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Physics of Mass and Force

Let me start by stating what a scientist does. He does three things.

1. He observes events: these events may occur on their own or he arranges for them to occur.
2. He analyses the observations.
3. From the observations he extracts simple but general rules which describe the observations and predict future events as well. These simple rules are the laws of science or the laws of nature.

Among the laws of nature, three of the most profound laws are Newton's laws of motion. They govern the dynamical behaviour of a large class of objects under very general conditions. In particular they introduce the profound concepts of *mass* and *force* which play an essential and important role in the dynamics of objects.

Before we introduce Newton's law of motion in general and the ideas of mass and force in particular, we must clearly understand the essential points of dynamical events. In a dynamical event, we observe some objects at some specific positions and also those positions at different times. For recording these events we need a *space-time reference frame*

A space-time frame of reference is defined in terms of three coordinate axes, e.g. x , y , z axes, and a clock. In terms of this frame of reference, we can describe the positions and the time evolutions of a set of objects. From the time evolution of positions we deduce two useful quantities. Firstly we have the velocity of an object which is the rate of change of position with respect to time. In practice we get the velocity by observing positions at two times close to each other and dividing the change in the position vector by the time interval. Similarly, we have acceleration of an object which is the rate of change of the velocity with respect to time, again obtained by dividing the change in the velocity vector by the time interval.



Newton *analysed* the observations of the motions of objects made over centuries and deduced three basic laws of nature which govern them. Now we state Newton's first law of mechanics.

A body continues to remain in a state of rest or moves with a constant velocity unless it is acted on by an unbalanced force.

This law introduces a qualitative description of the effect of a force. We describe force as pull or push. The law effectively states that an unbalanced force on a body causes a change in its velocity. Of course, the statement is qualitative, with no specific numbers. It just provides an intuitive feeling about the effect of a force, that it causes a change in the velocity. The quantitative aspects of the force are provided by the second law. The second law effectively states:

The net force on a body is equal to mass of the body times its acceleration, $\mathbf{F} = ma$.

This law introduces two important quantities, the mass of a body and the force. And it introduces them quantitatively. It states that the two are related in terms of the acceleration of the body. But what precisely does it tell us? In a sense it tells us nothing.

It is like equating two unknown quantities; for example, suppose I say bink of a body is equal to the tink. This statement does not mean any thing. And yet the statement that mass times acceleration is equal to force is one of the most profound statements and it acquires profundity because of some additional properties mass and forces possess. These properties are:

1. Mass is an intrinsic property of a body. This was implicitly assumed by Newton in writing his second law.
2. The force has an independent representation, not just as mass times acceleration. For example, the force exerted by a spring depends only on the amount of the extension. To a good approximation it is proportional to the amount of the extension.



We have Coulomb and gravitational forces which are proportional to the inverse square of the separation between objects.

Newton also observed another general property which he stated in his *third law*:

Forces exerted by two bodies on each other are equal and opposite to each other. With these additional properties of the *mass* and *force*, Newton's second law becomes very powerful and in particular provides a clear definition of mass and force.

(a) We can determine the relative masses of bodies once for all, by subjecting them to forces of known form, e.g. if we subject them to a spring force with the same extension, they are subjected to the same force. We then have

$$\begin{aligned}m_1 a_1 &= F \\m_2 a_2 &= F \\m_2 &= m_1 a_1/a_2\end{aligned}$$

This result is based on a *specific property* of forces, e.g. spring forces.

(b) We can determine the magnitude of a force by observing the acceleration it produces in a body of known mass:

$$F = m_1 a_1.$$

This result is based on the property that the *mass* determined before *remains unchanged* in the new situation.

Thus the concepts of mass and force become meaningful in the context of the second law of Newton with the additional properties that the masses are an intrinsic property and the forces have independent representations.

In the discussion of Newton's laws there is one background concept which requires careful handling. This is the concept of a frame of reference. To record the positions and movements in a meaningful way, we need a frame of reference which can be taken to be three coordinate axes, x, y, z axes, and a time clock.

However, it is obvious that different frames of reference require different descriptions of forces. Consider two frames of reference, one attached to an observer on the platform of a station, another attached to a moving train. Now if the driver of the moving train suddenly applies brakes, the passengers are thrown in the forward direction, as if there was an additional force in the forward direction. The need for the additional force is dictated by the sudden deceleration of the train. Thus the form of the required forces is different in the reference frame on the platform and in the reference frame attached to the train. A frame in which the forces are the simplest (for example, forces are absent when all the other bodies are far away) is called an inertial frame.

The importance of an inertial frame is illustrated by the following simple experiment. Consider a cool, clear, winter night and you go out in the open field. You see a clear sky with stars shining. Let there be a circular platform which can rotate about the vertical axis. You get on the platform, go to the centre and stand there with closed eyes. Now let the platform start rotating. You feel your arms being pulled out. As the speed of rotation changes you feel different amounts of the outward force which of course is the so-called centrifugal force. At some stage you again feel that there are no forces. So you say, this is the *best frame* with no forces other than the gravitational forces. Now you open your eyes, and find that the stars are not rotating. Is this a coincidence, that the best frame, the inertial frame, is the one which is not rotating with respect to the stars and the galaxies? *Mach* felt that it was not a coincidence. He went ahead and suggested that the inertial frame is the one which is at rest with respect to the mass distribution in the universe.

Clearly, mass and force are beautiful concepts, whose importance is stated in Newton's laws of motion supported by some additional properties that mass is an intrinsic property and forces have independent representations. In the understanding of the deeper properties of the mass and force, the meaning of inertial or best frames of reference in the universe plays an important role.

