distribution with the two parameters: mean life and standard deviation (*Figure 3*).

The statistical concepts are similar to those used by insurance analysts or actuaries. At this stage, we shall state the formal definition of reliability:

“Reliability of an equipment, component or system is the probability of its functioning at a given time under specified operating conditions with the environmental conditions imposed.”

Figure 3. Lives of brakes (in thousands of miles).



The definition points to the fact that the reliability of an equipment is not a constant but a function of time. It also stresses the importance of operating conditions and the environmental effects. For instance, a truck regularly overloaded is likely to exhibit lower reliability; likewise an equipment (say a radio) used on board a ship is likely to exhibit poor reliability due to vibration and exposure to corrosive salty air in the sea compared to the one used, say inside a home away from the sea.

The continuous probability distributions are often used for the models of reliability. If $f(t)$ is the probability density function (pdf) of the distribution, then $f(t)dt$ is the probability that the equipment would fail in the time

interval t to $t+dt$. Then it follows that the probability of failure over the interval $[0, T]$ is given by the integral:

$$P(T) = \int_0^T f(t)dt.$$

Note that, by definition

$$\int_0^{\infty} f(t)dt = 1.$$

We have several choices for the distributions that can be used. We shall consider only two distributions here: the exponential distribution and the Weibull distribution (Box 1).

For exponential distribution, the pdf is as follows:

$$f(t) = \lambda e^{-\lambda t}, \lambda > 0$$

and the mean of this distribution = $1/\lambda$

The reliability or the probability of equipment not failing be-

Box 1. Pioneers in Reliability Theory and Engineering

Reliability engineering has a rich background of theory and practice. The basic concepts were developed by statisticians, the most prominent being the Russian B Gnedenko. Y K Balyaev did pioneering work in applying the theory. In the West, Philip M Morse, a theoretical physicist at MIT, provided reliability analysis for military equipment during World War II.

The importance of reliability of electronic equipment was realised in the early fifties and a landmark report by the Advisory Group on Reliability of Electronic Equipment (AGREE) outlined the testing for reliability, especially for radiotubes, radar components and electronic parts.

Waloddi Weibull [1887-1979] made singular contributions to reliability theory and practice. A Swedish engineer, Weibull served in the Swedish military research and later became professor at the Royal Institute of Technology, Stockholm. He is credited with the inventions relating to ball bearings and electric hammers. He developed the statistical distribution named after him in 1951. He also studied fatigue and fracture of metal for aircraft components and later served as Professor at Columbia University, New York. Weibull distribution is probably the most widely used one in reliability field.

Much of the more recent works in reliability are related to space research and nuclear reactor safety issues.

The term 'MTTF' is perhaps the most common measure of reliability used by engineers.

tween 0 and t is given by

$$R(t) = 1 - P(t) = 1 - \int_0^T f(t)dt.$$

For exponential distribution $R(t)$ is expressed by the simple relation:

$$R(t) = e^{-\lambda t}.$$

This is, perhaps, the most widely used equation in reliability engineering.

The mean value is called the 'mean time to failure' MTTF:

$$\text{MTTF} = 1/\lambda.$$

The hazard rate or instantaneous failure rate $h(t)$ is given by the relation:

$$h(t) = f(t)/R(t).$$

Hence, (for exponential distribution) $h(t) = \lambda$, a constant. Therefore exponential distribution is used for modelling the constant failure rate or middle region of the bathtub curve. The term 'MTTF' is perhaps the most common measure of reliability used by engineers. A related term, 'mean time between failures' MTBF is the sum of MTTF and mean time to repair (MTTR);

$$\text{MTBF} = \text{MTTF} + \text{MTTR}$$

For wear-out stage, Weibull distribution is often used. Weibull applied this distribution for fatigue failure of aircraft components in 1939 (see *Box 1*).

The Weibull distribution requires two parameters to express the relationship. [Sometimes a three-parameter equation is also used.] Based on this distribution, the reliability function is :

$$R(t) = \exp \left(- (t/t_0)^n \right)$$

where t_0 is called characteristic time (a scaling factor for time) and n , the shape factor. When $n=1$, Weibull reduces to the



exponential distribution and t_0 is the same as MTTF.

For many mechanical systems, (bearings, gears, wing structures, etc.) n can be in the range of 2 to 4; thus the reliability decreases rapidly with time or the failure rate increases sharply. When $n=3.5$, Weibull density function resembles the normal (Gaussian) density function.

The Weibull distribution is often used for a histogram as shown in *Figure 2* for ball bearings failure data.

Physics of Reliability

The method to improve reliability at component level is to find the failure modes and mechanisms observed in field use and laboratory tests and to combat the failure modes. For instance, if a component fails due to oxidation or corrosion, suitable protective coatings can be applied. Sometimes the failure modes are complex, involving corrosion, wear and fatigue, which are studied in detail and proper methods are applied. Often a change of material could be suggested. More complex mechanisms like the electro-migration operate in integrated circuits, leading to voids in metallic thin films.

Since most reactions are thermally activated, the failure rates can be empirically described in terms of Arrhenius equation for reaction rates, as taught in chemistry classes:

$$\text{Failure Rate} = Ae^{-Q/RT}$$

where A and Q are constants and R is the gas constant and T -temperature in Kelvin and Q is the activation energy. This equation implies that failure rates can be accelerated by increase of temperature. Most electronic components, IC's for example are tested for failure at 80-120°C for short duration. These are called as accelerated tests. The failure times at the ambient temperature (say 25°C) would be several years!.

Many failure models are based on a power-law equation such as: $(t_1/t_2) = (\sigma_2/\sigma_1)^m$ where t_1 and t_2 are failures times at stress σ_1 and σ_2 , respectively and m is a constant. For instance the life of

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ceramic capacitors are related to voltage by such a relationship.

Engineers' Approach to Reliability

Often the approach of 'physics of failures' is not easy to apply because of complexity of failure mechanisms. Moreover, the physics approach is available only for certain modes of failures. The engineers, meanwhile employ several design approaches – for instance most aircraft components failing due to fatigue are improved by reducing areas of stress concentrations. Furthermore for many systems/sub-systems, temperature and humidity greatly reduce reliability. Suitable 'thermal design' of equipment is essential. This is done by using heat sinks (metal pieces attached to IC's and transistors) or by cooling with air or water ducts. Often engineers derate the components, i.e. use them at lower values of voltage or current or power to increase their lives.

For many others, reliability of the system as a whole is improved by using redundancy – such as using stand-by power generators, parallel computing systems, twin engine systems for aircraft and so on.

To improve maintainability, i.e., to reduce maintenance times, engineers build sub-systems/components in a modular fashion – for quick diagnosis of fault and for easy repair/replacement. Though reliability may not be improved, availability is increased by rapid repair procedures. In mathematical terms, steady-state availability as t tends to infinity is expressed as follows :

$$\text{Availability } (\infty) = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$$

General Comments

With increasing use of sophisticated equipment, which are highly automated and dependent on complex software, reliability becomes an important quality attribute. [Software reliability has become a subject of its own.] Designing systems for

Box 2. I S O – 9000 Quality System and Reliability

I S O – 9000 Quality System includes several reliability activities as follows:

4.4.5 : Design outputs : “identify those characteristics of the design that are crucial to the safe and proper functioning of the product”.

4.9 : Process Control : “monitoring and control of suitable process parameters and product characteristics” and

4.20 : Statistical Techniques : “the need for statistical techniques required for establishing, controlling and verifying process capability and product characteristics”.

high reliability and ensuring reliability are major concerns of engineers (see Box 2). Reliability evaluation has become important for safe-critical equipment (such as medical equipment and nuclear reactors) and for mission-critical equipment (such as aircraft, satellites and submarines.)

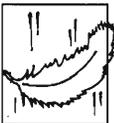
Reliability engineering, with its heavy accent on mathematics and physics of materials is an essential branch of engineering and is an interdisciplinary field with high degree of specialisation. The growth of this field is a positive indicator for industrial and social development in any country.

Suggested Reading

- [1] P D C O'Connor, *Practical Reliability Engineering*, 3rd Edition John Wiley, New York, 1991.
- [2] A Birolini, *Quality and Reliability of Technical Systems - Theory - Practice - Management*, 2nd Edition Springer-Verlag, New York, 1997.
- [3] R Billinton and R N Allan, *Reliability Evaluation of Engineering Systems*, Pitmans, London, 1983.

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People must understand that science is inherently neither a potential for good nor for evil. It is a potential to be harnessed by man to do his bidding.

– Glenn T Seaborg