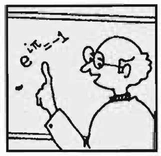

In this section of Resonance, *we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.*

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Poincaré Sphere

Introduction

In devices that exploit the optical anisotropy of materials we have to work with polarised light. In going through such devices the polarisation state of the incident light changes. The *Poincaré sphere* is very helpful in working out the state of polarisation of the emergent light. Unfortunately many textbooks do not discuss this beautiful contribution of Henri Poincaré to polarisation optics. Here we dwell upon some of the charming features of this sphere.

We must first of all understand what we mean by the polarisation state of light. We briefly address ourselves to this question here. Light is an electromagnetic wave. The electric and magnetic fields associated with it will be oscillating in mutually perpendicular directions. These directions are in turn perpendicular to the direction of propagation. The electric field is responsible for most of the phenomena attributed to light. The way the electric field evolves in space and time describes the polarisation state of light. For example, in an elliptically polarised light the electric vector traces out an ellipse at any point in space. Further, at any instant of time the tips of the electric vector lie

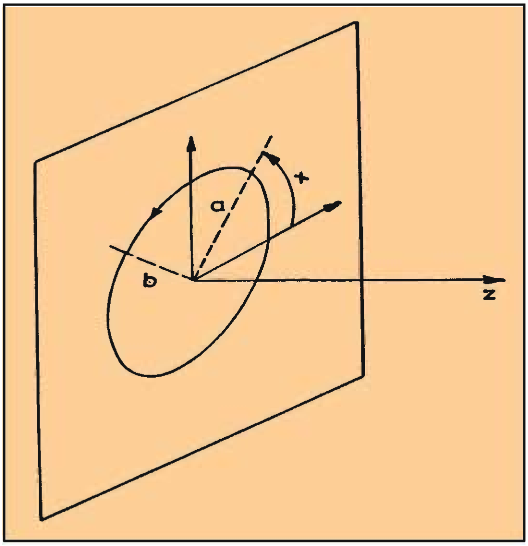


Figure 1.

in space on a helix of elliptical cross-section. It is obvious that both the linearly and circularly polarised states are special cases of the elliptic state. An elliptical vibration can be uniquely described by giving its (i) intensity-I, (ii) azimuth-X and (iii) ellipticity, $e = \tan^{-1}(b/a)$(see *Figure* 1). The ellipticity is taken to be positive (negative) for a left handed (right handed) ellipse which has its electric vector rotating in an anticlockwise (clockwise) direction as seen by a person looking at the source of light.

The Problem

As said already, the state of polarisation of light gets affected when it passes through some optical devices. Interestingly some familiar materials often imitate such devices. Here, we present two well-known examples.

1. A scotch (cello) tape is practically colourless. But between crossed polaroids it appears generally as a band of uniform colour. The colour depends on the thickness of the tape[1]. As the tape is turned in its own plane, in two perpendicular settings it becomes 'extinct'. An analysis of this colourful phenomenon implies that the linearly polarised light beam (produced by the polaroid) on entering the tape gets resolved or 'split' into two base states. These base states travel in the same direction but

[1] See article by Vilas Gohad, Polarization of Light: An Experimental Approach, *Resonance*, Vol. 2, No.9, 1997.

with different velocities. Also, they have their electric fields oscillating in two mutually perpendicular directions. Then it can be shown (try this problem) that the two vibrations recombine, on emergence, to yield an elliptically polarised light. There are many media behaving like a scotch tape. All such media are said to be linearly birefringent.

2. Our second example is a solution of sugar in water. It has a spectacular optical property. A linearly polarised light undergoes a continuous rotation of its plane of polarisation as it travels through the medium. It was Fresnel who demonstrated in 1817, that any incident state of polarisation gets resolved into right and left circularly polarised light travelling with different velocities. Again there are many examples of media exhibiting this property. These are said to be circularly birefringent or possess optical activity. Calculation (try this problem also) in this case shows that the ellipticity of the incident beam is unaltered but its azimuth changes as it emerges from the medium.

If we consider absorbing media we get complications due to polarisation dependent attenuation of light intensity. Such media are said to be dichroic. There are also cases wherein linear birefringence, circular birefringence and dichroism get combined. We can in principle again work out the emergent state of polarisation if we know the optical characteristics of the device. The procedure, though straightforward is very elaborate. It is here that the Poincaré sphere comes in handy. It helps us to quickly arrive at the essential ingredients of the answer. Of course we can get even the exact answer provided we are prepared to work with spherical trigonometry.

Representation of Polarisation States

Poincaré pointed out in 1892, that the state of polarisation can be uniquely represented by a point P on a sphere of radius I. The *longitude* of this point is $2X$ and its *latitude* is $2e$. The upper hemisphere represents positive and the lower hemisphere the negative values of e. *Figure* 2 depicts this sphere, called the

Poincaré sphere. It is instructive to look at some special points on the sphere. The 'north' pole represents a left circularly (L) polarised state and the 'south' pole a right circularly (R) polarised state. Any point on the equator represents linearly polarised state and two diametrically opposite points represent orthogonal states of polarisation.

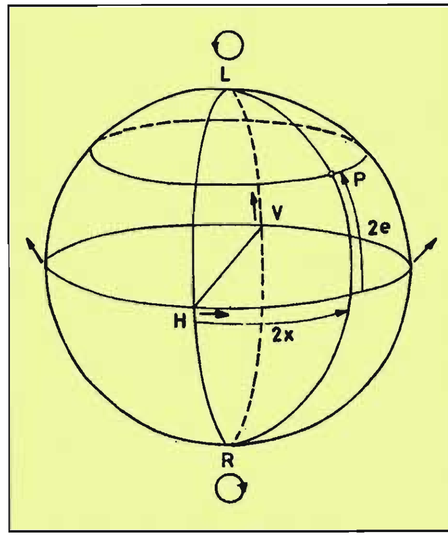


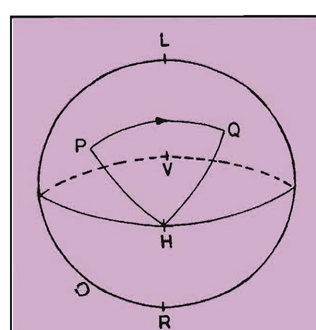
Figure 2.

How to Play with the Poincaré Sphere

It is natural to wonder what utility this representation has. We answer this question here with the two phenomena already alluded to.

Linear Birefringence: In this case the two orthogonal linear base states are represented by the diametrically opposite points H and V on the equator of the Poincaré sphere. For simplicity, assume that the H state travels slower than the V state. Let the incident light be elliptically polarised. This is represented by the point P on the sphere. Now, rotate the point P about the HV axis in the clockwise direction (as seen from H to V) through an angle equal to the phase difference between the H and V vibrations. This process is shown in *Figure* 3. If the H state travels faster than the V state then the rotation will be anticlockwise. The point Q obtained at the end of this operation represents the elliptic light emerging from the medium. Obviously if the incident light is one of the base states (H or V in the present case) its state of polarisation remains unaltered. Further, linearly polarised light emerges as elliptically polarised light. Now we come to the colourful phenomena seen in a scotch tape. The birefringence of the tape depends on colour. Maximum light is transmitted between crossed polaroids when the phase difference is 180°. Thus the tape appears with this colour.

Figure 3.



Circular Birefringence: In this case the orthogonal base states are circular and they are represented by the points R and L on the Poincaré sphere. The effect of the medium on an incident state of polarisation P is given by rotation about the RL axis. The rotation is again through an angle equal to the phase diffe-

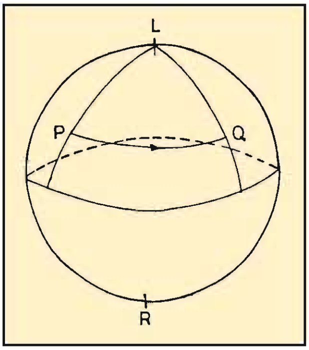


Figure 4.

rence between the R and L states. Thus only the longitude of the point P (and hence of the incident light) changes. *Figure* 4 depicts this operation. It is not difficult to see that for incident linear light this process leads to a rotation of the plane of polarisation in a solution of sugar.

A Double Trouble

What if the medium has both linear and circular birefringence? Crystalline alpha-quartz has this feature. From the recipe described earlier, the incident state will have to rotated about the HV axis followed by rotation about the RL axis or vice versa. This is quite a messy procedure. There is an easy way out. It is not difficult to show (try this exercise) that under the combined influence of both these operations, two diametrically opposite points A and B, on the great circle HLVR shown in *Figure* 5 representing orthogonal elliptic states, remain unaltered in position. Thus these are the base states for the medium. Hence, the effect of such a medium on any incident state is given by rotating the incident state about the AB axis through an angle equal to the phase difference between A and B states.

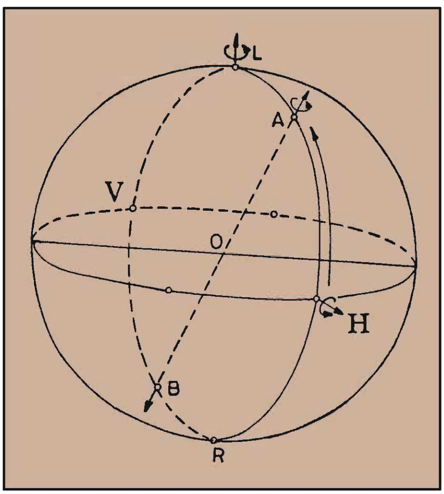


Figure 5.

Concluding Remarks

We end this article on a note that will hopefully inspire young readers. S Pancharatnam while he was a research student working under C V Raman discovered the operation, on the Poincaré sphere, that represented dichroism. He also discovered what is now well known as the Pancharatnam phase, which can be easily visualised on a Poincaré sphere. But a separate 'Classroom' piece would be necessary to highlight his important results .

One may be a mathematician of the first rank without being able to comput . It is possible to be a great computer without having the slightest idea of mathematics.

Novalis

Quantum Chemistry and Spectroscopy