Norbert Wiener and Control Engineering

Wiener's association with control engineering began when he undertook a project on control of anti-aircraft fire from the National Defense Research Council of USA in 1940, with the second world war in full swing. The problem was to estimate the position of an aircraft at a fixed time in future given noisy measurements of its past trajectory. An important innovation of Wiener here was to cast the problem as a statistical problem, bringing over the machinery of 'time series analysis' of statistics to bear upon control and communication engineering.

In the abstract, the problem can be stated as : Given a stationary random process consisting of a (desired) signal plus (undesired) noise, 'filter out' the noise in an appropriate sense and recover a process of estimates of the signal. That is, one seeks to design a box that takes the former as input and yields the latter as output, subject to natural constraints such as causality: The box is not allowed to anticipate future inputs. Wiener worked in a linear framework, i.e., looked for a linear transformation of the input. Such transformations are completely characterized by their output for a unit impulse input, as the output for other inputs can then be deduced by superposition. Equivalently, one looks for the Fourier transform of this 'impulse response', the so called transfer function. Wiener sought to minimize the mean square error between the signal and its estimate subject to causality and characterized the optimal transfer function in terms of the associated spectral quantities. Interestingly, this led him to an integral equation he had already encountered, the Wiener–Hopf equation.

In all fairness, it must be mentioned that this work was preceded by a closely related work by Kolmogorov in the USSR. But it was Wiener's book, *Extrapolation, interpolation and smoothing of stationary time series, with engineering applications* which caught the engineers' fancy, despite its abstruseness (which, along with its yellow jacket, earned it the title of 'yellow peril').

Wiener later tried to handle nonlinear problems as well, using the 'Volterra series' approach, a kind of 'power series' for nonlinear operators. Though this was not as successful and later got eclipsed by other approaches, it led to significant mathematical developments and remains a landmark in the subject.

But the kind of status Wiener enjoys in the control and communication engineering community is not because of a clever solution of a single hard problem. It is because of the two mental leaps implicit in the solution that revolutionized the field. The first is the systematic use of abstract statistical paradigms in engineering. The second, even more important, is the notion of casting an engineering design problem as an abstract optimization problem that seeks to optimize a performance measure subject to static or dynamic descriptions of the underlying system as a constraint. This way of thinking has now become second nature to control and communication engineers, to an extent that it may be hard for them to believe that it could ever have been otherwise.

Wiener went on to work in what he called 'control and communication in the animal and the machine', a field he dubbed 'cybernetics'. Looking at his influential book on the subject, one finds several topics currently in vogue discussed there, such as learning and self-organization. Although Wiener was by no means the first or the only person to have thought about such things, he is one of the early ones to have held a panoramic view of this circle of ideas, shaping them into a well-defined field of scholarly enquiry.

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Norbert Wiener and Harmonic Analysis

Norbert Wiener made profound contributions to harmonic analysis. In fact, even if Wiener's considerable discoveries in other branches of mathematics are ignored, his contributions to harmonic analysis alone would still place him among the twentieth century greats in the area of mathematics! It would be absurd even to attempt a description of Wiener's contributions to Fourier analysis (another name for harmonic analysis) in a short article. Instead, we will be content to describe here just *one* of his famous results about Fourier transforms, a result which has had considerable impact on mathematical analysis, especially in the area of *spectral synthesis*. In fact, the origins of the subject of *spectral synthesis* can be traced back to the seminal ideas of Wiener and (independently) A Beurling in the 1930's.

An *integrable* or L^1 function f on the real line $\mathbb{R}$ is a measurable function f for which the quantity $\int_{-\infty}^{\infty} |f(x)|\, dx$ is finite. A natural way of measuring the distance between two integrable functions f and g is to consider the quantity $\int_{-\infty}^{\infty} |f(x) - g(x)|\, dx$. One of Wiener's beautiful discoveries is the following: Suppose f_0 is an integrable function such that its Fourier transform $\widehat{f_0}(\lambda)$ is *never* zero, $\lambda \in \mathbb{R}$. Roughly speaking, in physical terms, this says that if we view f_0 as a signal, then *every* frequency $\lambda \in \mathbb{R}$ is present in the Fourier decomposition of $f(x)$. (See the article by Sitaram and Thangavelu in *Resonance*, October 1998.) Examples of such functions are $f_0(x) = e^{-x^2}$ and $f_0(x) = \frac{1}{1+x^2}$. (Exercise: What is $\widehat{f_0}(\lambda)$ in each of these cases?) Then *any* integrable function h can be approximated to within *any* prescribed level of accuracy by finite linear combinations of translates of f_0, i.e. given $\varepsilon > 0$, there exist complex numbers $c_1, c_2, \cdots, c_k$, and real numbers $x_1, \cdots, x_k$ such that

$$\int_{-\infty}^{\infty} |c_1 f_0(x - x_1) + \cdots + c_k f_0(x - x_k) - h(x)|\, dx < \varepsilon$$

This remarkable result is known as the Wiener–Tauberian theorem for L^1 functions[1].

[1] This theorem was also discovered independently by A Beurling, although Beurling couched his result in somewhat different language.