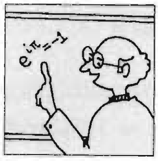

In this section of* Resonance, *we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

Doctor's Dilemma

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The following data relate to a medical trial concerning two treatments A and B for a disease. Data are given separately for male and female patients participating in the trial. At the end of the trial, each patient is classified as 'Success' (i.e., cured) and 'Failure' (i.e., not cured).

		Success	Failure	Total
Male	Treatment A	60	20	80
	Treatment B	100	50	150
Female	Treatment A	40	80	120
	Treatment B	10	30	40

Now, if you examine the data, you will easily see that for both males and females, Treatment A is better than Treatment B (in the sense of yielding higher success rate). For males, Treatment A results in a success rate of 60/80 = 75%, while Treatment B results in a success rate of 100/150 = 66.67%. For females, these rates are respectively 40/120 = 33.33% and 10/40 = 25%.

Now, pool the data for males and females to obtain

	Success	Failure	Total
Treatment A	100	100	200
Treatment B	110	80	190

Here, then, the success rates of Treatments A and B respectively are 100/200 = 50% and 110/190 = 57.9%. Thus Treatment B seems to be better.

So, for males, Treatment A is better, for females also, Treatment A is better, but on the whole, Treatment B is better! This certainly is paradoxical!! If you are a physician, would you prescribe Treatment A if you know the gender of the patient but Treatment B if you do not? This of course is absurd.

What is the explanation for this apparent paradox? What treatment should actually be prescribed under what circumstances?

When you pool the male – female data and work out the overall success rate, you are computing the weighted average of the male and female success rates, the weights being the relative male – female numbers of patients as follows:

Treatment A: $3/4 \times 0.4 + 1/3 \times 0.6 = 0.5$
Treatment B: $2/3 \times 0.79 + 1/4 \times 0.21 = 0.579$

since the weights for Treatment A are 80/200 = 0.4 and 120/200 = 0.6, for Treatment B 150/190 = 0.79 and 40/190 = 0.21.

The overall comparison between Treatments A and B is done on the basis of weighted averages but with quite different weight structures – 0.4: 0.6 for Treatment A and 0.79: 0.21 in the case of Treatment B. This evidently is not a valid way to compute weighted averages for the purposes of comparison or otherwise. That is the reason for the apparent paradox.

If you really want meaningful overall success rates for the two treatments, you should use common meaningful male:female weights, maybe 1/2: 1/2 or a ratio which is likely to arise while prescribing treatments, irrespective of the male:female ratio used in the trial. Ideally, in the trial, the male:female ratio should have been 1/2:1/2, but often in trials of this kind these quantities cannot be controlled, since one cannot get patients the way you want them.

It goes without saying that if the results of the trial are of the kind seen here, then the physician should prescribe Treatment A for all, unless he has other medical evidence to go by. However, in genuine medical research, since some patients at least are cured by Treatment B, further research would be carried out to understand the circumstances under which one treatment is better than the other.

This paradox is called *Simpson's Paradox*. Here is another data set, in which the investigation pertains to murder trials in the USA, in which the victims and defendants are classified by Race (as 'Black' (B) and 'White' (W)), the outcome being 'Convicted' or 'Not Convicted'.

		Convicted	Not convicted	Total
Black victim	Black defendant	6	97	103
	White defendant	0	9	9
White victim	Black defendant	11	52	63
	White defendant	19	132	151

You will notice the same kind of paradox here if you pool the data on black and white victims.

It is tempting to think that the name of the paradox is derived from the recent O J Simpson case. No, this predates O J Simpson and is named after the statistician E T Simpson, who first observed this paradox.

Suggested Reading

[1] A Agresti, *Analysis of Categorical Data*, New York, John Wiley & Sons, 1992.

[2] R Christensen, *Log-linear Models and Logistic Regression*, New York, Springer-Verlag, 1997.

[3] E T Simpson, The interpretation of interaction in contingency tables, *Journal of the Royal Statistical Society*, B13, 238–241, 1951.