Second Sound

2. The Role of Elastic Waves

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In Part 1[1] we described the propagation of temperature and entropy as waves in superfluids emphasizing the experimental demonstration of their existence. Here we dwell upon a microscopic view of these waves and point out that they can exist even in crystals at low temperatures.

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[1] Part 1. Waves of Entropy and Temperature, *Resonance*, Vol. 4, No.3, 1999.

Excitation Spectrum of Liquid He II

In Part 1 we discussed the two fluid model for superfluid helium. What are the two fluids postulated in Tisza's model? In a system obeying Bose statistics, below a temperature T_c, a large fraction of the atoms condense into the ground state. This condensed state has zero entropy. Atoms in this Bose condensed ground state constitute the superfluid component. LHe II will have excitations to higher energy states in which groups of atoms move together. These are called collective excitations. At a finite temperature, some of the atoms are excited to these collective excitations. They account for the thermal properties like entropy and specific heat. These atoms in the excited states constitute the normal fraction. Since at any temperature, atoms will be present both in the ground and excited states in thermal equilibrium, it is not possible to separate the normal and superfluid components by any means.

To explain superfluidity in LHe II Landau postulated two types of collective excitations in LHe II. The collective excitations are waves, each wave having a frequency ν. For normal sound waves we know that $\nu = c/\lambda$, where c is the wave velocity of sound and λ is the wavelength of the sound wave. One often uses $\omega = 2\pi\nu$ and $q = 2\pi/\lambda$ as parameters describing the wave. For a sound wave a plot of ω against q will be a straight line. The graph of ω as a function of q is called a dispersion curve. Landau postulated

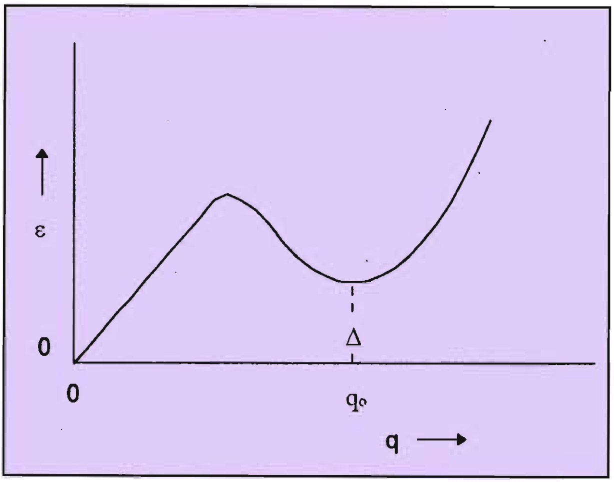

Figure 1. Dispersion curve in LHe II.

a dispersion curve for liquid He II having the shape shown in *Figure* 1. For values of q near zero, the dispersion curve is linear as in the case of familiar sound waves. Just as light waves sometimes behave as particles called photons, sound waves sometimes behave like particles called *phonons*. They have a velocity $c_1 = \omega/q$. These excitations are the normal or first sound waves. They involve fluctuations in the total density of the liquid. There is a second type of excitation around a wave vector q_0. For these excitations the energy varies with q as a parabola around the point q_0. These excitations are called *rotons*. Landau's surmise about these excitations in LHe II has been verified experimentally.

The phonons and rotons contribute to the thermal energy and entropy of the liquid. As the temperature falls below 1K, the phonons are present in greater numbers than rotons which have a much higher energy. At such temperatures the contribution to the thermal properties of the liquid comes entirely from the phonons.

Phonon–Phonon Interaction

Generally mechanical vibrations are simple harmonic for small amplitudes of oscillation as the restoring force will be proportional to the displacement in such a case. But as the amplitude of vibration increases, the restoring force deviates from this linear behaviour in displacement. This is called *anharmonicity*. Due to anharmonicity there will be interactions between the waves, or equivalently, the phonons. Each phonon can be considered as a particle carrying an energy $h\omega$ and momentum hq. The lowest order interaction is a collision involving three phonons. In such interactions (collisions) energy and momentum are conserved. Such a process of scattering is called a *normal* process or N process. Phonons are mechanical vibrations of the atoms. With

There are two types of collective excitations in liquid He II, *phonons and rotons.*

increase in temperature the atoms vibrate more vigorously leading to an increase in the number of phonons per unit volume. As the number density of phonons increase with the temperature, the probability of collisions will also increase.

Below 1 K the dominant contribution to the thermodynamic properties of LHe II is from phonons.

Second Sound in Phonon Gas

If the temperature is uniform, then the number density of phonons is independent of position. If there are temperature non-uniformities then the number density depends also on position. Because of gradients in the number density there will be a flow of phonons. If we fix our attention on a small volume element around a point, there can be three contributions to the time rate of change of the number density at the point, namely, (a) an intrinsic time rate of change due to local creation or destruction of phonons, (b) a change arising from the unequal outflow and inflow of phonons from the volume element, and (c) a rate of change due to collision processes. In steady state the three processes together will be in equilibrium. A detailed consideration shows that fluctuations in the number density of phonons propagate as waves. As already stated a phonon is a mechanical wave in a medium. But what we are describing here is wave in the number density of phonons. Such waves are called *second sound. Second sound can be propagated in phonons only if collisions between phonons conserve momentum and energy. To experimentally see second sound one has to work at temperatures in the liquid helium range.* At such temperatures normal liquids will solidify.

Phonons in Crystals

Very similar arguments hold in crystals as well. They also have phonons which interact with one another due to anharmonicity. Here also we have momentum conserving collisions. In addition the periodic character of the crystal lattice leads to another collision process in which the lattice can either absorb or contribute momentum when three or more phonons collide. In such a process the sum of the momenta of the phonons after

The normal (*N*) collisions among the phonons conserve energy and momentum.

Second sound in a phonon gas corresponds to the propagation of fluctuations in the number density of phonons.

collision is different from the total momentum of the phonons before collision. This is a momentum non-conserving process. Such a momentum non-conserving process also leads to a flow of phonons from a high density to a low density region. But this is a *diffusive* process like the spread of ink in water. These momentum non-conserving processes are called *Umklapp* processes. They are important at *high temperatures.* At low temperatures (below about 1K) the N processes dominate. So one can only hope to see *second sound* in relatively *pure* crystals at very low temperatures.

Experimental Results in Solid ^{4}He

Second sound was seen in solid ^{4}He crystals by Ackermann and others in 1966. ^{4}He will not solidify even at absolute zero of temperature unless one applies a pressure in excess of 25 atmospheres. Helium crystals were grown at a pressure of 52 atmospheres. Out of the 13 crystals grown, four were found to be single crystalline. A temperature wave was produced by applying a pulse of current to a heater at one end of the crystal. The pulse width τ varied from 0.1 to 5 micro-second. Such a pulse is composed of Fourier components with frequencies from zero to $1/\tau$. The temperature rise was detected at a distance d from the heater. The maximum temperature increase in the detector was less than 20mK. When the temperature of the specimen was above 0.7K, the detector exhibited a gradual rise in temperature. When the time derivative of the detector signal was plotted, it exhibited a broad maximum. The arrival time of the pulse at the detector was temperature dependent decreasing as the temperature was lowered. Above 0.7K the mean free path of phonons due to Umklapp processes was less than d. The heat was being transported diffusively. It can be shown theoretically that the arrival time increases as the temperature is lowered in such a case. The different Fourier components in the pulse travel with different velocities and are attenuated differently. So the detected pulse is broad.

Phonons are present in crystals. The crystal periodicity gives rise to momentum non-conserving collisions called *U* processes among phonons. These are dominant at high temperatures and give rise to a diffusive flow of phonons.

When the detector response was seen at a temperature of 0.54 K, the derivative of the response was sharp indicating that all the frequencies in the pulse were travelling nearly with the same velocities. This sharp response indicated that the temperature wave was propagated as second sound. An echo of the second sound pulse was seen corresponding to a reflected wave from the detector going back to the transmitter, getting reflected from there and reaching the detector after traveling a distance $3d$. If the sound had been propagated diffusively, it would have suffered so much attenuation and the pulse would have spread out so much due to the different frequency components travelling with different velocities that one would not have seen the echo. The observation of the echo is a positive indication that all the frequencies in the pulse are propagated with nearly the same velocity and suffer very little decay. The velocity of second sound calculated from d and the time of travel was about 160 m/s and agrees with the theoretical value. The velocity of second sound was also nearly independent of temperature below 0.5 K. Thus second sound has been seen in crystals of solid helium.

Second sound was detected in crystals of solid ^{4}He below 0.54 K.

Conclusion

Second sound was first seen in liquid ^{4}He below 2.17K. In superfluid helium second sound corresponds to the oscillation of the superfluid component against the normal component. The superfluid component corresponds to atoms in the ground state and the normal component to atoms in excited states of energy of the liquid. According to Landau, the excited states are of two types, phonons and rotons. At temperatures below 1K, second sound is the propagation of number density fluctuations in the phonons. Such number density fluctuations in phonons should also be present in crystals. But second sound in crystals can only be seen in relatively pure crystals in a narrow temperature range below 1K. In crystals of ^{4}He prepared under a pressure of 52 atmospheres second sound has been unambiguously detected below 0.54 K.

Suggested Reading

[1] D R Tilley and J Tilley, *Superfluidity and Superconductivity*, Chapter 3, Third Edition, Adam Hilger, Boston and New York, 1990.

[2] J Wilks, *Properties of Liquid and Solid Helium*, Clarendon Press, Oxford, Chapter 6, 1996.

[3] Zygmunt M Galasecwicz, *Helium 4*, Pergamon Press, Paper E 12, page 166, 1971.

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