Great Experiments in Physics

3. Measuring Diameters of Stars: The Hanbury Brown–Twiss Effect

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Accurate measurement of angular sizes of stars became possible with the advent of stellar interferometry pioneered by Michelson and Pease which was limited to only the brightest giant stars. Only with the discovery of a new effect, by Hanbury Brown and Twiss, the measurement of stellar sizes became possible for a large number of stars. This method named after them had far reaching applications in nuclear and high energy physics.

All of us have sometimes looked up at the night sky and been fascinated by the twinkling stars. A common way to tell a star apart from the planets is that stars twinkle and the planets do not. This is commonly attributed to the stars appearing as nearly point objects whereas the planets appear to present to us a finite size. But surely the stars must have finite extensions too. In fact, many of them are very much larger than our very own *Sun*. How have the sizes of stars been measured?

Previous articles in this series –
1. Discovery of Transistor Effect that changed the Communication World. *Resonance*, Vol.3, No. 9, 1998.
2. Tunnelling in Superconductors: The Josephson Effect. *Resonance*, Vol.3, No.11, 1998.

Let us go over briefly the history of measurement of stellar sizes. Back in the 16th century before the advent of telescopic observations, Tycho Brahe reached the conclusion that a first magnitude star presents a disc of 120 seconds of arc in diameter and a fifth magnitude star, roughly the faintest star one can see with the naked eye, has a disc of 30 seconds of arc. When Galileo used his telescopes, he found that Tycho was quite wrong. The stars, even when magnified, still looked like points of light. Galileo tried to measure the angular size of the bright star Vega by using a fine silk thread at a distance to occult the star and then measuring the distance required and the diameter of the silk thread. He concluded that this value was 5 seconds of arc. About 350 years later the angular size of Vega was measured at the Narrabri Observatory in New South Wales and Galileo's result

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Articles on topics in the undergraduate curriculum, especially those which students often consider difficult to understand, new classroom experiments, emerging techniques and ideas and innovative procedures for teaching specific concepts are particularly welcome. The submitted contributions should not have appeared elsewhere.

Manuscripts should be submitted in *duplicate* to any of the editors. Authors having access to a PC are encouraged to submit an ASCII/MS Word/Latex/Word Perfect version on a floppy diskette. If necessary the editors may edit the manuscript substantially in order to maintain uniformity of presentation and to enhance readability. Illustrations and other material if reproduced, must be properly credited; it is the author's responsibility to obtain permission of reproduction (copies of letters of permission should be sent). In case of difficulty, please contact the editors.

Title Authors are encouraged to provide a 4-7 word title and a 4-10 word sub-title. One of these should be a precise technical description of the contents of the article, while the other must attract the general readers' attention.

Author(s) The author's name and mailing address should be provided. A photograph and a brief (in less than 100 words) biographical sketch may be added. Inclusion of phone and fax numbers and e-mail address would help in expediting the processing of manuscripts.

Summary and Keywords Provide a 2 to 4 sentence summary and a few keywords.

Style and Contents Use simple English. Keep the sentences short. Break up the text into logical units, with readily understandable headings for each. Do not use multiple sub sections. Articles should generally be 1000-2000 words long.

Illustrations Use figures, charts and schemes liberally. A few colour illustrations may be useful. Try to use good quality computer generated images, with neatly labelled axes, clear labels, fonts and shades. Figure captions must be written with care and in some detail. Key features of the illustration may be pointed out in the caption.

Boxes Highlights, summaries, biographical and historical notes and margin notes presented at a level different from the main body of the text and which nevertheless enhance the interest of the main theme can be placed as boxed items. These would be printed in a different typeface. Such a boxed item should fit in a printed page and not exceed 250 words.

Suggested Reading Avoid technical references. If some citations are necessary, mention these as part of the text. A list of suggested readings may be included at the end.

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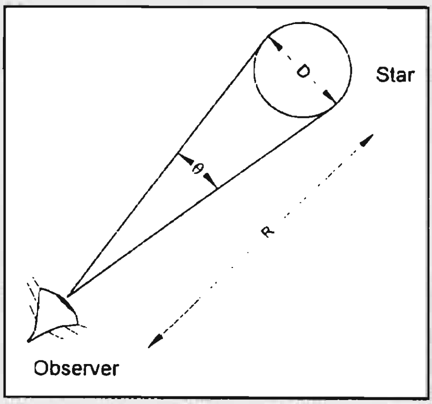
was found to be 1500 times too large. Isaac Newton, though, got the right answer by assuming the stars to be bodies similar to the *sun* and calculated how far away the *sun* would have to be so that it gave us as much light as a first magnitude star. He found that at that distance the angular size of the *sun* would be $\sim 2 \times 10^{-3}$ seconds of arc which is a reasonable estimate of the angular size of a first magnitude star.

The first successful measurement of the angular size of a star was made by Michelson and Pease in 1920 using an interferometer. They measured the angular diameter of the red super giant Betelgeuse (α Orionis) to be 0.047 seconds of arc. The measurements using Michelson's interferometer were limited only to a few bright stars in the sky. It was only in the 1950s with the construction of a new type of interferometer by R Hanbury Brown and R Q Twiss, that it became possible to measure the apparent angular sizes of stars and in some cases the distribution of intensity of light across their diameters. This information helps in determining the distance of a star and its size based on its nature.

The principle of this effect is intensity interference between

Box 1

Stars are classified by the intensity of light received from them at earth. The Greek astronomers Hipparchus and Ptolemy made a catalogue of stars and devised a magnitude system for stars, with the brightest given the magnitude 1 and the faintest the magnitude 6. The magnitude system for stars has survived to this day and astronomers now define the magnitude M of a star by the relation to its luminosity L or the intensity of light received,



M = 2.5 log (L) + constant. The constant depends on the wavelength of the light in which the starlight is received.

By angular size, θ, we mean the number of degrees, minutes or seconds of angle which the star subtends at our eye as shown in the *figure* . If we know the angular size, θ, and the distance, R, of the star, we can know its actual diameter, D, from the relation $D = R\theta$.

light emitted, whereas previous interferometers all relied on the principle of interference of amplitudes.

Why was it necessary to use an interferometer and not just a telescope? Even a telescope with aperture diameter 'a' of the objective can resolve a minimum angle, θ, given by the relation, $\theta = 1.22\ \lambda/a$, which is identical to the resolving power of a Michelson's interferometer. A bright planet has typically angular diameter of 0.5 min of arc. With an wavelength of 0.0005 mm of light, we need a lens with diameter, $a \sim$ 4mm before we can see that the planet is not a point source of light. This is roughly the size of our eye lens. For a typically bright star with angular size of 0.0005 sec of arc, we find that the lens should have a diameter of $\sim$200 m. However, even if such a monstrous telescope could be built, we still would not succeed in measuring the stellar sizes with it. Unfortunately, the light from the stars is randomly bent and delayed on its way to earth by the turbulence in earth's atmosphere. The characteristic size of the turbulent elements is about 10 cm and they move with the wind. These elements differ in refractive index and introduce random patches of phase and amplitude into the starlight falling on the telescope and limit its resolving power roughly to that of a telescope with 10 cm diameter. Consequently, the actual image of a star seen through a telescope, no matter how large, is seldom less than 1 second of arc. This is, of course, one of the many reasons why astronomers would love to build their telescopes on the moon.

Now let us look into the functioning of the Michelson's stellar interferometer and its limitations for measuring the angular sizes of stars. A simplified outline of the interferometer is shown in *Figure* 1.

The mirrors M_1 and M_3 are movable and mounted on a beam so that their separation can be altered. The mirrors are so oriented that the two images of the star are superimposed in the focal plane and when the instrument is properly adjusted, this image is crossed by alternate bright and dark fringes caused by interference of the light from the star seen in the two mirrors.

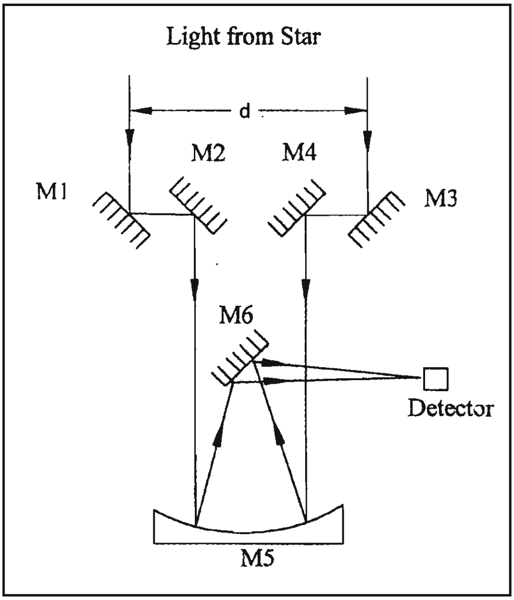


Figure 1. A schematic arrangement of Michelson's interferometer.

The contrast or 'visibility' of these fringes is a measure of the initial 'mutual coherence' of the light beams received at the two separated mirrors. If the spacing between the mirrors is made very small, so that they are effectively in the same place then the mutual coherence has the value unity, i.e., the two light beams are identical and visibility of fringes is high. As the mirrors are separated they get different 'views', so to speak, of the star and mutual coherence and hence visibility of fringes decreases until they disappear. The exact relationship between fringe visibility and mirror separation depends on the wavelength of light used, the angular size of the star and the way in which light is distributed across the diameter of the star. This is explained in the accompanying box item.

The Michelson interferometer worked because the two mirrors were comparable in size with the 10 cm turbulent elements so that the phase and amplitude of the light across them was reasonably uniform, at least for short periods of time. However, the observed visibility of the fringes depended on the state of the atmosphere. The mechanical precision required for adjusting the separation between the two mirrors is quite high. The

[1] This term strictly refers to the range of frequencies $\Delta\nu$ present in the light, but can also be used for the range of wavelengths $\Delta\lambda$ since we can always calculate $\Delta\nu$ from $\Delta\lambda$, λ and c.

maximum difference in path length that can be tolerated depends on the bandwidth[1] of light. The larger the bandwidth, tighter is the tolerance. For example, for optical bandwidth of 100 nm, fringes will vanish if two paths differ by 5 wavelengths of light. The separation between the two mirrors required for measuring angular diameters in the range of 10^{-3} sec of arc would be ~100 m. The largest such instrument that operated successfully had a maximum spacing between the mirrors of about 6 m and measured angular sizes >0.02 sec of arc for only about 6 or 7 bright stars. These limitations prevented further development of the Michelson interferometer.

Working on the nature of radio sources at the Jodrell Bank experimental station of the University of Manchester in 1954, R Hanbury Brown and R Q Twiss showed that instead of adding the amplitudes of the signals if one multiplied the intensities of light from the mirrors (or in case of radio waves, the parabolic dish antennae), the correlation between the signals was still meaningful and the diameter of the star or the radio source could be measured.

For interference with addition of amplitudes (see *Box* 2), the intensity variation is given by

$$I = 2I_0(1 + \cos\delta), \text{ where } \delta = \omega\tau \text{ is the phase difference.}$$

In a correlation interferometer where the intensities are multiplied followed by a low pass filter to allow only the difference frequency to pass, the output of the correlator is proportional to

$$C(\tau) \propto I_0^2 / T \int_0^T \cos\omega t \cos\omega (t-\tau)\, dt$$

For $T >> 2\pi/\omega$, the value of the integral will not be significantly different from the average value over one cycle, $2\pi/\omega$.

Thus,

Box 2

The amplitudes of the light received by the mirrors are added in a Michelson interferometer to get the resultant signal. Due to the separation, d, of the mirrors (these could be two telescopes as well) the signal amplitudes U_1 and U_2 from the two mirrors would be

$$U_1 \propto E\,\exp(j\omega t) \text{ and } \quad U_2 \propto E\,\exp\{j(\omega t - \tau)\}$$

where τ is the delay between the two signals, ω is the circular frequency of light, t is the time and j is the imaginary number equal to $\sqrt{(-1)}$. Thus U_1 and U_2 are complex numbers.

On adding these signals the resultant intensity would be

$$I = \langle (U_1 + U_2)(U_1 + U_2)^* \rangle$$
$$= \langle U_1 U_1^* \rangle + \langle U_2 U_2^* \rangle + \langle U_1 U_2^* \rangle + \langle U_2 U_1^* \rangle$$
$$= I_1 + I_2 + \sqrt{(I_1 I_2)}\,(\exp j\omega\tau + \exp -j\omega\tau),$$ I_1 and I_2 being the intensities of light received by the individual mirrors.

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,\cos\omega\tau$$
For $I_1 = I_2 = I_0$, $I = 2I_0\,(1 + \cos\omega\tau)$

Taking the case of a linear extended source as shown in *Figure* a, the path difference, d, between two rays reaching the observer from the two mirrors is given by,

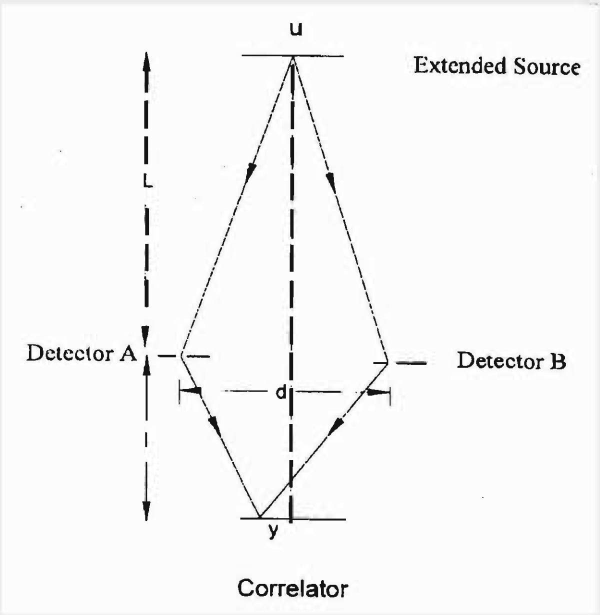


Figure a. Extended source observed by an interferometer.

$$\Delta = \sqrt{(l^2 + (y + d/l)^2)} - \sqrt{(l^2 + (y - d/l)^2)} + \sqrt{(L^2 + (u + d/L)^2)} - \sqrt{(L^2 + (u - d/L)^2)}$$

Keeping terms only up to 1st order in d/l and d/L, we have

$$\Delta = (d/l)y + (d/L)u,$$

and the corresponding phase difference is

$$\delta = (kd/l)y + (kd/L)u,$$

where $k = 2\pi/\lambda$ denotes the wave number.

If $i(u)\,du$ is the intensity of light emitted by a segment du of the source at point u, the contribution of this segment to the overall intensity of the interference pattern at a point is given by

$$dI(y) = 2i(u)\,(1 + \cos\delta)\,du$$

Box 2 Continued...

Then, $I(y) = 2\int i(u).\,(1 + \cos\delta)du$

Using the expression for δ, we can write after expanding,

$$I(y) = Q + C\cos(kdy/l) - S\sin(kdy/l) = Q + \sqrt{(C^2+S^2)}\cos(kdy/l + \alpha)$$

where, $\cos\alpha = C/\sqrt{(C^2+S^2)}$, $\sin\alpha = S/(C^2+S^2)$ and $I_{max} = Q + \sqrt{(C^2+S^2)}$, $I_{min} = Q - \sqrt{(C^2+S^2)}$

The quantities Q, C and S are defined by

$$Q = 2\int i(u)du;\quad C = 2\int i(u)\cos(kdu/L)du;\quad S = 2\int i(u)\sin(kdu/L)du$$

Then the visibility of an interference pattern is defined by,

$$V = (I_{max} - I_{min})/(I_{max} + I_{min}) = \sqrt{(C^2+S^2)}/Q$$

In case of the distribution being uniform, u_0, the expression for intensity reduces to

$$I(y) = 4I_0\,u_0\,[1 + \sin(kdu_0/L)/(kdu_0/L)\cos(kdy/l)$$ and for visibility
$$V = |\sin(kdu_0/L)/(kdu_0/L)|$$

From this relation it follows that the visibility of the fringes depend on the distance between the slits and the angle at which the source is seen from the midpoint between the slits. If $y = 0$ then, for a given d in the Michelson interferometer,

$$I = 4I_0\,u_0\,[1 + \sin(kd\phi/2)/(kd\phi/2)],\quad \text{where } \phi = 2u_0/L$$

and $$V = |\sin(kd\phi/2)/(kd\phi/2)|$$

the first zero of the fringes are thus at $\phi = 2\pi/(kd) = 1/d$. A typical visibility and correlation function is plotted in *Figure b*.

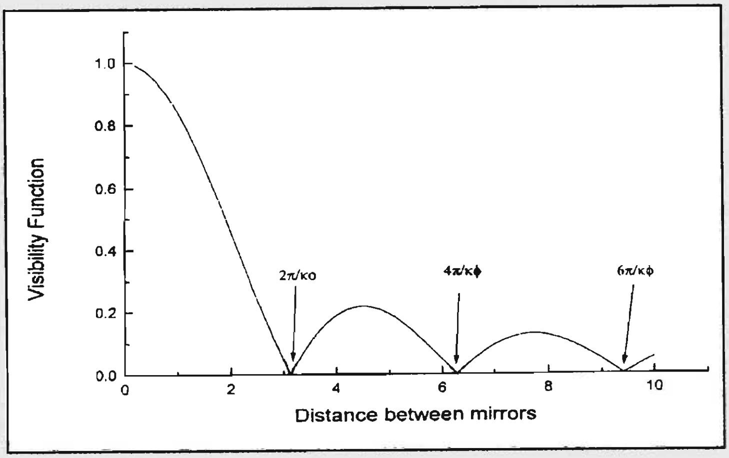


Figure b. Depiction of the visibility and correlation function.

$$C(\tau) \propto (\omega/2\pi) I_0^2 \int_0^{2\pi/\omega} \cos\omega t \cos\omega(t-\tau) \mathrm{d}t$$

$$\propto (\omega/2\pi) I_0^2 \int_0^{2\pi/\omega} \cos\omega t [\cos\omega t \cos\omega\tau + \sin\omega t \sin\omega\tau] \, \mathrm{d}t$$

or, $C(t) \propto I_0^2/2 \cos \omega\tau = \mathrm{I}_0^2/2 \cos \delta$.

Thus we see that the interferometer output varies periodically with the delay time and hence with the separation between the two mirrors or antennae, much as in case of the Michelson's interferometer and hence the same analysis can be performed for the visibility function. This new interferometer was developed at Jodrell Bank experimental station of the University of Manchester by R Hanbury Brown and R Q Twiss in 1950 while investigating the nature of radio sources. In particular they were studying the two strongest sources in Cassiopeia and Cygnus. For the radio analogue of Michelson's interferometer they figured that the baseline would have to be about 20000 km to get a resolution of 0.02 sec of arc and their thoughts turned to the intensity interferometry. How does intensity interferometry overcome the problems associated with the Michelson's interferometer? This instrument does not need the extreme mechanical precision in maintaining the distance. The path lengths from the detectors to the correlator have to be equal as in the case of Michelson's interferometer, but the precision required is much less, about a million times lower. This is because for addition of amplitudes the bandwidth is of the order of frequency of light (about 6×10^{14} Hz for visible region) and for the intensity interferometry the relevant bandwidth is of the order of the fluctuation frequency (about 10^8 Hz) in the currents transmitted to the correlator. Thus the spacing between the mirrors or the antennae can be made very large. As the turbulence and scintillations have higher frequencies associated with them, this method works well even if these are present.

The device they built can be schematically described as in

Figure 2. Schematic arrangement of the intensity interferometer of Hanbury Brown and Twiss.

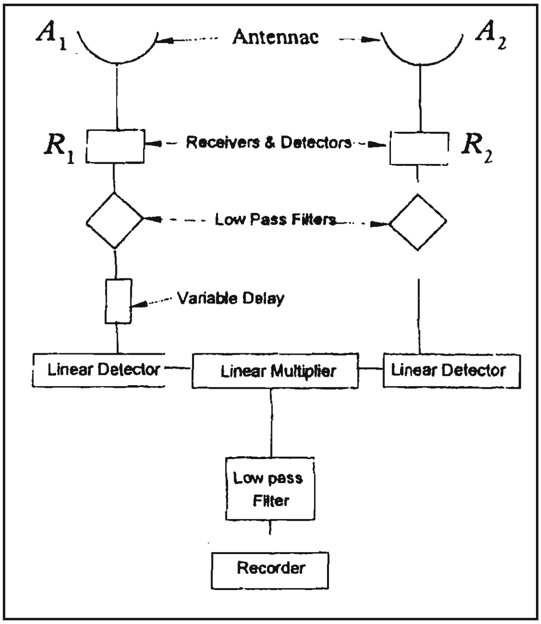


Figure 2. Two antennae A_1, A_2 separated by a suitable distance were connected to two independent superheterodyne receivers R_1 and R_2 which had similar bandwidths and were tuned to same carrier frequency(*Box* 3). A narrow band of low frequencies (1 – 2.5 kHz) in the outputs of their square law detectors was selected by band-pass filters and these two low frequency signals were brought together by radio link or by telephone line.

Box 3.

For transport of signals of a certain frequency e.g., audio or video signals, they are superimposed in a certain way on a much higher frequency wave(modulation) called the carrier wave. This is done to prevent neighbouring signals overlapping with each other and also increase the bandwidth of transmission. The detection of modulated radio waves is usually done using the superheterodyne receivers, where after the initial amplifying stage, the signal is mixed with another radio wave of slightly different frequency (local oscillator) and the resultant wave with the difference frequency is amplified in the next stages. For superheterodyne radio receivers this frequency is always kept at 455 kHz and is known as the intermediate frequency. When we tune a radio set, all we do is to adjust the local oscillator frequency so that it differs from the broadcasting frequency of the particular station by 455 Hz.

Their correlation, or in other words their similarity was measured by multiplying them together in a linear multiplier[2]. This correlation, $C(d)$ when normalised suitably by the signal levels, equals the square of fringe visibility, $V^2(d)$. Thus by measuring $C(d)$ as a function of the separation, d, between the antennae we can find the angular size of a radio star. The first instrument built by Hanbury Brown and Twiss worked at a frequency of 125 MHz. This was used by R Hanbury Brown, R C Jennison and M K Dasgupta to resolve radio sources at Cygnus and Cassiopeia with a baseline of 4 km and the aerials had an area of 500 m^2. Soon after, Hanbury Brown and Twiss extended the same technique to the case of visible light. This created an apparent controversy with some physicists claiming that the method would not work for photons. This was cleared by E M Purcell as well as by Hanbury Brown and Twiss (*Box* 4). For collecting the light signals they employed parabolic mirrors with

[2] If two signals are very similar, then when they are multiplied the product usually has a positive sign and so does its average. But when they are unrelated, the average of the product tends to zero because it is equally likely to be + or – .

Box 4.

Hanbury Brown and Twiss proposed the method of intensity interferometry for visible light in their laboratory experiment in which light from a pinhole source was split by a semi-transparent mirror into two beams falling onto two phototubes. As long as the two light beams were partially coherent, a finite correlation between the photocurrents was measured by them. This experiment was criticised by E Brannen and H I S Ferguson who carried out a somewhat different test, in which the simultaneous emission of photoelectrons were recorded in a coincidence counter and no difference was observed by them for coherent or incoherent light beams. A similar experiment by A Adam, L Janossy and P Varga also obtained a null result. They also suggested that the results of Hanbury Brown and Twiss were due not to true correlation between arrival times of quanta but to some other causes such as intensity fluctuation in the light source. Hanbury Brown and Twiss correctly diagnosed the failure to observe correlation in the experiment of Brannen and Ferguson as due to the large bandwidth of the light falling on the photocathodes coupled with low intensity of the light source. They repeated the experiment with a brighter source and using a narrow band of light using optical filters and got a positive correlation. E M Purcell showed that the confusion arose if photons were treated as classical particles. But photons are indistinguishable particles and obey Bose–Einstein statistics which allowed more than one photon to exist in the same quantum state. This would give rise to bunching of photons emitted within the coherence time of the light which is given by $(\Delta\nu/c)$. This effect was responsible for the positive correlation in Hanbury Brown and Twiss` experiment. For electrons which are fermions, there would be an antibunching effect and for strictly classical particles there would be no such correlation.

The group in Australia first planned a bigger version of the intensity interferometer to study fainter stars with higher resolution. After reviewing technical aspects, the decision was made to go back to a modernised Michelson interferometer. Hanbury-Brown himself participated in this! The Sydney University Stellar Interferometer (SUSI) now operates at 80m baseline and will ultimately go to 400m. This is achieved with separate small mirrors. Computer control and laser measurements make path adjustment possible.

photodetectors at their foci. The reflectors were old army searchlights having backsilvered paraboloids of borosilicate glass of 156 cm diameter and 65 cm focal length and could focus the star image to a rather crude patch of 8 mm. The photodetector outputs were multiplied and the resultant beat frequency of about 100 MHz was used. Thus path differences varying upto 30 cm for the two receivers could be tolerated and scintillation effects were negligible. The first measurement was made on Sirius at Jodrell Bank observatory. Over a 5 month period the observation time was only 18 hrs limited to clear sky conditions. This measurement, however confirmed that the intensity interferometer was substantially unaffected by atmospheric scintillations and the technique was capable of giving a high resolving power. With the encouraging results obtained from this measurement, Hanbury Brown and co-workers looked for a place where clear sky conditions existed for a large part of the year and built an working stellar interferometer at Narrabri in Australia, consisting of two 6.5m reflectors each composed of several hundred smaller mirrors. The reflectors were mounted on trolleys on a circular track of 94m in radius and provided a variation of the distance between the reflectors in the range 10–196 m. A maximum resolution of 0.0005″ of arc was achieved with this interferometer and the angular sizes of few hundred bright stars were determined.

The technique of intensity interferometry pioneered by Hanbury Brown and Twiss finds application in many other branches of physics specially in high energy and nuclear physics. The problem of finding the size of the hot zone in nuclear collisions poses similar problems as found in determining the sizes of stars although the dimensions involved here are extremely small. Nuclear sizes are usually measured in a unit called the fermi which is equal to 10^{-15}m. By observing the correlated emission of radiation(γ-γ, π-π, n-n, etc.) in nuclear collisions, it has been possible to infer the size of the hot source zones in the collision region from where these particles are emitted.

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