The Nobel Prize in Physics 1999

Rohini M Godbole

Rohini M Godbole is a Professor (and currently Chairperson) at the Centre for Theoretical Studies, Indian Institute of Science, Bangalore. She works on high energy physics phenomenology. She hopes that the fact that she can see eye to eye with some of the younger *Resonance* readers, might help her in communicating to them what this strange sounding field of research really means.

The last Nobel Prize of the millenium in Physics has been awarded jointly to Gerardus 't Hooft of the University of Utrecht in Holland and his thesis advisor Martinus J G Veltman of Holland. According to the Academy's citation, the Nobel Prize has been awarded for 'elucidating the quantum structure of electroweak interaction in physics'. It further goes on to say that they have placed particle physics theory on a firmer mathematical foundation. In this article, we will try to understand both these aspects of the award. The work for which they have been awarded the Nobel Prize was done in 1971. However, the precise predictions of properties of particles that were made possible as a result of their work were tested to a very high degree of accuracy only in this last decade. To understand the full significance of this Nobel Prize, we will have to summarise briefly the development of our current theoretical framework about the basic constituents of matter and the forces which hold them together. In fact the path can be partially traced in a chain of Nobel Prizes starting from one in 1965 to S Tomonaga, J Schwinger and R P Feynman, to the one in 1979 to S L Glashow, A Salam and S Weinberg and then to C Rubbia and Simon van der Meer in 1984 ending with the current one.

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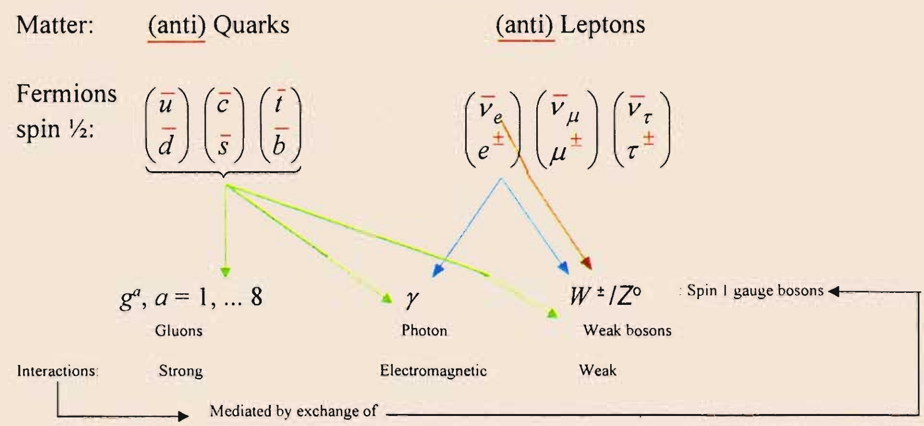
In the article on 'Search for a final theory of matter' in the previous issue, Ashoke Sen has described the 'Standard Model (SM)' of particle physics, wherein he has listed all the elementary particles according to the SM. These consist of the matter particles: the quarks and leptons along with various vector bosons γ, $W^{\pm}$, Z^0 and gluons g which mediate the various interactions between them. *Box* 1 summarises them here again for the sake of completeness. As explained in that article, one of the conceptual cornerstones of the current description of particle physics is the fact that an interaction (say Coulomb) between two elementary particles (say electrons) can be understood *either* (i) as

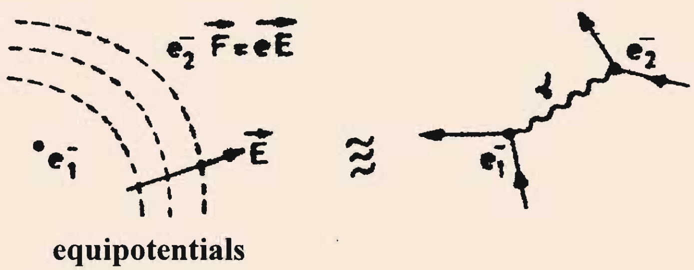
Box 1.



the effect of the force field generated by one of them on the other one *or equivalently* (ii) as arising due to an exchange of the carrier of the force (photon in this case) between them. The photon is the 'quantum' of the electromagnetic field. The range as well as the dependence of this force on the relative spins and positions of the particles is correlated with the properties of this 'quantum' (exchanged particle) and this can be established in a well defined mathematical framework. *Box* 2 depicts this equivalence in a pictorial manner.

Box 2.

In the picture on left e^-_2 moves in the electric field generated by e^-_1. In the picture on right e^-_1, e^-_2 interact by exchanging a photon γ.



We know that just as Newtonian mechanics is the right mathematical framework to describe the terrestrial and celestial motion, quantum mechanics is the right language to describe the motion of molecules, atoms, electrons, neutrons/protons at the molecular/subatomic and subnuclear level. If we want to describe, in addition to these, creation and annihilation of particles, e.g. as it happens in the spontaneous emission of a photon by an atom, we need to further extend this mathematical framework to the next higher level of sophistication called 'Quantum Field Theory' (QFT). In QFT not only do we employ fields to describe the carriers of interactions, the matter particles are also described by matter fields.

Another cornerstone of our theoretical understanding of the fundamental particles and their interactions is the realization of the important role played by symmetries/invariances. The idea of symmetries can be understood in the following way. Laws of physics, let us say $\mathbf{F} = m\,(d^2\mathbf{x} / dt^2)$, should be the same no matter which point in the universe we choose as the origin of our coordinate system. This means that the physics is unchanged under a change of the origin of the coordinate system. This is expressed by saying that the system is invariant under a transformation of coordinates involving translations in space. As per our current understanding, underlying fundamental invariance principles actually dictate the forms of interactions. Let us understand it by taking the example of gravitation. Newton deduced the law of gravitation from the observation of motion. On the other hand Einstein wrote down the general theory of relativity by postulating that the description of motion should be the same for two observers employing two coordinate systems which are related to each other by a general transformation. In particular the transformation can be different at different points in space-time. The non-relativistic limit of this theory (i.e. when objects move at speeds much lower than light) contains Newton's theory of gravitation. Thus the 'general coordinate invariance' 'explains' the laws of gravitation 'deduced' by Newton. So in some sense we have a theoretical understanding of an observed law of nature in

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terms of a deeper guiding principle. The tenet of current theoretical description of particle physics is that the quantum field theories which have certain invariances are the correct theoretical frameworks for this description.

The invariance that is most relevant for the discussion here is the so called 'local gauge invariance'. Without going into the details of the idea, let us just note that this is basically a generalization of the idea that in electrostatics the electric field and hence the electrostatic force depends only on the *difference* in potential and not on the actual values of the potential, i.e. setting of zero of potential scale is arbitrary as far as the force is concerned.

Quantum field theories, though now *the* accepted framework for describing particles and their interactions, were in the doghouse for a long time in the 30's and 40's because they used to predict nonsensical, infinite results for properties of particles when one tried to compute them accurately. The difficulties arise essentially because of the nontrivial structure that the vacuum has in QFT. This can be visualized by thinking about the effect that a medium has on particle properties; e.g. the transport of an electron in a solid can be described more easily by imagining that its mass gets changed to an 'effective' mass. Another example is the polarization of the charges in a dielectric medium caused by a charged particle. This polarization can cause a 'change' of the charge of the particle. In QFT, vacuum acts as a nontrivial medium. The troublesome part, however, is that when one tries to calculate this change in the charge due to the 'vacuum polarisation', one gets infinite results. Tomonaga, Schwinger and Feynman (who shared the Physics Nobel Prize in 1965) put the quantum field theoretic description of the electron and photon (quantum electrodynamics) on a firmer mathematical footing They showed how one can use the theory to make sensible, testable predictions for particle properties (such as a small shift in the energy level of an electron in the hydrogen atom due to the effect of vacuum polarization), in spite of these infinities. If this can be done always in a consistent manner, then the corresponding QFT is said to be *renormalizable*. The point to

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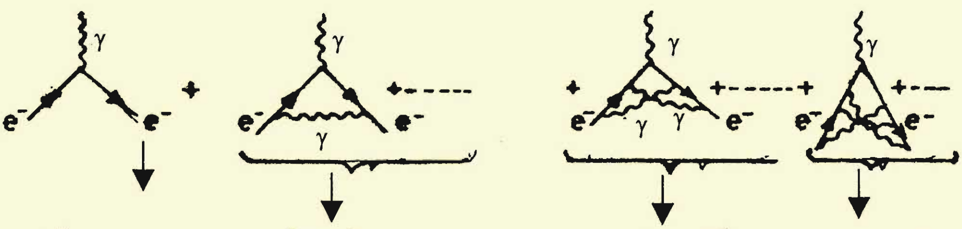
note is that the 'local gauge invariance' mentioned earlier was absolutely essential for the proof of renormalizability of quantum electrodynamics (QED).

The best example where the predictions of this theory were tested to an unprecedented accuracy is the measurement of gyromagnetic ratio[1] of the e^- viz. g_e. This is predicted to be based on a quantum mechanical equation which is written down with the requirement that the description of the e^- is the same for two observers moving relative to each other with a constant velocity. (Dirac equation for the cognoscenti). The experimentally measured value is close to 2 but differs from it significantly. In QED one can calculate the corrections to the value of $g_e=2$ coming from effects of interaction of the electron whereby it emits a γ and absorbs it again in a systematic fashion. *Box* 3 indicates some

[1] This is the ratio of the magnetic moment to the angular momentum, both being in suitable units.

Box 3.

Diagramatic representation of corrections to $((g-2)/2)_e$. Not all the diagrams at each order are drawn.



$$\left(\frac{g-2}{2}\right)_e^{th} = 0 + \left(\frac{\alpha_{em}}{2\pi}\right) + (-0.328478965)\left(\frac{\alpha_{em}}{\pi}\right)^2 + 1.17611(42)\left(\frac{\alpha_{em}}{\pi}\right)^3$$

$$-1.434(138)\left(\frac{\alpha_{em}}{\pi}\right)^4 + \delta$$

$$= 1159652140\ (28) \times 10^{-12} \qquad (1)$$

$\alpha_{em} = 1/(137.06)$ is the fine structure constant. Only a representative diagram at each order has been drawn. The contributions written in the first line of (1) are of course the sum of all the diagrams at a given order. Here δ represents corrections due to particles other than just e^-s and photons. The numbers in brackets at the end of the coefficient of the third and fourth term are the theoretical errors. Note that this prediction agrees with the experimentally measured value upto 10 decimal places as shown below.

$$((g-2)/2)_e^{expt} = (1159652193 \pm 10) \times 10^{-12}$$

of these corrections. The measured value agrees with the theoretical prediction to 11 significant digits as shown in *Box* 3.

Thus to summarize so far, the electromagnetic interactions between the electron and photons can be described in terms of a QFT. The description has immense predictive power due to the property of renormalizability that the theory has. The theory has this property only because of its invariance under a set of transformations called U(1) local gauge transformations.

With this, we come to a point in history in 1971 when particle physicists had a unified description of electromagnetic and weak interaction in terms of exchange of γ, $W^{\pm}$ and Z^0. S Weinberg, A Salam and S Glashow later shared the Nobel Prize in 1979 for putting forward this EW model. Just as unification of electricity and magnetism by Maxwell had predicted the velocity of light 'c' in terms of the dielectric constant ε_0 and magnetic permeability and μ_0 of the vacuum, this unification predicted values of masses M_W, M_Z in terms of the ratios of two coupling strengths, called $\sin^2\theta_W$. These coupling strengths are the analogue of the electric charge in QED. Details of these relations are displayed in *Box* 4. C Rubbia and Simon Van der Meer shared the Nobel Prize in 1984 for discovering the $W^{\pm}$ and Z_0 bosons with masses and decays as predicted by the EW model.

Left: Gerardus 't Hooft

Right: Martinus J G Veltman

Box 4.

SM relation between M_W, M_Z and $\sin^2\theta_W$:

$\sin\theta_W$ is defined in terms of a ratio of two coupling strengths, which are the generalisations of electric charge e of the electron. At the lowest order

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \qquad (1)$$

Including the dominant correction due to top quark (1) becomes

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 + \rho_t = 1 + \frac{3G_F m_t^2}{8\sqrt{2}\pi^2}\,. \qquad (2)$$

G_F here is the Fermi coupling constant related to lifetime of the muon μ and is known to be 1.166×10^{-5} GeV^{-2}. Corrected prediction for ρ then is

$$\rho^{\text{th}} = 1 + 0.0031\left(\frac{m_t}{100\text{GeV}}\right)^2. \qquad (3)$$

Value of ρ determined experimentally is

$$\rho^{\text{expt}} = 1.011 \pm 0.006\,. \qquad (4)$$

Using the measured value of m_t this measurement can then check EW theory just as $((g\text{–}2)/2)_e$ of *Box* 3 checks QED.

The $W^\pm$, Z^0 bosons were found to have nonzero masses (M_W = 80.33 ± 0.15GeV, M_Z = 91.187 ± 0.007GeV where 1 GeV is approximately the mass of a proton). As a result the early efforts to cast this electroweak model in the framework of QFT by using a more complicated gauge invariance suggested by a generalisation of QED met with failure. Their nonzero mass makes a QFT incorporating these bosons noninvariant under these gauge transformations. This makes the theory nonrenormalizable. This means calculating corrections to the relation 1 in *Box* 4 is again riddled with infinities.

At around the same time P Higgs and others had proposed a way to write a QFT of *massive* $W^\pm$, Z^0 bosons, where the mass term did not spoil the gauge invariance of the theory. This required existence of an additional particle called the Higgs boson. This is where 't Hooft and Veltman stepped in. 't Hooft demonstrated, in his thesis work and the paper published in *Nuclear Physics* B in 1971, that the QFT with massless $W^{\pm}$ and Z^0 was renormalizable and the invariance of the theory under more complex noncommutative local gauge transformations was essential for that. He further showed that a QFT containing *massive* $W^{\pm}, Z^0$ bosons would be renormalizable (i.e., coefficients of infinite corrections would vanish identically) *inspite* of nonzero masses as long as the mass was generated through the mechanism suggested by P Higgs. Together 't Hooft and Veltman developed new methods of calculation for the higher order corrections to particle properties, which explicitly preserved this gauge invariance. This work opened the floodgates of the prospects of using the electroweak theory to make accurate predictions and test the theory to a similar degree of accuracy as the QED (*cf Box* 3). Veltman led the program of calculation of various higher order corrections to EW quantities, having established that the results were guaranteed to be finite. He actually developed a computer program called 'Schoonship' to use the computer to do these very complicated analytical calculations specific to theoretical high energy physics. This is the sense in which the work of 't Hooft and Veltman put the EW theory on a firm mathematical footing. This work was enough to convince particle theorists that gauge theories with Higgs mechanism was the way to go to describe EW interactions.

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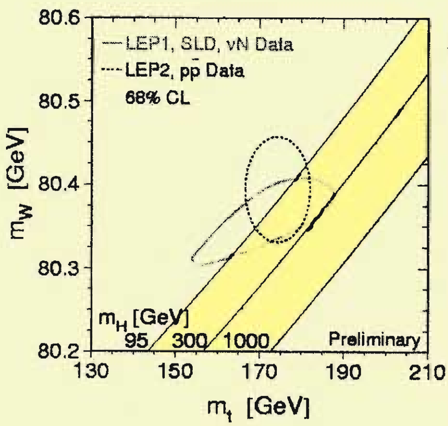
In QED the corrections (e.g. to $((g-2)/2)_e$ shown in *Box* 3) depend only on the mass and charge of an e^-, whereas in EW theory they depend on the free parameters of this theory viz. the masses of various quarks and leptons. The corrections are dominated by the top quark due to its large mass. *Box* 4 shows the leading corrections predicted in the EW theory to the ratio $\rho = M^2_W / M^2_Z \cos^2 \theta_W$. The measurement of M_W / M_Z and $\sin^2 \theta_W$ in 1984 were consistent with $\rho = 1$ which was the analogue of $g=2$ prediction of QED. The measurements then were not precise

Box 5.



The region enclosed by the red curve is the region in m_t–m_W plane allowed in the SM using all the EW measurements excluding directly measured value of m_t from $\bar{p}p$ collider experiments. The region enclosed by the green circle is obtained when this information is included. The overlap of the two clearly vindicates the SM. Blue straight lines indicate dependence on the mass of the Higgs m_H.LEP1, LEP2 correspond to two sets of experiments at the large electron positron collider LEP.

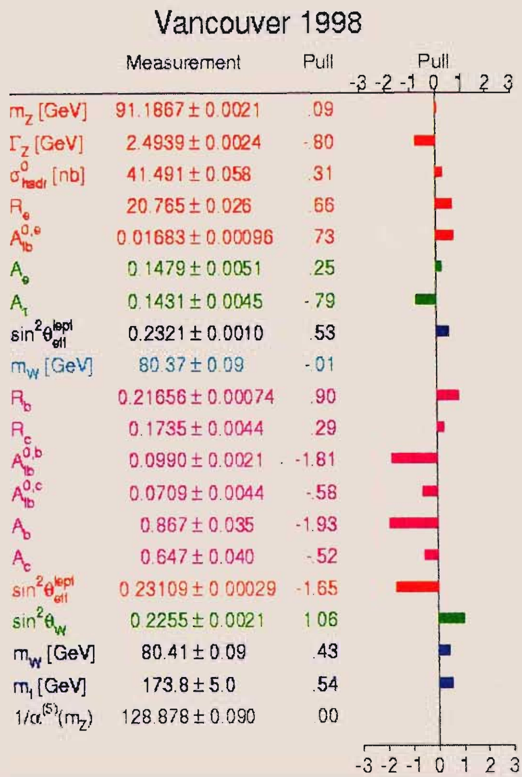
enough to decide the deviation of the experimentally measured value of ρ from 1. In the decade since then, a detailed study of the properties of these bosons has been possible using the 10 million Z^0 bosons created at the Large Electron Positron Collider (LEP) in Geneva and thousands of $W^{\pm}$ bosons at the $\bar{p}p$ collider Tevatron at Fermilab in Chicago. By 1993 ρ was found to be 1.011± 0.006. This implied, as can be seen from the *Box* 4, that the top quark, which was not discovered till 1995, must have a mass M_t ~180GeV. Finding the top quark in 1995 with a mass consistent with this value indeed tested the predictions of the EW theory to high accuracy. The precision of these measurements meant that if one did not use the corrected expressions, the values of M^2_W, M^2_Z and $\sin^2\theta_W$ would not be consistent with each other within the SM.

Even though not shown in *Box* 4, the corrections to this ratio also depend on the mass of the *only particle in the SM which is as yet undiscovered* viz. the Higgs boson, albeit very weakly. The figure in *Box* 5 shows the region in the $M_W - M_t$ plane that is indicated by measurements today. The straight line shows predictions of the SM for different values of the Higgs boson mass. So just as five years ago, one used these measurements to 'determine' values of M_t (which was then not measured) now particle physicists are using them to 'determine' the mass of the elusive Higgs particle. These precision measurements narrow down the mass range where the Higgs boson is likely to be found if SM is indeed completely correct. Hunt for this will be on at the Large Hadron Collider (LHC) which will go into action in 2006.

The EW theory predicts a slew of measurable quantities in terms of the basic parameters of the theory viz. the couplings and masses of quarks/leptons. The figure in *Box* 6 shows a compari-

Box 6.

First column lists various observables measured at the large electron-positron (LEP) collider in Z^0 decays and the second column currently measured values (taken from presentation of LEP collaborations). The third column gives differences between the measurements and predictions of the SM in units of standard deviation of each measurement. e.g. central value of m_t differs from the SM prediction by $0.54 * 5 = 2.70$GeV.



son of the predictions of the SM (corrected for these loop effects) with data. The numbers in the third column indicate the differences between the prediction and measurement in units of the standard deviation. It is this agreement, which would be nowhere as excellent if we do not include the higher order corrections, that has proved that the EW interactions are correctly described in terms of a QFT whose renormalizability was established by 't Hooft and Veltman's work. Their Nobel Prize is also the recognition of the success of QFT and Gauge Principle which are the two cornerstones of the mathematical description and understanding of the electromagnetic, weak and strong interactions among fundamental particles. The only part of this edifice that is as yet not honoured with a Nobel Prize is QCD or Quantum Chromodynamics: Gauge theory of strong interactions. Who knows, in a few years we may be reading about the work of D Gross, H Politzer and F Wilczek in a similar article!

Address for Correspondence
Rohini M Godbole
Centre for Theoretical Studies
Indian Institute of Science
Bangalore 560 012, India.
Email:rohini@cts.iisc.ernet.in