Solar Neutrinos

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1. Introduction

Revathi finished her MSc Physics from IIT, Powai in 1998. She has just started the graduate program in physics at the University of Texas at Austin.

See also Siddheshwar Lal, Neutrino Oscillation, *Resonance*, Vol.3, No.12, 1998.

The neutrino, which means the little neutral one in Italian, is a very special elementary particle. It is a spin half, chargeless and almost massless particle and therefore eluded detection for a long time. However, the sun is a rich source of neutrinos and physicists have studied the reactions in the interior of the sun, in order to understand neutrinos better. The main reactions are as follows:

Reactions	Energy of neutrino E_ν
$p+p \longrightarrow {}^2\mathrm{H}+e^+ + \nu$	≤ 0.42MeV (*pp* neutrinos)
${}^7\mathrm{Be}+e^- \longrightarrow {}^7\mathrm{Li}+\nu$	$= 0.86$MeV (beryllium neutrinos)
${}^8\mathrm{B} \longrightarrow {}^8\mathrm{Be}+e^+ + \nu$	≤ 15MeV (boron neutrinos)

Many ingenious experiments have been devised to detect solar neutrinos. The Chlorine, Kamiokande and the SAGE/ GALLEX experiments are the most important ones and the type and energy range of the neutrinos they detect is shown in the table below [1,7,8]. The Chlorine experiment, located in the Homestake Gold Mine in Lead, South Dakota, was the first solar neutrino experiment to be set up. A tank of 10^5 gallons of perchloroethylene in which the electron neutrino reacts with chlorine to produce argon is used to detect them. The GALLEX and SAGE experiments are both radiochemical experiments; they use gallium as a detector, which converts to Germanium on reacting with an electron neutrino. These two experiments, set-up in Italy and Russia, can detect the low energy *pp* neutrinos too. In contrast, Kamioka is a three kiloton water Cerenkov detector in the Japanese Alps. The new SuperKamioka experiment, which has been built in the same mine as Kamioka, is a huge fifty kiloton water Cerenkov detector.

Expt.	Neutrinos	Threshold Energy	Capture Rate in SNU Expected	Observed[a]	
Chlorine	^7Be, ^8B	≥ 0.8 MeV	8.0 ± 0.33	2.05 ± 0.3	
Gallium	pp, ^7Be, ^8B	≥ 0.2 MeV	132^{20}_{-17}	$69.7^{3.9}_{-4.5}$	Gallex
				69 ± 10	Sage
^8B	^8B	≥ 7.5 MeV	5.69^b	2.8 ± 0.33	Kamioka
				2.51 ± 0.18	Super K

a. Observed flux

$\phi(pp) = 0.9\phi^{SSM}(pp)$
$\phi(^7Be) = 0$
$\phi(^8B) = 0.5\phi^{SSM}(^8B)$

SSM - standard solar model

b. Observed rate = $(0.492^{0.034}_{-0.33}$ (stat) $\pm$ 0.058 (syst)) $*$ (Expected Rate) [7] and no error is estimated.

Table 1.

However, when the data were analysed from the three experiments, it was found that the number of neutrinos or events/sec of solar neutrinos arriving on earth is much less than that predicted by the standard solar model with the standard electroweak theory.[1] This is the solar neutrino problem. A slight modification to the above model does seem to provide a likely solution to the problem.

Although, today, physicists believe that they have a much better understanding of various solar processes and the neutrino problem than before, more experimental evidence is needed to conclusively test their theories. The new generation of neutrino experiments in progress may soon provide us with many exciting answers on the flux, energy and flavour of solar neutrinos.

2. Neutrino Propagation in Vacuum

Assuming that the mass eigenstates and the flavour eigenstates are not the same and are related by an orthogonal transformation, we have

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_v & \sin\theta_v \\ -\sin\theta_v & \cos\theta_v \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{1}$$

or

$$|f\rangle = U_v|m\rangle$$

where $|f\rangle$ is the flavour eigenstate and $|m\rangle$ is the mass eigenstate.

Neutrino propagation is diagonal in the mass basis and therefore Schrödinger's equation is stated as follows [1]

$$\begin{aligned} \frac{i\partial}{\partial t}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} &= \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \\ &= \begin{pmatrix} \sqrt{p^2+m_1^2} & 0 \\ 0 & \sqrt{p^2+m_2^2} \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \\ &= [pI + \tfrac{1}{2p}\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}]\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \text{ assuming } p^2 \gg m_1^2. \end{aligned} \tag{2}$$

If we use in (2), the orthogonal relation between the mass and the flavour eigenstates (1), the equation for neutrino propagation in vacuum in the flavour basis is given by

$$\frac{i\partial}{\partial t}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = [pI + (\tfrac{m_1^2+m_2^2}{4p})I + \tfrac{\Delta m^2}{4p}\begin{pmatrix} -\cos 2\theta_v & \sin 2\theta_v \\ \sin 2\theta_v & \cos 2\theta_v \end{pmatrix}]\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \tag{3}$$

where $\Delta m^2 = m_2^2 - m_1^2$.

The probability of finding an electron neutrino ν_e after propagating for time t in vacuum is [1]

$$|\langle\nu_e|\nu(t)\rangle|^2 = 1 - \sin^2 2\theta_v \sin^2 \frac{(E_2 - E_1)}{2} t. \qquad (4)$$

From (4), the predicted suppression of neutrino flux in vacuum is very small, as θ_v, the vacuum mixing angle is considered to be small and hence the vacuum oscillation solution is thought to be viable but not very likely. This leads to the study of neutrino propagation through solar matter.

3. Neutrino Propagation in Matter

The equation for neutrino propagation in matter on the flavour basis can be written by modifying the corresponding equation in vacuum. While both ν_e and ν_μ interact with electrons via neutral current, only ν_e interacts with electrons via charged current. This extra interaction of ν_e with matter is accounted for by including the interaction term $2A$ in the first entry of the mass matrix for propagation in matter. A, the forward scattering amplitude for $|\nu_e\rangle$ is defined as

$$A = 2\sqrt{2}Gn_e(r)E$$

where G = Fermi's coupling constant, $n_e(r)$ = number density of electrons and E = neutrino energy.

So, Schrödinger's equation in matter takes the form [2]

$$\frac{i\partial}{\partial t}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\Delta m^2 \cos 2\theta_v + 2A & \Delta m^2 \sin 2\theta_v \\ \Delta m^2 \sin 2\theta_v & \Delta m^2 \cos 2\theta_v \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

$$= [\frac{AI}{4E} + \frac{1}{4E}\begin{pmatrix} -\Delta m^2 \cos 2\theta_v + A & \Delta m^2 \sin 2\theta_v \\ \Delta m^2 \sin 2\theta_v & \Delta m^2 \cos 2\theta_v - A \end{pmatrix}]\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. (5)$$

Just as neutrino propagation in vacuum is characterized by θ_v, the vacuum mixing angle, neutrino propagation in matter is charaterized by θ_m, the matter mixing angle. The relation between the two is obtained by diagonalising the mass matrix in matter (the matrix in (5)) with

$$\begin{pmatrix} \cos\theta_m & -\sin\theta_m \\ \sin\theta_m & \cos\theta_m \end{pmatrix}.$$

The result is [2]

$$\tan 2\theta_{\rm m} = \frac{\Delta m^2 \sin\, 2\theta_{\rm v}}{(\Delta m^2 \cos\, 2\theta_{\rm v} - A)}. \tag{6}$$

Thus, it is the relation between A and $\Delta m^2 \cos 2\theta_{\rm v}$ that determines the nature of the mass matrix in matter. The special case when $A = \Delta m^2 \cos\, 2\theta_{\rm v}$ (say at core) is called resonance. In this condition, the diagonal terms of the mass matrix are equal (here equal to zero), $\theta_{\rm m} = \frac{\pi}{4}$ and maximal mixing between the states takes place. If $A \gg \Delta m^2 \cos\, 2\theta_{\rm v}$ at the core, the condition of resonance will be satisfied at some distance away from the core since A is a monotonically decreasing function with distance from the core. If $A < \Delta m^2 \cos\, 2\theta_{\rm v}$ at core, the resonance condition is never satisfied and electron neutrinos remain electron neutrinos, after propagating through matter. However, the mixing angle changes slowly from its value in matter to that in vacuum.

In order to understand resonance and how it affects the probability of finding an electron neutrino after it travels a distance x, it is important to know how fast $\theta_{\rm m}$, the matter mixing angle, changes with time. To find this, we write the equation for neutrino propagation in matter

$$\begin{pmatrix} \nu_1^{\rm m} \\ \nu_2^{\rm m} \end{pmatrix} = \begin{pmatrix} \cos\,\theta_{\rm m} & -\sin\,\theta_{\rm m} \\ \sin\,\theta_{\rm m} & \cos\,\theta_{\rm m} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \tag{7}$$

Using (7) in (5), the equation on the mass basis is

$$\frac{i\partial}{\partial t} \begin{pmatrix} \nu_1^{\rm m} \\ \nu_2^{\rm m} \end{pmatrix} = \begin{pmatrix} E_1^{\rm m} & \frac{-i d\theta_{\rm m}}{dt} \\ \frac{i d\theta_{\rm m}}{dt} & E_2^{\rm m} \end{pmatrix} \begin{pmatrix} \nu_1^{\rm m} \\ \nu_2^{\rm m} \end{pmatrix}. \tag{8}$$

It can be seen that if $\frac{d\theta_{\rm m}}{dt} \ll |E_2 - E_1|$, the matrix in (8) is instantaneously diagonal and is similar to the equation for propagation in vacuum. Both ν_1 and ν_2 will continue to remain the same even after propagation through matter. This is called the adiabatic condition. However, this condition may not hold good at all places during neutrino propagation through solar matter for a known density profile of the sun and the nonadiabatic condition has to be considered, especially in the resonance region where the matter mixing angle $\tan 2\theta_{\rm m}$ changes rapidly. This implies that there exists a finite probability P_x in the nonadiabatic case of ν_1 converting to ν_2 and vice versa on propagating through the resonance region in matter. Therefore, the survival probability of an electron neutrino after travelling a distance x is given by [3]

$$P_{\rm ee} = \tfrac{1}{2} + (\tfrac{1}{2} - P_x)\cos 2\theta_{\rm v}\cos 2\theta_{\rm m}, \qquad (9)$$

where P_x can be approximated by an exponential term of the form $e^{-E_{\rm NA}/E}$. The nonadiabatic threshold energy $E_{\rm NA}$ is, therefore, the energy, beyond which nonadiabatic effects play a role.

If the neutrino energy at the core is less than the adiabatic threshold, $P_{\rm ee}$ is one and if it greater than the adiabatic threshold but less than the nonadiabatic threshold $P_{\rm ee}$ is zero. For $A < \Delta m^2 \cos\theta_{\rm v}$, $P_{\rm ee} = 1$, i.e this holds for $E < \frac{\Delta m^2 \cos\theta_{\rm v}}{2\sqrt{2} G n_e(r)}$. The above inequality defines the adiabatic threshold. If the neutrino energy is much greater than the adiabatic threshold energy, P_x becomes important and $P_{\rm ee} \simeq P_x$.

4. A Likely Solution to the Problem

In the theory of the solar neutrino problem discussed, there are two free parameters involved – Δm^2 (mass difference between the neutrinos) and $\theta_{\rm v}$ (the vacuum mixing angle). The number of solar neutrinos observed on earth, i.e. events/sec, is [1,6]

$$\int (\text{cross} - \text{section} * \text{flux}) \mathrm{d}E. \qquad (10)$$

However, according to the theory developed for propagation of solar neutrinos in matter,

$$\text{events/sec} = \int (\text{cross} - \text{section} * \text{flux} * \text{survival probability}) \mathrm{d}E \qquad (11)$$

Referring to *Table* 1, the Kamioka experiment detects only ^{8}B neutrinos greater than 7.5 MeV and there is a suppression factor of 0.5 in these neutrinos. In the Chlorine experiment, which has a threshold of 0.8 MeV, Be neutrinos (energy = 0.86 MeV) completely oscillate away. The Gallium experiment (which has a threshold of 0.2 MeV) detects 0.9 times the low energy *pp* neutrinos (energy ≤ 0.42 MeV). A graph of P_{ee} vs E which quantitatively explains the solar neutrino problem should have $P_{ee} = 0.9$ for $E \leq$ 0.42MeV, zero at 0.86 MeV and 0.5 around 12 MeV.

If we find the values of Δm^2 and θ_v which satisfies the ratio

$$\frac{\text{Events/sec with oscillation}}{\text{Events/sec without oscillation}}$$

for all the three experiments, a possible solution to the solar neutrino problem emerges. For this purpose, a computer program was written and values obtained from the program for Δm^2 and θ_v were used to plot a graph of P_{ee} vs E. The form used for P_{ee} (9) to obtain the following results contains the jump probability P_x and so includes nonadiabatic effects. Estimates of the density at core and its rate of change have also been used.

The small angle solution obtained is:

$$\Delta m^2 = 4.5 * 10^{-6}\text{eV}^2$$
$$\theta_v = 2.5\,\text{deg}\,.$$

The large angle solution obtained is

$$\Delta m^2 = 8.0 * 10^{-6}\text{eV}^2$$
$$\theta_v = 30\,\text{deg}\,.$$

Summary and Conclusions

The standard solar model with the standard electroweak theory does not explain the solar neutrino problem. However, a slight modification to this theory, in which there is a mass difference in the neutrinos and mixing between them, does seem to explain the experimental observations. However to test conclusively the theory and oscillation effect, a detector which can detect separately the charged current interactions of the electron neutrinos and neutral current interaction of both the neutrinos with matter and also give accurate measurements of the two interactions is to be built.

The next generation of solar neutrino experiments aims to test the above theory. One experiment, which proposes to detect neutrinos through both neutral and charged current interactions, is the Water Cherenkov Detector of the Sudbury Neutrino Observatory (SNO) at Ontario, Canada. Another Water Cherenkov Detector,the massive Super Kamiokande detector, has just become operational and the analysis of the data being collected from the detector are being reported in many conferences. The scientists at Kamiokande have very recently reported at the Neutrino '98 Conference the exciting result that the neutrino does have a mass. They are now clear that they have reached a level of understanding in Neutrino Astrophysics, where neutrino oscillation solutions can be discussed.

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Suggested Reading

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