Systems and Control Engineering

3. Control Systems – Analysis and Design

A Rama Kalyan and J R Vengateswaran

In this part we describe how systems are modelled and analyzed mathematically and explain how control systems are designed.

The four components of system theory may be summarized as follows: physical modelling, mathematical modelling, analysis and design.

Some aspects of physical modelling and mathematical modelling were explained in Part 2[1] of the series. The role of mathematical modelling is obvious for the last two components. A mathematical model should be simple enough to analyse. On the other hand, it should not miss essential features for the sake of mathematical simplicity. We give a simple example below.

An Analysis Example

Quite often we travel in automobiles. For a comfortable ride, the role of shock absorbers is quite evident. A physical model of the suspension system of an automobile is depicted in *Figure* 1.

Applying elementary mechanics, we obtain the following equation of motion,

$$m\ddot{x} + k_2\dot{x} + k_1 x = u. \tag{1}$$

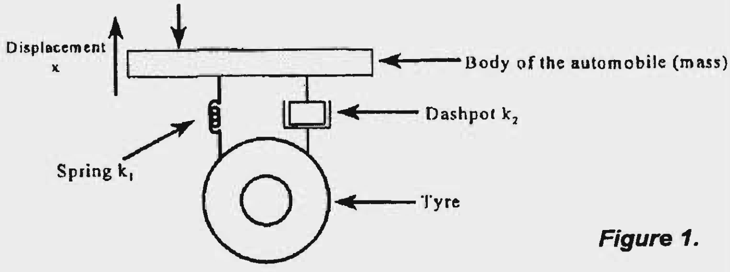


Figure 1.

Rama Kalyan is currently with the Department of Instrumentation and Control Engineering, Regional Engineering College, Tiruchirapalli. He is deeply interested in the concepts of control wherever they exist, be they physical, biological, or abstract systems. The mathematics behind control theory always fascinates him. His other interests are Carnatic music and philosophy.

J R Vengateswaran is presently a third year student of the same Department. He would like to work in control engineering and its applications in robotics and bio-medical engineering.

Here, we have assumed (for the sake of simplicity) the spring to be a linear one, though in fact, it does exhibit non-linear behaviour. Similarly the dashpot has been assumed to be linear. Let us further assume that $m = k_1 = k_2 = 1$. If the automobile suddenly hits a pot hole on the road, the force experienced by the shock absorbing system may be modelled by a impulse function $\delta(t)$ (Dirac's delta). (What are the poles of the system ?)

Previous parts of this article are:
1. Notions of Control, *Resonance*, Vol.4, No.1, 45–52, 1999.
2. Basic Concepts of Systems, *Resonance*, Vol.4, No.3, 45–52, 1999.

Case (i): All is well

$$\begin{aligned} \ddot{x}+\dot{x}+x &= \delta(t)\forall t \geq 0 \\ \Rightarrow (s^2+s+1)X(s) &= 1 \\ \text{or, } x(t) &= Ae^{-0.5t}\sin\frac{\sqrt{3}}{2}t \end{aligned} \tag{2}$$

Thus, the response is a damped sinusoid. Physically, this means that the oscillations caused due to the sudden jolt exponentially die out. This phenomenon is usually called *cushioning*. Indeed, this is the purpose of a shock absorber.

Case (ii): Suppose the spring fails i.e., $k_1 = 0$

$$\begin{aligned} \ddot{x}+\dot{x} &= \delta(t) \\ \Rightarrow (s^2+s)X(s) &= 1 \\ \text{or, } x(t) &= 1-e^{-t} \end{aligned} \tag{3}$$

The constant term in the characteristic polynomial is missing. The response indicates that though at the instant the automobile hits the pot hole, there is no jolting felt but there is exponential rise in the displacement. Physically, we feel being pushed downwards. An unpleasant experience indeed.

Case (iii) : Suppose the dashpot fails i.e., $k_2 = 0$

$$\begin{aligned} \ddot{x}+x &= \delta(t) \\ \Rightarrow (s^2+1)X(s) &= 1 \\ \text{or, } x(t) &= \sin t \end{aligned} \tag{4}$$

The first degree term is missing in the characteristic polynomial. We leave the consequences to the imagination of the reader.

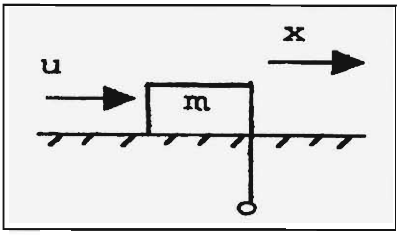


Figure 2.

The purpose of this example is to show that the mathematical model need not be too complicated as long as the analysis carried out is appropriate for the physical situation.

A Typical Design Example

Consider a body of mass m resting on a surface as shown in *Figure* 2. Lets assume there is no contact friction. A force of u pushes the mass by x units. In other words, u is the controlling signal.

$$u = m\ddot{x} \tag{5}$$

gives the dynamical equation of motion of the body.

We will fix the origin O at the right hand bottom corner of the body. If we wish to displace the body by x_d units along the surface, in the positive X-direction, what is the amount of force required? This is our control system problem.

The dynamical equation does not provide any answer directly! To make a beginning, let us resort to our ingenuity and intuition. First, we shall see if this problem is familiar to us in some form. A moment's reflection would take us to the carrom board. (We have encountered control problems since our early childhood without our knowledge!). Yes, the board is a smooth surface and we use boric acid to minimize the friction. A coin would serve as the body of mass m. The game demands us to push each coin into a hole, usually along a straight path. We use a striker to hit the coin with the necessary force. With some experience we seldom fail. But then how do we determine the force? Imagining that we are playing the game for the first time, what do we do?

If we observe carefully, we follow the sequence :

(1) We try to estimate the distance between the coin and the hole, i.e., $x_d - x$.

(2) We wish the coin to start moving at once, approach the hole *steadily* and drop into it, i.e., we would like to compute $\dot{x}_d - \dot{x}$ (the subscript d denotes desired quantities).

Putting these together, we may compute the required force u as

$$u = k_p(x_d - x) + k_v(\dot{x}_d - \dot{x}), \tag{6}$$

where k_p and k_v are constants of appropriate dimensions. Notice here that the desired velocity $\dot{x}_d$ is a nonzero constant but the desired acceleration $\ddot{x}_d$ should be zero to ensure uniform motion. Our requirement is $x \rightarrow x_d$ and $\dot{x} \rightarrow \dot{x}_d$.

With this u in (6) as the applied force, the dynamics appear as

$$m\ddot{x} = k_p(x_d - x) + k_v(\dot{x}_d - \dot{x}). \tag{7}$$

If we define an error $e = x_d - x$, the dynamics may be rewritten as

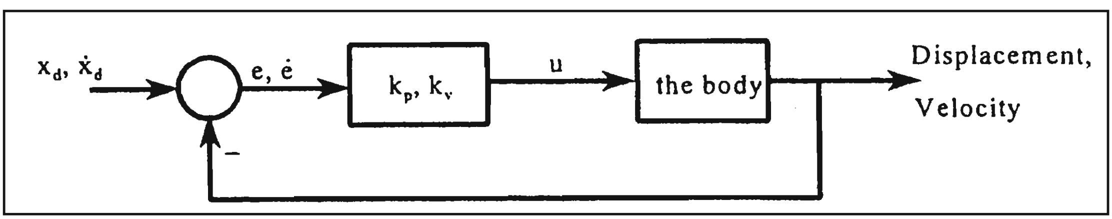
$$m\ddot{e} + k_v\dot{e} + k_p e = 0 \tag{8}$$

and $e(0) = x_d$ and $\dot{e}(0) = \dot{x}_d$ serve as the initial conditions.

From the nature of the equation (8), it is evident that the error tends to zero asymptotically with an appropriate choice of k_p and k_v, and hence our intuitive choice is correct (otherwise, the coin would not drop into the hole). A schematic diagram of this design is shown in *Figure* 3.

Since the required control signal, i.e., the force, is proportional to the displacement as well as the velocity, it is called

Figure 3.



A system is said to be stable if and only if every bounded input produces a bounded output.

proportional + derivative (or, PD) compensation. Physically, this compensator is implemented using a dashpot and a spring. The choice of the word *compensation* is appropriate from the context. Alternatively, we also use the word *controller.* Several compensators and their corresponding design procedures are available in the literature. Interested readers may refer to the suggested readings.

Stability

In the analysis example we saw earlier, what exactly could be the difference between case (i) and the other two cases? In other words, why is it that we could call the first case as "All is well?" Apparently, in the later two cases, one or the other term is missing. Does this really matter? We shall explore it in the next paragraph. Before that, let's look at this. We all know that with a gentle push we enjoy a ride on a bicycle down a hill slope. But is it desirable to choose a bicycle without brakes?

The answer to the above question and the like lies in the concept of *stability.*

Bounded Input & Bounded Output [BIBO]

If the input applied to a system is finite in magnitude, we call it a *bounded input.* Similarly, a *bounded output* may be defined. A system is said to be *stable* if and only if *every* bounded input produces a bounded output. The stability problem was originally posed by Maxwell. A brakeless cycle pushed down the slope is an example of a system whose output is not bounded (the speed increases at every instant!), while the input (the gentle push) is bounded. So, naturally, we do not like unstable systems.

How do we investigate stability? Do we have to apply every possible bounded input and see if the corresponding output is also bounded? This is a formidable task. Fortunately, both the necessary and sufficient conditions for stability are given, in terms of the characteristic polynomial of the system, by Routh. The basic idea is as follows. Since the roots of the characteristic polynomial (i.e., the poles) determine the nature of the response, we would not like them to have positive real parts for they would cause an exponentially growing response instead of a bounded response. This is the sufficient condition. A necessary condition on the polynomial would be that all of its coefficients are of the same sign and none of them is zero. This is to say that all the poles are restricted to lie only in the left half of the complex s-plane. Routh's criterion provides an easy means of determining the stability of a system without having to solve the polynomial (of n^{th} order, in general) for its roots. Another criterion by Nyquist studies the stability of a system in the frequency domain and provides useful insights in terms of stability margins. Stability overrides all other requirements in designing a control system.

Stability overrides all other requirements in designing a control system.

Concluding Comments

In this article, we have tried to show a mountain in a mirror. We believe that others understand better what control engineers do.

Control is all Around us

Control is a common concept, since there always are variables and quantities, which must be made to behave in some desirable way over time. In addition to the engineering systems, variables in biological systems such as blood sugar and blood pressure in the human body are controlled processes that can be studied using control theory. Variables such as unemployment and inflation, which are controlled by government fiscal decisions can also be studied using control methods.

Challenges in Control

The ever increasing technological demands of society impose needs for new, more accurate and novel problems. Typical examples are controlling passenger aircraft and automobiles. At the same time, the systems to be controlled often are more complex, while less information may be available about their dynamical behaviour; for example large flexible space structures. The development of control methodologies to meet these challenges will require novel ideas and interdisciplinary approaches, in addition to further developing and refining existing methods.

Emerging Control Areas

The increasing availability of vast computing power at low cost and the advances in computer science and engineering, are influencing developments in control. For instance ideas from expert systems have been used to design intelligent control systems. There is significant interest in better understanding and controlling manufacturing processes typically studied in disciplines such as operations research. Fuzzy control logic and neural networks are other examples of methodologies control engineers are examining to address the control of very complex systems.

Future Control Goals

The future looks bright. We are moving toward control systems that are able to cope with and maintain acceptable performance levels under significant unanticipated uncertainties and failures, systems that exhibit considerable degrees of autonomy. We are moving toward autonomous underwater, land, air and space vehicles; highly efficient and fault tolerant voice and data networks; reliable electric power generation and distribution; seismically tolerant structures; and highly efficient fuel control for a cleaner environment.

Control systems are decision-making systems where the decisions are based on predictions of future behaviour derived via models of the systems to be controlled, and on sensor-obtained observations of the actual behaviour that are fedback. Control decisions are translated into control actions using control actuators. Developments in sensor and actuator technology influence control methodology, which is also influenced by the availability of low cost computers.

Suggested Reading

[1] C T Chen, *Analysis and Design of Control Systems*, Saunders College Publishing, 1995.

[2] K Ogata, *Modern Control Engineering*, 3/e, Prentice Hall of India, 1997.

[3] W A Wolovich, *Automatic Control Systems*, Saunders College Publishing, 1994.

Address for Correspondence
A Rama Kalyan and
J R Vengateswaran
Department of Instrumentation and Control Engineering
Regional Engineering College,
Tiruchirapalli 620 015, India.
Email: vkalyn@rect.ernet.in