Second Sound

1. Waves of Entropy and Temperature

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Introduction

We all know that in a fluid (liquid or gas), pressure or density waves can be propagated. If we have a loud speaker activated in air, the vibrating diaphragm of the loud speaker pushes the air in front of it. This causes changes in pressure and density of the air in front of the diaphragm. This fluctuation in pressure is transmitted through the air. The velocity of sound in a fluid medium depends on the *adiabatic* bulk modulus and the density of the medium. As long as the frequencies are within a certain range, the velocity of the wave is independent of the frequency. The product of the frequency and wavelength of the wave is the velocity of the wave. Since pressure fluctuations take place adiabatically, these will be associated with temperature fluctuations in the gas. Such density fluctuations with accompanying temperature fluctuations are a collective excitation of the fluid. We shall call this *first sound.*

Second sound is a form of collective excitation in which entropy (and hence temperature) fluctuations *without accompanying density fluctuations* are propagated in a medium. Second sound was first observed in superfluid liquid ^{4}He. It was later realised that second sound can also be observed at very low temperature in crystals. In this article we shall first give a brief account of the superfluid transition in liquid helium and then explain the origin of second sound on the two fluid model. A brief discussion of second sound in crystals will be presented in the next part.

Superfluid Liquid ^{4}He

Helium has two common isotopes, ^{4}He with mass number 4 and ^{3}He with mass number 3. ^{4}He is the most abundant isotope.

Hereafter we use He to stand for ^{4}He.

Below 2.17 K liquid helium shows unusual physical properties.

Liquid He boils under one atmosphere pressure (1 bar) at 4.22 K. When the liquid is cooled below 2.17 K, it shows very strange behaviour. We shall denote liquid helium with a temperature above 2.17 K as LHe I and liquid helium below 2.17 K as LHe II. The thermal conductivity of LHe II is a million times more than the thermal conductivity of LHe I. When the viscosity of the liquid is measured by a capillary flow method, LHe II does not show any viscous behaviour. LHe II is capable of flowing freely through narrow channels. But if viscosity of the liquid is measured by the damping produced on a set of oscillating discs immersed in the liquid, the viscosity of LHe II appears comparable to that of LHe I.

The fountain effect in LHe II was first discovered by Allen and Jones in 1938. The *Figure* 1 illustrates the essentials of their experiment that demonstrated the fountain effect. A wide bore U tube has a long capillary of narrow bore fused at one end. The U tube is tightly filled with carborundum powder of grain size of about 1μm. The channels in such a tight plug of carborundum powder are so narrow that they will not allow LHe I to flow through unless one applies a large pressure to the liquid. If this arrangement is suspended in a bath of LHe II so that the top of the capillary tube is above the liquid level while the free end of the U tube is below the liquid level, liquid He flows freely

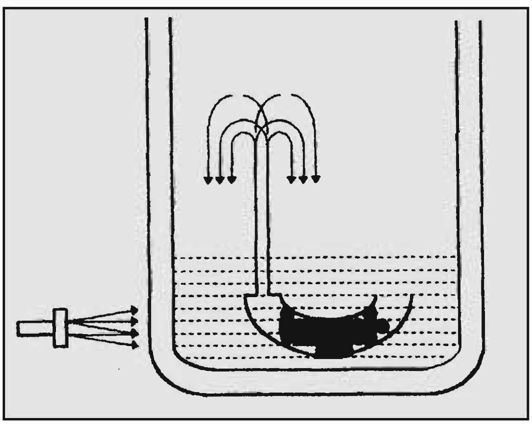


Figure 1. Fountain effect in LHe II.

To account for these properties the two fluid model was proposed by Tisza.

through the carborundum plug to equalise the levels in the two limbs of the liquid. If a torchlight beam falls on the carborundum powder nearer the capillary a fountain of liquid gushes out of the top of the capillary tube. More careful experiments show that the height to which the fountain rises is proportional to the temperature difference across the carborundum powder.

Two Fluid Model of LHe II

Tisza proposed in 1938 a two fluid model to account for the strange behaviour of LHe II. The fact that LHe II can flow through narrow channels without viscous resistance but at the same time produces a damping on a set of oscillating discs immersed in it suggested to Tisza that LHe II is made up of two components. These components are chemically indistinguishable and LHe II cannot be separated into its components by any method. But they have different physical properties. One component has no viscosity and is responsible for superfluid flow through narrow channels while the other has viscosity and is responsible for damping the oscillating discs. The first component is called the superfluid component and has a density ρ_s; the second component called the normal component has a density ρ_n. The total density ρ of LHe II is the sum of the densities of the two components. The fraction of the normal component ρ_n/ρ decreases from unity, at the transition temperature, to zero as the temperature tends to absolute zero. On the other hand the fraction of the superfluid component ρ_s/ρ increases from zero, at the transition temperature, to unity as the temperature is reduced to absolute zero.

When a temperature gradient is established along the carborundum plug in the fountain effect experiment, a concentration gradient of the normal and superconducting fractions is also established. The superconducting fraction is more at the low temperature end of the plug than at the high temperature end. So the superfluid component rushes through the narrow channels of the plug without hindrance. The normal component has a larger volume fraction at the high temperature end of the plug. But the concentration gradient cannot drive the normal fraction in counterflow to the superfluid component as the normal component is viscous. So a pressure builds up at the bottom of the capillary which forces the liquid to gush out as a fountain from the top of the capillary.

In the presence of a temperature gradient there is a counterflow of superfluid and normal components of LHe II.

Tisza also suggested that the superfluid component has no entropy. It is in a highly ordered state. The normal component carries the entire entropy ρS per unit volume of liquid He II. If a temperature gradient is maintained across a wide tube connecting two vertical tubes containing LHe II, the liquid level in the warmer tube rises above that in the colder tube (see *Figure* 2). The pressure builds up to such an extent that there is no acceleration of the superfluid or normal components. This will happen when the pressure difference and temperature difference are related by

$$\Delta P = \rho S \Delta T \qquad (1)$$

Thus the height to which the liquid rises in the fountain effect experiment is proportional to the temperature difference across the carborundum plug.

The pressure difference ΔP causes the normal component to flow by Poiseuille's flow through the wide channel. In equilibrium this flow of the normal component is cancelled by the counterflow of the superfluid from the cold to the warm tube. But the normal component carries away entropy (and hence heat) from the warm to the cold end. The counterflow of superfluid and normal helium is somewhat analogous to convection currents in a fluid. This accounts for the very large heat conductivity of LHe II.

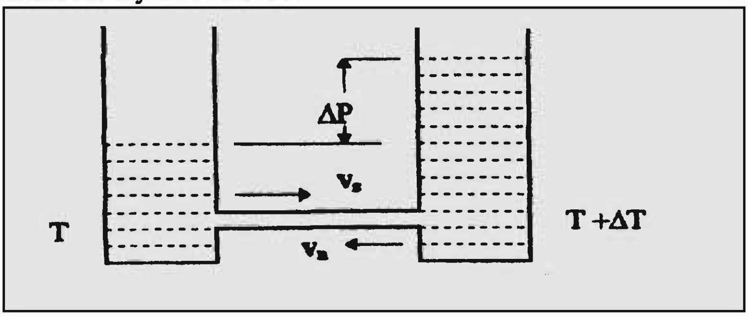


Figure 2. Two cylinders connected at the bottom with a wide bore tube. The two cylinders are filled with LHe II. If a temperature gradient is established between the tubes, the level in the warmer tube rises to build a steady hydrostatic pressure difference ΔP proportional to ΔT. Superfluid component flows from the cold to the warm tube while an equal amount of normal component flows in the reverse direction.

Density fluctuations in the liquid are propagated as first sound.

Propagation of First Sound

If a piezoelectric crystal is put in a bath of LHe II and is excited at a certain frequency, it will push both the normal and superfluid components in front of it. This causes density and pressure fluctuations which propagate as first sound in the liquid with a velocity given by

$$C_1^2 = K_s/\rho. \tag{2}$$

Here K_s is the adiabatic bulk modulus of the liquid. In any liquid, sound waves will propagate with this velocity. This is called first sound. Unless the amplitude of pressure or density fluctuation is large, the associated temperature fluctuations are very small and can be neglected.

Second Sound

In the two fluid model *Figure* 2 a temperature gradient causes a counterflow of the superfluid and normal components with a net mass flow ρv given by

$$\rho v = \rho_n v_n + \rho_s v_s = 0. \tag{3}$$

If we produce an oscillatory temperature at one point in the liquid this will cause an oscillatory counterflow of the superfluid and normal components at that point. Due to this oscillatory counterflow density will not change as there is no net mass flow. So pressure will not change. However the oscillatory counter flow will produce fluctuations in entropy. Oscillations in entropy will be reflected as oscillations in temperature. Tisza showed that these fluctuations propagate with a velocity different from that of first sound. The velocity of second sound is given by

Entropy or temperature fluctuations in the superfluid are propagated as second sound.

$$C_2^2 = (\rho_s/\rho_n)\, S^2\, (\partial T/\partial S)_\rho = (\rho_s/\rho_n)\, S^2\, (T/C_v), \tag{4}$$

where C_v is the specific heat at constant volume of the liquid.

The velocity will be temperature dependent since all the factors in the above equation for velocity are significantly temperature dependent.

One may give an analogy to second sound from lattice vibrations in crystals. If we take a lattice of diamond, the unit cell contains two chemically indistinguishable but crystallographically non-equivalent carbon atoms. One can have two types of long wave length vibration modes. In the first mode both the carbon atoms in the unit cell move together. This is the acoustic mode corresponding to the propagation of sound. There is a second mode in which carbon atoms of one type vibrate against the carbon atoms of the second type. This is the optic mode. The vibration of the superfluid against the normal fluid in second sound is analogous to the optic mode in diamond. However the analogy should not be pushed too far. There is a finite restoring force proportional to the displacement even at very long wavelengths in the optic mode of diamond leading to a high frequency of vibration. In second sound the restoring force on the superfluid component is proportional to the square of the wave number (reciprocal of the wavelength λ) and tends to zero as the wavelength tends to infinity just as for long wavelength sound. So the frequency is proportional to the wave number, the constant of proportionality being the velocity of second sound.

To produce second sound one immerses a heater in LHe II through which an alternating current of low amplitude is used. If the frequency of the alternating current is f, it will produce temperature fluctuations at a frequency $2f$ because the heating rate depends on the square of the current. The temperature fluctuations produced at a frequency $2f$ in the superfluid can be picked up by sensitive resistance thermometers.

Experimental Verification

The first experimental verification of Tisza's prediction was made by Peshkov in 1946. He used a resonator like an organ pipe. Suppose we have a cylindrical tube, closed at one end, containing a liquid as shown in *Figure* 3a. To study first sound we may put at the bottom a piezoelectric crystal to produce sound waves. These sound waves will propagate upwards and get reflected at the liquid surface. The original and reflected waves will form a

Propagation of second sound in LHe II was experimentally demonstrated by Peshkov.

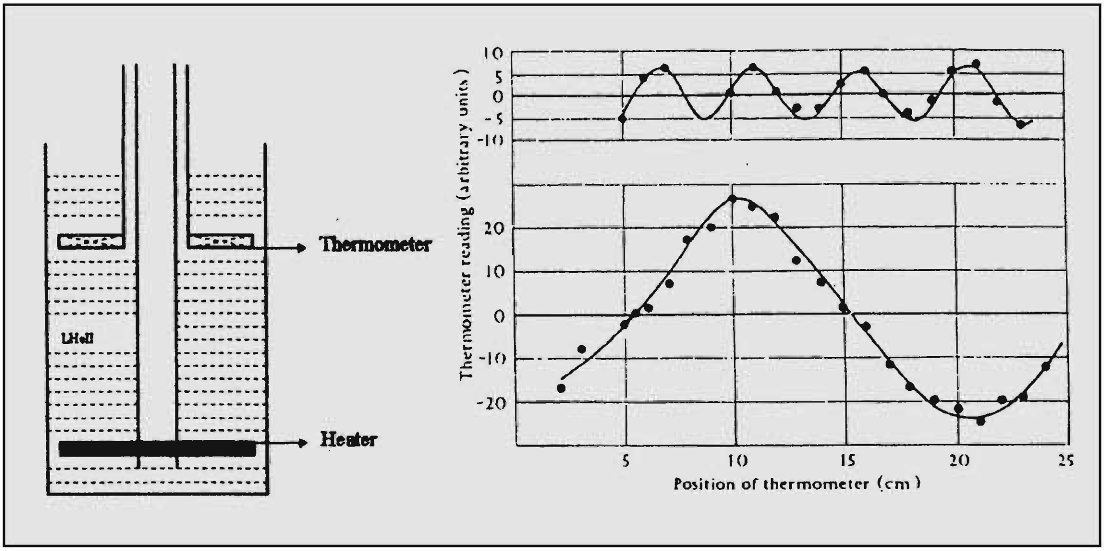
standing wave pattern in the liquid with nodes and antinodes succeeding each other at intervals of $\lambda/4$. If we have a moveable pressure sensor, then it will detect an oscillatory variation of pressure with maxima and minima. Replace the piezoelectric transmitter with a heater, and the moveable pressure sensor with a moveable sensitive temperature sensor. We then have a set-up to detect standing waves of second sound. It was with such a set-up that Peshkov measured the standing temperature wave (*Figure* 3b). From a measurement of the wavelength of the temperature wave and the frequency of the heater current one can calculate the velocity of second sound.

A second method is to send a pulse of current through the heater. This generates a pulse of temperature near the heater. This pulse travels through the liquid and reaches a sensitive thermometer. By measuring the time taken for the pulse to travel from the transmitter to the receiver and the distance between the two one can determine the velocity.

The temperature variation of the velocity of second sound as measured by Peshkov is shown in *Figure* 4. The sound velocity is nearly independent of temperature from 1 to 2 K and has a value of about 20 m/s. The near temperature independence arises because the increase in (ρ_s/ρ_n) as the temperature falls is

Figure 3a. Peshkov's resonator for measuring velocity of second sound in LHe II.

Figure 3b. Typical standing wave pattern of second sound observed by Peshkov in LHe II.



nearly compensated by the decrease in the other factors, S, T and C_v. This low velocity of second sound between 1 and 2 K is to be compared with the velocity of first sound which has a value of approximately 230 m/s. Below 1 K the velocity of second sound increases rapidly because of the rapid increase of the ratio (ρ_s/ρ_n). It becomes difficult to detect second sound below 1K.

Second sound should always be observed in superfluids.

It should be clear that second sound should always be observed in *superfluids*. The observation of second sound can be taken as a confirmation that superfluid state has been achieved. Second sound has been observed in the superfluid states of mixtures of liquid ^4He and liquid ^3He Pure liquid ^3He becomes a superfluid below 7 mK and the observation of second sound in liquid ^3He below 7 mK was clinching evidence for superfluidity in the liquid.

Difference with Diffusive Temperature Wave One can See in a Liquid

If one produces an oscillatory variation in temperature in any liquid or solid, a temperature wave is propagated in the medium.

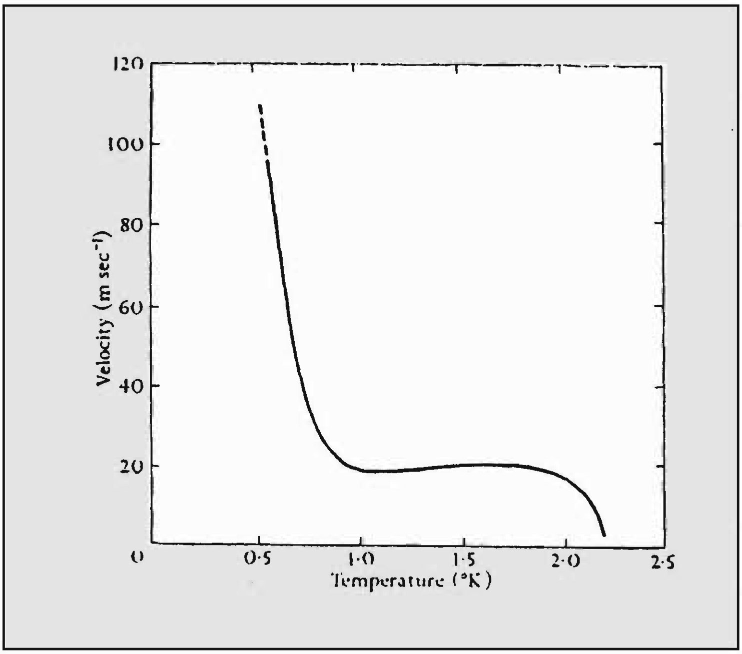


Figure 6. Velocity of second sound as a function of temperature of LHe II as observed by Peshkov.

In second sound the velocity of the temperature wave is independent of the frequency. In a diffusive temperature wave seen in all fluids and solids, the velocity will be strongly dependent on frequency and the attenuation will be high.

The propagation arises due to diffusion of heat through thermal conductivity. We may recollect here the Angstrom's experiment for determining the thermal conductivity of a metal rod. One end of a long metal rod is periodically heated and cooled and the temperature is measured as a function of time at various positions along the length of the rod. The temperature profile is Fourier analysed to find the wavelength of the fundamental wave. The velocity of the wave is found by dividing the wavelength by the period of the heating and cooling. It is found that the velocity is dependent on the wavelength. The waves are also attenuated with the amplitude decreasing exponentially with length. On the other hand the velocity of second sound is independent of frequency. If a heat pulse is applied to any liquid, the different Fourier components of the heat pulse travel with different velocities through the liquid. When they arrive at the detector, the different Fourier components would have acquired different phases and they have also been attenuated by different degrees. So the shape of the temperature pulse recorded by the detector is very different from the shape of the initial temperature pulse. In second sound the velocity is independent of frequency and the attenuation is weak. So the pulse recorded by the detector has the same shape as the original temperature pulse. In the language of mathematics, second sound is the solution of a partial differential equation which is of second order both in time and in space co-ordinates. On the other hand the diffusive temperature wave in a liquid is the solution of a differential equation which is of first order in time and second order in space co-ordinates.

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In the next part of this article, we will discuss a more microscopic view of second sound.

A monument to Newton! A monument to Shakespeare! Look up to the Heaven, look into the human heart. Till the planets and the passions, the affections and the fixed stars, are extinguished their names cannot die.

John Wilson
Quantum Chemistry and Spectroscopy