Complex Variables – Introduction and Applications

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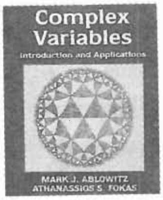

Complex Variables – Introduction and Applications
M J Ablowitz and A S Fokas
Cambridge Texts in Applied Mathematics
Cambridge University, (South Asian Edition published by Foundation Books, New Delhi), pp. 647, Price: Rs 275/-

One of the popular books on science in my college days was George Gamow's *One, Two, Three,... Infinity*. In the section dealing with complex numbers, we find the following quotation from Euler's book on algebra: "All such expressions $(-1)^{1/2}$, $(-2)^{1/2}$ as etc. are impossible or imaginary numbers, since they represent roots of negative quantities, and of such numbers we may truly assert that they are neither nothing, nor greater than nothing, nor less than nothing, which necessarily constitutes them imaginary or impossible."

But inspite of all these excuses and abuses, as Gamow says, complex numbers have now become as unavoidable in mathematics as fractions or radicals and one could practically not get anywhere without them. Both as a research discipline in its own right, and as an analytic tool with numerous applications, complex analysis is now a topic of study in the mainstream of mathematics.

Today, complex analysis is an integral part of any reasonable mathematics curriculum and we have numerous texts on this subject. Thus, writing yet another text capable of making a mark becomes increasingly difficult. The book under review is an addition to this growing collection of texts on the subject and its novelty and value stem from its emphasis on applications.

The book is divided into two parts. The first part covers what is now generally accepted as basic material in any introductory text on complex variables – complex numbers, elementary functions, analytic functions, complex integration, Cauchy's theorem, series and product developments, residue calculus and so on.

Applications form an integral part of this book and start appearing from the very beginning. Differential equations are introduced as early as in section 1.4 and the velocity potential and stream function of ideal fluid flows appear just after the derivation of the Cauchy–Riemann equations. Series solutions to differential equations with regular singular points are discussed in chapter 3. This last mentioned section ends with a slightly more esoteric topic – Painleve equations. Computational issues are frequently addressed.

Simpler proofs are given wherever possible at the expense of generality but in the interest of faster development of ideas. However, all the major results are stated and rigorously proved. For instance, a very short proof of Cauchy's theorem (Theorem 2.5.2) based on Green's formula is first given using the additional assumption that an analytic function is continuously differentiable, a fact later deduced from the Cauchy integral formula (Theorem 2.6.3). A proof without this extra hypothesis, due to Goursat, is outlined in section 2.7.

Discussions of a more abstract or purely theoretical nature are often relegated to a separate section entitled something like 'Theoretical Developments' in several chapters. These sections are starred and can be skipped if one is not interested in such discussions and at the same time they make the book complete as a work of reference.

The second part of this book makes it really very different from other books on this subject. It is devoted to applications of complex analysis. It consists of three chapters (chapters 5–7). Of course, chapter 5 is on a standard topic, conformal mappings but covers a very wide range of applications.

Chapter 6 is indeed one of the most useful ones in the book. It deals with the asymptotic evaluation of integrals, a tool very necessary for analysts. It discusses a variety of techniques such as the Laplace method, the stationary phase method, the method of steepest descent, the WKB method and the Mellin transform method, to quote a few.

Given two function f and g and a closed contour C, the Riemann–Hilbert problem is to find analytic functions ϕ^+ inside C and ϕ^- outside C such that

$$\phi^+(t) - g(t)\,\phi^-(t) = f(t) \text{ on } C.$$

This general problem covers a host of problems from diverse areas like integral equations, partial differential equations, inverse scattering theory and the inversion of the Radon transform. A closely related problem is the DBAR problem which involves solving the equation

$$\partial\phi(x,y)/\partial\bar{z} = g(x,y),\ z = x + iy \in D$$

for a given function $g(x, y)$ in D, a region in the complex plane. These problems occur in connection with some multidimensional inverse problems and some nonlinear partial differential equations. The last chapter discusses the Riemann–Hilbert problem and also the DBAR problem and some applications.

Throughout the book, numerous examples are worked out. There are a wealth of helpful illustrations. Each section has a fairly large collection of exercises and home-work problems. The book also has a useful index and a bibliography.

The book is certainly a must for every library as it is valuable both as a text and as a work of reference. It can be used as a text for a course at the master's level in India. Many Indian universities have now introduced a course entitled MSc (Applied Mathematics). Given the bias towards applications in this book, I would strongly recommend it as a text in complex analysis for such courses.

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