times in his lectures. This assessment is indeed correct. The second theme is that India should become a developed nation. In fact, the last sentence of the book is along these lines. The plan to make India an economically strong nation is vital for India to be rated well along with the super powers. For this to happen, many other aspects – the enormous moral decadence leading to corruption in higher levels of society, the inability to prevent this from affecting the common man's life, the inability to improve the quality of life of 70% of the population residing in the villages, the inability to reduce the pace of urbanization and the inability to reduce the inequity between the rich and the poor need to be accounted for. These are of course beyond the preview of one worthy citizen whose life is described in this book.

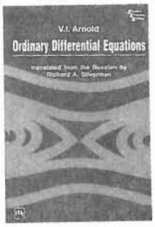
The last chapter entitled 'Contemplation' contains a condensation of ideas and thoughts arising out of his colourful life, the awards that he received (the highest honour, namely Bharata Ratna that he received does not find a mention in this book) and some messages for the future generation.

The book is worthy of being read by every Indian who can read!

H S Mukunda, Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560 012, India.

Ordinary Differential Equations

Shiva Shankar



Ordinary Differential Equations
V I Arnold
(reprinted by Prentice-Hall of India from the MIT Press translation by R Silverman of Obyknovennye Differentsial'nye Uravnenia), Price: Rs.95

"Space and time are commonly regarded as the forms of existence of the real world, matter as its substance. A definite portion of matter occupies a definite part of space at a definite moment of time. It is in the composite idea of motion that these three fundamental conceptions enter into intimate relationship". So declared Hermann Weyl, and this declaration of the supreme importance of the study of motion is at the same time also a declaration of the supreme position that the theory of differential equations holds in mathematics and physics. For, since Newton, it has been clear that the "shape of motion" is determined by local data, encapsulated in what we call a differential equation, and the study of motion is thus synonymous with the study of differential equations. But the shape of motion equally clearly depends on the global shape of (phase) space, and the study of differential equations must therefore be geometric in its methods. This geometric nature of the study of motion was of course clear to Newton (as even a superficial browsing of the *Principia* will confirm), but had to be reaffirmed by Poincaré after a 200 year interregnum, perhaps inevitable, during which time Lagrange could even exult that he had banished figures from the study of mechanics!

The book under review is a first level text in the geometric spirit of Newton and Poincaré and contains 259 illuminating figures. But how is one to start such a study of differential equations? After all, people observed things in motion for many thousands of years before they realized that these trajectories were solutions to differential equations. Naturally then does Arnold start with the formal properties of solutions or phase curves, viz. the concept of a one-parameter group of diffeomorphisms of phase space. This is the collection of all solutions considered (and pictured in one's mind) as a single geometric object. The differential equation itself can then be pictured as a collection of vectors (arrows), at every point tangent to the phase curves (i.e. as a section of the tangent bundle, or a vector field). Thus given a differential equation (i.e. a vector field), to solve it is to fill up phase space with curves, all tangent to the vector field.

With this geometric picture in mind, understanding and progress come easy. One can visualize how the shape of a vector field must change under the action of a diffeomorphism (change of coordinates); one can even convince oneself (without the crutch of a formal proof) the validity of the 'basic theorem of the theory of ordinary differential equations', viz. that in some small neighbourhood of a nonsingular point (i.e. a point where the vector field does not vanish), one can by a diffeomorphism carry it to a constant vector field. Even if the vector field vanishes at some point, one can by adding the equation $dt/dt = 1$, reduce this situation to the one above. Local properties of the flow (existence, uniqueness, dependence on a parameter etc.) can now be easily deduced from the corresponding properties of the trivial constant vector field. A formal proof of this basic theorem, including a discussion on the contraction mapping principle and other related material, is collected in a separate chapter towards the end of the book – one therefore makes rapid progress simply assuming these statements.

Arnold then goes on to discuss in detail the important and transparent case of a one-parameter group of linear automorphisms of euclidean space, i.e. linear differential equations. Starting with the definition of the exponential of a linear map, he goes on to discuss topological conjugacy of linear hyperbolic systems, Lyapunov stability, nonautonomous systems, systems with periodic coefficients and a host of other topics. The book ends with a chapter on differential equations on manifolds, including a beautiful discussion (leading up to almost a proof) of the Hopf index theorem for a flavour of the topological nature of the global theory.

Let me illustrate the beauty and the simplicity of the geometric methods in Arnold's book by the following example. The problem is to solve the initial value problem for the nonhomogeneous linear equation

$$\frac{dx}{dt} = Ax+h(t),\ x(t_0)=c,$$

where x is in $\mathbb{R}^n$ and A is a linear map on $\mathbb{R}^n$. The method is the variation of constants, whose usual explanation of replacing constants with functions in the solution of the homogeneous equation always left me mystified. But consider Arnold's explanation: the solution of the homogeneous equation

$$\frac{dx}{dt} = Ax,\ x(t_0)=c,$$

is $e^{tA}c$. Thus acting on $\mathbb{R}$ at time t by the linear automorphism e^{-tA} moves this solution of the homogeneous equation to the constant curve c, i.e. to the solution of

$$\frac{dx}{dt} = 0,\ x(t_0)=c.$$

As this is a linear change of coordinates, the derivative of this automorphism is again e^{-tA}. Under it the nonhomogeneous equation becomes

$$\frac{dx}{dt} = e^{-tA}h(t),\ x(t_0)=c,$$

whose solution is clearly $x(t)=c\int_{t_0}^{t} e^{-\tau A}h(\tau)d\tau$. Returning by the inverse of this automorphism, i.e. by e^{tA}, yields the solution of the original equation i.e. $e^{tA}c+\int_{t_0}^{t} e^{(t-\tau)A}h(\tau)d\tau$!! Can one praise enough this masterly book by a master mathematician? I cannot. I learnt the subject reading this book and I prescribe it even to engineering students (for courses on control). This is the definitive introductory book on ordinary differential equations; there is no need to read (or write) another.

Shiva Shankar, Dept. of Electrical Engineering, Indian Institute of Technology, Mumbai 400076, India.

ANNOUNCEMENT

An instructional conference on Number Theory will be held at the University of Hyderabad during the period 1–21 December, 1999. The conference is sponsored by the National Board of Higher Mathematics (NBHM). The conference is meant for research scholars and college teachers interested in Algebra/Number Theory. The conference meets the requirements of a course for career advancement. TA/DA will be paid for most participants though research scholars who have fellowships are encouraged to use their contingency grants. Boarding and lodging expenses will be met for all selected participants by the NBHM.

The course will cover basic algebraic number theory and some applications. An attempt will be made to show how Fermat's Last Theorem motivated the development of the subject.

Applicants may apply on a plain sheet of paper giving the following information:
1) Name, 2) Age, 3) Address, 4) Sex, 5) Educational qualifications, 6) Present position, 7) Institution currently studying in or serving, 8) two letters of recommendation, 9) whether accommodation is required (the university campus is about 15 km outside the city of Hyderabad), 10) whether the applicant can bear his/her travel expenses, 11) other information that may be useful to the organising committee. Applications must be sent to: Coordinator NBHM conference on Number Theory, Department of Mathematics and Statistics, University of Hyderabad, Hyderabad 500 046. The last date for submitting applications is 1–11–1999.