André Weil

André Weil (6th May 1906 – 6th August 1998) will be remembered as one of the towering and influential figures of 20th century mathematics. In addition to fundamental contributions to areas as diverse as point-set topology, differential topology, algebraic geometry, representation theory and number theory, he played a crucial role in *defining* how mathematics is to be done. He was also a scholar of classical works in Greek and Sanskrit and had a command of many languages. The combination of all these skills and his acerbic wit made him a formidable person indeed. It is thus with some relief that we find that he had a soft spot for India and Indians; perhaps partly due to some of the people he met during his sojourn at the Aligarh Muslim University (1930–1932).

In spite of his many interests, it can be asserted with some accuracy that Weil's abiding interest remained number theory. Note however that his idea of what constitutes number theory and what doesn't was very much like an artist's view of art; he quoted Housman (an English poet) to explain how he decided what is number theory and what isn't:

> If you ask a fox-terrier to define a rat, he may not be able to do it,
> but when he smells one he knows it.

In a way, this view of his also applies to recognising what constitutes good mathematics and what doesn't. This 'artistic' notion of quality in mathematics should be compared with the approach of Wiener and von Neumann (featured earlier in *Resonance*). At the same time Weil was very categorical regarding what constitutes mathematics (good or bad); it is that which offers proofs. He once boasted that with the advent of Bourbaki (a 'mathematician' created by Weil, Cartan and others) there was no longer any need for someone to talk of a complete proof – either there was a proof or there wasn't!

Returning to number theory, Weil was first inspired by Mordell's theorem on the rational solutions to cubic equations in two variables (see earlier articles in *Resonance*) and Mordell's conjecture on the number of rational solutions to equations of higher degree. He obtained a generalisation of Mordell's theorem and in the process developed a new technique (called Weil pairing) which he hoped would point the way to tackle the Mordell conjecture. He did not heed Mittag–Leffler's advice that he delay publishing his results until he had proved Mordell's conjecture; which was lucky since the final proof by Faltings in 1982 had to wait for many more developments! Weil then collaborated with Chevalley, Zariski and Chow in setting up the foundations of algebraic geometry in a manner that makes explicit the geometric nature of the problem of finding integer and rational solutions of polynomial equations in many variables; these are called Diophantine problems and some would say that these are the most fundamental problems of mathematics. It was ironic that these foundations were being rewritten by Grothendieck even as the ink was drying on the print copies of Weil's book on the *Foundations of Algebraic Geometry*; in fact the irony is doubled by the fact that Grothendieck was in a sense more Bourbaki than Bourbaki.

The geometric nature of Diophantine problems (or in reverse the arithmetic nature of geometrical invariants) was beautifully realised in the now famous Weil conjectures posed by Weil in 1949. He saw these conjectures as the 'real reason' behind numerous results proved earlier by Gauss, Hasse, Davenport and himself on Diophantine problems. An article by V Srinivas and K Paranjape in this issue gives more details for the curious.

A related area of mathematics is the study of automorphic forms (which generalise modular forms). Weil and Tate showed how these objects arise in a natural way when one attempts to tie up representation theory with number theory and/or geometry. These ideas were further developed by Harish-Chandra and Langlands and were finally formulated as the Langlands programme. To see the breadth of this programme one needs only to note that the proof by Wiles of Fermat's Last Theorem is only the first step in this programme.

Lest the reader get the impression that Weil's contributions were fundamental only in number theory and algebraic geometry, we now list *some* of his contributions to other areas. The interested reader is encouraged to examine the Notices of the AMS [1] to read about these in more detail. It is the misfortune of us all that the way mathematics is taught and written, the beautiful (and indeed artistic) work of this genius must currently remain accessible to so few.

Analysis: The construction of the Bohr compactification of almost periodic functions.
Representation theory: Foundations of harmonic analysis of locally compact abelian groups.
Complex geometry: The definition and use of holomorphic bundles and the foundations of Kählerian geometry.
Point set topology: The foundations of the study of uniform spaces and measure spaces.
Complex analysis: Generalisation of Cauchy's integral formula that anticipates the Silov boundary.
Differential geometry: Generalisation of the Gauss–Bonnet theorem and the systematic introduction and use of characteristic classes.

We have restricted ourselves largely to the mathematical work of Weil. His life is a subject of a very interesting autobiography [3]. Among the numerous humorous and curious things he did one should perhaps mention the following (the quotes below have been paraphrased by the author):

1. The 'creation' of Bourbaki and the spirited rebuttal of an article by Ralph Boas talking about the 'group of French mathematicians called Bourbaki':

> *There is no mathematician called Boas. Actually BOAS is an acronym for Big Organisation of American Scientists.*

2. The application to the Institute of Advanced Study by N Boileau (a French writer from the seventeenth century) for a permanent position:

> *You surely know that, once per century, our ruler Pluto grants us leave during which we are allowed to re-incarnate ourselves on the earth.*

3. A letter from the Hades (or 'hell'), purportedly by R Lipschitz, a mathematician of the 19th century, addressed to the *Annals of Mathematics* in 1959:

> *It is sometimes a matter of wonder, to us in the Hades, that what we had believed to be our best work remains buried under thick layers of dust in your libraries, while the very talented young men in the mathematical world of the present day strive manfully against problems which are by no means as novel as they think.*

4. Weil's review of Chevalley's book on algebraic function fields in one variable:

> *This is algebra with a vengeance. Were one not better informed one would conclude, upon reading this book, that Professor Chevalley has never heard of algebraic curves.*

Sources

1. *The Notices of the American Mathematical Society*, Vol.46, No. 4, 1999.
2. *Collected Papers of André Weil* (1964–1978), in three Volumes, Springer Verlag.
3. *The Apprenticeship of a Mathematician*, Birkhauser, Basel 1992, translated by Jennifer Gage from the French original *Souvenirs d'Apprentissage*, Birkhauser, Basel 1991.

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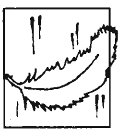

"Weil is a luminous figure in twentieth-century mathematics. The beauty of his discoveries and the clarity and consistency of his vision have been the sources of continued inspiration for two generations of mathematicians. But there is much more to him. His autobiography gives some wonderful insights into the mind of a profound thinker who was supremely creative, even when everything around him was collapsing, who had a full understanding of the world and yet stayed aloof from it. In his mind everything fitted perfectly – mathematics, philosophy, and politics. In this book he presents this world view with great forthrightness and eloquence. It will be a worthy addition to the personal library of every mathematician whose interests go beyond the merely mathematical."

V S Varadarajan, concluding his review of the autobiography of André Weil in Notices of the AMS, April 1999.