The EPR Paradox: Einstein Scrutinises Quantum Mechanics

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Einstein, Podolsky and Rosen (EPR), in their famous paper, argued that the quantum-mechanical description of physical reality is incomplete. They showed that one can envisage physical situations whereby 'an element of physical reality' can be located such that it does not have a counterpart in quantum theory. A simple description of the EPR paradox is presented in this article, using the example of two spin-1/2 particles.

Einstein contributed key ideas to the early development of quantum theory. However, he did not think that quantum mechanics was the final answer to the question of theoretically mapping objective physical reality. His tersely worded, yet extremely lucidly written, criticism of quantum theory is formulated in the EPR paper.We intend to give a simple description of the EPR objections to quantum theory.

Objective reality exists independent of any theory. Hence, any theory that seeks to describe reality has to operate with concepts which have a correspondence with this reality. Within this framework we can demand that, for any theory to be satisfactory, the following criteria must be fulfilled: (a) the theory must be correct, i.e. its predictions should be in agreement with experimental results and (b) the theory must be complete, i.e. "*every element of physical reality must have a counterpart in the theory*" The EPR paper shows that there exist elements of reality which do not have any counterpart in the quantum theory and hence, while quantum theory may be correct, it is incomplete.

Before proceeding further, we need to be able to define elements of reality for a given physical situation. This is a tricky question and can lead to long philosophical arguments! Instead of trying to identify a complete set of elements of reality for a physical system, EPR suggested a means of identifying an element of physical reality: "*If without in any way disturbing a system we can predict with certainty (i.e. probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*"

This prescription of identifying elements of physical reality can be applied to classical as well as quantum systems. The goal here is to use the above criterion and to come up with at least one element of physical reality which does not have a corresponding counterpart in quantum theory. If this can be achieved, then one has shown that quantum theory is incomplete.

Elements of Physical Reality

The fact that quantum theory predicts only the probabilities and not the outcomes of individual measurements, and that the state of the system is altered by the measuring procedure are two key features of quantum measurement theory. What happens if the state of the system is one of the eigenstates of the operator being measured? For an eigenstate $|a\rangle$ of the hermitian operator $\hat{A}$, only the eigenvalue a will be obtained after the measurement, with a probability equal to unity. The state too will not suffer any change after the measurement process. We can thus predict the outcome of the measurement with a probability equal to one and without disturbing the system in any way. Therefore, by the EPR criterion, *for an eigenstate, the value of the corresponding observable is an element of physical reality.*

Example of a Two-level System

Consider a spin-1/2 particle as an example of a two-level system, the two levels being characterised by the spin being in the $|\uparrow\rangle$ or the $|\downarrow\rangle$ state. The three spin components of this system are represented by the hermitian operators

$$S_x = \frac{\hbar}{2}\sigma_x, \quad S_y = \frac{\hbar}{2}\sigma_y, \quad S_z = \frac{\hbar}{2}\sigma_z,$$

with σ_x, σ_y and σ_z being the standard Pauli spin ma-

Box 1. Measurement in Quantum Theory

Consider a quantum system in the quantum-mechanical state $|\alpha\rangle$. Assume that we have set up an apparatus to measure the physical observable A. The first task is to identify the hermitian operator $\hat{A}$, corresponding to the observable A. Proceeding step-wise in order to interpret the results of quantum measurements on the observable A:

- One has to find the eigenvalues and eigenvectors of the operator $\hat{A}$,

$$\hat{A}|a\rangle = a|a\rangle.$$

  These states form a complete set, so that every quantum state can be expanded as a linear combination of them.

- The possible outcomes of the experiment are the set of eigenvalues $\{a\}$. In an individual experiment, only one of the eigenvalues $a \in \{a\}$is obtained.

- After every measurement, the state of the system is transformed into an eigenstate of the operator $\hat{A}$, corresponding to the eigenvalue obtained.

- If we repeat the experiment on the state $|\alpha\rangle$a large number of times, we can obtain the relative frequency of different eigenvalues.

- To calculate these relative frequencies from quantum theory, we expand the state $|\alpha\rangle$ in terms of the eigenfunctions of the operator $\hat{A}$,

$$|\alpha\rangle = \sum_a C_a|a\rangle$$

  giving us the probability of finding the eigenvalue a as

$$P_a = |\langle a|\alpha\rangle|^2 = |C_a|^2.$$

trices. These operators are the only three independent observables in this system.

Consider the eigenstates of S_z: $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$ with eigenvalues $+\frac{\hbar}{2}$and $-\frac{\hbar}{2}$, respectively. The value of the observable corresponding to the z component of the spin is an element of reality for these states. Further, we can construct states for which S_x and S_y are the elements of physical reality, which are just the eigenstates of these operators, respectively. For a general state of the spin-1/2 system,

$$|\alpha\rangle = \cos(\theta/2)|\uparrow\rangle_z + \sin(\theta/2)e^{i\phi}|\downarrow\rangle_z$$

the component of spin along (θ, ϕ) is an element of physical reality.

Simultaneous Reality of Two Observables

Let us now consider two observables A and B for a quantum system.

Case 1: When A and B commute, i.e. when

$$[A, B] = 0,$$

the two observables have simultaneous eigenstates and hence, for these eigenstates, their values can be simultaneous elements of reality. We can predict their values with a probability equal to one, and can also measure them without disturbing the state of the system.

Case 2: When A and B do not commute, i.e. when

$$[A, B] \neq 0,$$

the two observables A and B do not have simultaneous eigenstates. Each eigenstate of one of the observables will, in general, be a linear combination of the eigenstates of the other. For no state does quantum theory predict precise values of these two observables together. Thus it is expected that no experiment will be able to determine the values of these observables simultaneously.

Repeated observations will in general lead to different values for at least one of the observables. Let us take another look at the example of the spin-1/2 particle. We will see here that as a direct consequence of the non-commutation of S_z and S_x $[S_z, S_x] = i\hbar S_y$, there is no quantum state for which both S_z and S_x are simultaneously elements of physical reality.

If we measure the x component of the spin for either of the eigenstates of S_z (for which S_z is an element of physical reality),both the values $\pm\frac{\hbar}{2}$ are equally probable and the state after measurement changes to the corresponding eigenstate of the S_x operator. This can be seen directly if we expand, say, the state $|\uparrow\rangle_z$ in terms of the eigenstates of S_x

$$|\uparrow\rangle_z = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_x + |\downarrow\rangle_x\}$$

and apply the quantum measurement theory given in *Box* 1. Thus, we are neither able to predict values for S_x with probability one, nor able to measure it without disturbing the particle. Therefore, quantum mechanics does not predict S_x to be an element of physical reality simultaneously with S_z.

EPR Argument for Two Spin-1/2 Particles

We now apply the above arguments to a composite system consisting of two spin-1/2 particles P_1 and P_2. In this case, the four eigenstates are given by

$$\begin{aligned}
|\uparrow\rangle_{1z}|\uparrow\rangle_{2z} &: \quad P_1 \text{ in state } |\uparrow\rangle_z, \text{ and } P_2 \text{ in state } |\uparrow\rangle_z,\\
|\uparrow\rangle_{1z}|\downarrow\rangle_{2z} &: \quad P_1 \text{ in state } |\uparrow\rangle_z, \text{ and } P_2 \text{ in state } |\downarrow\rangle_z,\\
|\downarrow\rangle_{1z}|\uparrow\rangle_{2z} &: \quad P_1 \text{ in state } |\downarrow\rangle_z \text{ and } P_2 \text{ in state } |\uparrow\rangle_z,\\
|\downarrow\rangle_{1z}|\downarrow\rangle_{2z} &: \quad P_1 \text{ in state } |\downarrow\rangle_z \text{ and } P_2 \text{ in state } |\downarrow\rangle_z.
\end{aligned}$$

All these states are eigenstates of the operators S_z^1 and S_z^2. (The superscripts 1 and 2 refer to P_1 and P_2.). This implies that, for these states, the z component of the spin of each particle corresponds to an element of physical reality.

Now, any normalised linear combination of these eigenstates is also a valid quantum state (using the *superposition principle* of quantum mechanics). Specifically, let us consider a state which is formed by taking a linear combination of two of these states, with equal weights and a phase difference of π

$$|\psi\rangle_{12} = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{1z}|\downarrow\rangle_{2z} - |\downarrow\rangle_{1z}|\uparrow\rangle_{2z}\}$$

This state is rotationally symmetric and will therefore retain the same form if we make a change of basis. In fact, it is the well-known singlet state, with zero total angular momentum. Re-writing $|\psi\rangle_{12}$ in the x basis,

$$|\psi\rangle_{12} = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{1x}|\downarrow\rangle_{2x} - |\downarrow\rangle_{1x}|\uparrow\rangle_{2x}\}$$

Here

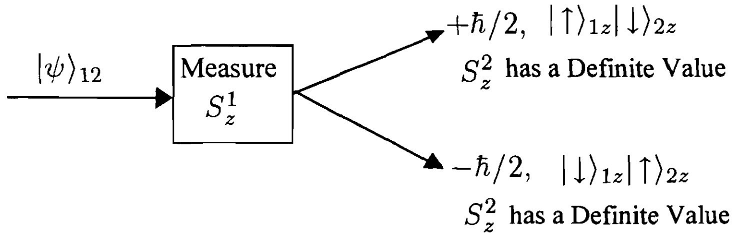
$$|\uparrow\rangle_{jx} = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{jz} + |\downarrow\rangle_{jz}\} \qquad |\downarrow\rangle_{jx} = \frac{1}{\sqrt{2}}\{|\uparrow\rangle_{jz} - |\downarrow\rangle_{jz}\}$$

are the eigenstates of the operator S_x^j with $j = 1,2$. Once the two particles are prepared in such a state, they remain so even after they cease to interact as total angular momentum is conserved.

We can now perform different types of measurements on the particles. Let us think of an experimental setup which measures S_z^1 for the particle P_1. There are only two possibilities: either we get the value $+\frac{\hbar}{2}$ and the state of the system changes to the corresponding eigenstate $|\uparrow\rangle_{1z}|\downarrow\rangle_{2z}$ or we get the value $-\frac{\hbar}{2}$ and the state changes to the eigenstate $|\downarrow\rangle_{1z}|\uparrow\rangle_{2z}$. Both these situations are equally probable.

In each case the second particle P_2, after the measurement on the first particle P_1, is forced into an eigenstate of the operator S_z^2. Therefore, if we make a measurement of S_z^2 we are bound to get a value which is predictable *a priori* and is $-\frac{\hbar}{2}$ for the first case and $+\frac{\hbar}{2}$ for the second case.

The particles P_1 and P_2 are non-interacting and can be very far away from each other (one could be on the moon and the other on the earth!). This ensures that the measurement on particle P_1, which allows us to predict the value of S_z^2, does not disturb the particle P_2 in any way. *We thus conclude that S_z^2 is an element of reality for the state $|\psi\rangle_{12}$.*



This prediction can be verified by actually measuring the value of S_z^2.

We have seen that the state $|\psi\rangle_{12}$ has the same form when written in terms of the eigenstates of S_x^j. Using identical arguments, we can conclude that S_x^2 is also an element of physical reality. As a matter of fact, the rotational symmetry of the state $|\psi\rangle_{12}$ allows us to establish that all spin components of particle 2 are elements of reality for this state.

Quantum mechanics however does not predict two-spin components as being elements of reality simultaneously, because of the non-commutation of different spin components. Therefore, these elements of reality we have identified do not have a correspondence in quantum theory and quantum theory seems incomplete. This is in essence, the main thrust of the EPR argument and the paradox immediately suggests the scope for a more complete theory. The original EPR paper considers positions and momenta of two particles and establishes the simultaneous reality of the position and momentum of one of them. The argument was later modified by David Bohm using spin components.

This piquant situation can be summarised in the following way: Let us assume that pairs of particles are being produced in the above-mentioned singlet state. From every pair produced, one particle comes to my lab (on earth) and the other goes to my friend's lab (on the moon). For every particle I get, I measure some spin component and my friend too measures the *same* spin component on her particle. For both of us, the values $\pm\frac{\hbar}{2}$ are equally probable and after making measurements on a large number of particles we do get a sequence with $\pm\frac{\hbar}{2}$ occurring randomly with equal probability. If we now compare our lists of readings we find that, whenever I got a value of $+\frac{\hbar}{2}$, she got a $-\frac{\hbar}{2}$ and vice versa, irrespective of the spin component chosen. Thus it appears as if my values were known to my friend even before I performed my measurement, in which case I can conclude that my values are all elements of physical reality!

Bohr's Reply and Later Developments

Soon after the EPR paper(*Box* 2), Niels Bohr wrote a paper as a rejoinder to the EPR argument, in which he detailed the Copenhagen interpretation of quantum mechanics. He argued that the EPR paradox does not prove the incompleteness of quantum theory and that the statement "*in no way disturbing the system*" has to be interpreted differently in quantum mechanics. A measurement on particle P_1 changes the very circumstances in which the later measurements on particle P_2 are being conceived. Therefore one cannot say that the values of spin components can be predicted without disturbing the particle. Though Bohr's rejoinder came within a few months of the EPR paper, it was not widely accepted and the controversy still continues! The EPR

Suggested Reading

[1] A Einstein, B Podolsky, and N Rosen, Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?, *Physical Review* 47, 777, 1935.

[2] N Bohr, Can Quantum-Mechanical Description of Physical Reality be Considered Complete?, *Physical Review*, 48, 696, 1935.

[3] Max Jammer, *The Philosophy of Quantum Mechanics*, John Wiley and Sons, New York, 1974.

[4] J J Sakurai, *Modern Quantum Mechanics*, Addison-Wesley, Massachusetts, 1994.

[5] J S Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, Cambridge, 1987.

Box 2. Origin of the EPR Paper

The EPR paper finds its origins in the discussions of Bohr and Einstein, on the foundations of quantum theory, which began in the 1920s and reached their pinnacle during the fifth and sixth Solvay Conferences. The Bohr–Einstein dialogue is one of the great scientific debates in the history of physics and ensued mostly in the unofficial sessions of the Solvay Conferences. During the latter half of 1933 Einstein joined his new position in Princeton and brought Walter Mayer from Berlin to work with him. However, Mayer soon obtained an independent position and Einstein began looking for the assistance of young scientists. Boris Podolsky, who had already worked with Einstein and Tolman in 1931 on issues dealing with the Uncertainty Principle and the distinction between the past and the present in quantum mechanics, had left shortly afterward to work with Vladimir A Fock (and Lev D Landau) on quantum electrodynamics. In 1933, he returned on a fellowship to the Princeton Institute and Einstein became interested in him. In 1934 Nathan Rosen, who had obtained his PhD in atomic physics from MIT under J C Slater, began to work at Princeton University. Venturing one day to enter Einstein's office, Rosen was surprised by the friendliness with which Einstein inquired about his work. When on the following day he met Einstein in the yard of the Institute, Einstein said to him: "Young man, what about working together with me?". Shortly thereafter Rosen became a research fellow at Einstein's department. This then is the story of how Podolsky and Rosen joined Einstein.

paradox has generated much debate and a large number of research papers.

People continue to work on the resolution of this paradox. In the beginning, it was felt that one ought to develop a theory which is complete, and such theories called hidden variable theories, were attempted. The most important development in this context has been through the work of John Bell. He investigated the possibility of constructing hidden variable theories which respect locality i.e. do not allow instantaneous action-at-a-distance. He found that such theories are inconsistent with the statistical results of quantum mechanics. It is now very clear that the quantum world is indeed very strange and is beyond our naive intuition in more ways than one.

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