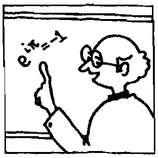

In this section of* Resonance, *we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

IMO 1999 Question Paper

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The problems which appeared in the 40th International Mathematical Olympiad held in Bucharest, Romania between 10-22 July, 1999 are reproduced here. (The Indian team won three silver and three bronze medals, (see *Resonance*, Vol.4, No.9.) Though the contest is meant for high-school students the problems are challenging and would require originality and ingenuity to be able to solve them; they require concentrated and sustained effort for hours or perhaps even days! The problems of the 40th IMO, especially the sixth problem, are regarded as tough nuts even by the IMO watchers; in fact, this year for the first time in the 40 year history of the IMO no student got a perfect score (42 out of 42)! Interested readers are encouraged to attempt these problems (undeterred by possible lack of success!) and send us the solutions.

1. Determine all finite sets S of at least three points in the plane which satisfy the following condition:

   for any two distinct points A and B in S, the perpendicular bisector of the line segment AB is an axis of symmetry for S. (Estonia)

2. Let n be a fixed integer, with $n \geq 2$.

(a) Determine the least constant C such that the inequality

$$\sum_{1\le i<j\le n} x_i x_j (x_i^2 + x_j^2) \le C \left(\sum_{1\le i\le n} x_i \right)^4$$

holds for all real number $x_1, \quad , x_n \ge 0$.

(b) For this constant C, determine when equality holds. (Poland)

3. Consider an $n \times n$ square board, where n is a fixed even positive integer. The board is divided into n^2 unit squares. We say that two different squares on the board are *adjacent* if they have a common side.

   N unit squares on the board are marked in such a way that every square (marked or unmarked) on the board is adjacent to at least one marked square.

   Determine the smallest possible value of N (Belarus)

4. Determine all pairs (n, p) of positive integers such that p is a prime, $n \le 2p$, and $(p-1)^n + 1$ is divisible by n^{p-1}. (Taiwan)

5. Two circles Γ_1 and Γ_2 are contained inside the circle Γ, and are tangent to Γ at the distinct points M and N respectively. Γ_1 passes through the centre of Γ_2. The line passing through the two points of intersection of Γ_1 and Γ_2 meets Γ at A and B. The lines MA and MB meet Γ_1 at C and D, respectively.

   Prove that CD is tangent to Γ_2. (Russia)

6. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x - f(y)) = f(f(y)) + xf(y) + f(x) - 1$$

   for all $x, y \in \mathbb{R}$. (Japan)