Neutrino Oscillation

Siddheshwar Lal

Oscillation as a quantum mechanical phenomenon between a neutral particle and its anti particle was first envisaged in 1955 to understand the $K^o - \overline{K}^o$ problem. In the study of nuclear interactions produced by high energy particles, coming from outer space in the form of cosmic rays, as well as those produced through accelerators in the laboratory, many new unstable particles like pions, muons, kaons etc., were discovered. Positive, negative and neutral charged K-mesons were also found. The neutral K-meson is produced through the following types of reactions:

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$$\pi^- + p \rightarrow \Lambda + K^o \quad (1)$$

$$\pi^+ + p \rightarrow K^+ + K^o + p \quad (2)$$

The strangeness quantum number S is zero for pions, nucleons (protons –neutrons), –1 for Λ and +1 for K^+. Conservation of strangeness, then demands $S = +1$ for K^o in (1) and –1 for K^o in (2). The K^o with $S = -1$ is written as $\overline{K}^o$ and called antiparticle of K^o. In such a case one could form states with linear combinations of the two:

$$| K_1 > = \frac{1}{\sqrt{2}} (| K^o > + | \overline{K}^o >)$$

$$| K_2 > = \frac{1}{\sqrt{2}} (| K^o > - | \overline{K}^o >).$$

The K^o and $\overline{K}^o$ are distinguished by their mode of production, whereas the K_1 and K_2 are distinguished by their mode of decay. K_1 decays into two pions with a lifetime of 0.9×10^{-10} sec. and K_2 decays into three pions with a lifetime of 0.5×10^{-7} sec. These decays have been observed experimentally. Such a scenario would lead to the process of regeneration. This means, a pure beam of K^o particles, after travelling partly through vacuum and partly through matter will contain a mixture of K^o and $\overline{K}^o$.

This was observed experimentally also and the phenomena were termed oscillations between $K^0 - \overline{K}^0$.

[1] See Anjan S Joshipura, Missing Neutrinos from the Sun. *Resonance*, Vol.2, No.8, 1997.

Neutrino[1]

The beta decay of nuclei at rest visually looked like a 2-body decay consisting of the emitted electron and the daughter nucleus in the final state whereas the continuous energy spectrum of the emitted electrons and considerations of spin and statistics indicated it as a 3-body decay. Pauli, in a desperate attempt to explain this apparent anomaly, suggested the existence of a new neutral particle with almost zero mass which could be emitted along with the electron to make it a 3-body decay. Fermi who worked out the theory for the beta decay christened this particle neutrino. Neutrinos are emitted along with electrons and muons and hence a subscript is added to indicate this like υ_e and υ_μ. Similarly, neutrinos emitted with positively charged particles are written simply as υ whereas those with negatively charged particles are called anti-neutrino and written as $\overline{\upsilon}$ Reines and Cowan [1] in a beautiful experiment carried out first at Hanford reactor and later at the Savannah River reactor confirmed the existence of $\overline{\upsilon}_e$ (anti-electron neutrino). Davis [1] through another beautiful experiment showed that υ_e (associated with electrons) is not the same as $\overline{\upsilon}_e$ (associated with positron). Pontecorvo then argued [2] that there could be possible oscillations between υ_e and $\overline{\upsilon}_e$. In 1962, Danby and others [3] demonstrated clearly that $\overline{\upsilon}_e \neq \upsilon_\mu$ that is neutrinos associated with electrons are not the same as those associated with muons.

Solar Neutrino

Energy production in the sun takes place through p-p and CNO chain of fusion reactions, which occur at a high temperature of about 1.7×10^7 K inside the sun. In both the chains protons fuse to form He, through various intermediate nuclear reactions, which produce high-energy photons and υ_e. For the last three decades scientists have been detecting these so-called solar neutrinos. It is found that in all the experiments the observed flux of solar neutrinos is only 50 – 60% of the flux expected on the basis of standard solar model (SSM)[2]. Since the parameters used for SSM have been found satisfactory from other considerations, it was imperative to look for different reasons for this deficiency in the flux of observed solar neutrinos. One suggestion was the existence of a new kind of weakly interacting massive particle (WIMP) which could affect the SSM, lowering the temperature in the regions inside the sun. The low temperature will slow down the rate of nuclear reactions and hence flux of neutrinos emitted by the sun may be less then expected. Since there are no other indications for the existence of WIMP, they have not been taken seriously. This left the possibility of oscillations between υ_e and υ_μ as a possible solution to the deficit in the observed flux of solar neutrinos. The neutrinos emitted in the fusion reactions inside the sun are all υ_e type and the detectors based on the earth are designed to detect υ_e only. Pontecorvo [4] suggested that since the υ_e emitted by the sun has to travel very large distance before being detected in the ground based detectors, some of them may turn into υ_μ through oscillation phenomena. These υ_μ will be missed by the ground-based detectors (which only detect υ_e).

[2] See B N Dwivedi, Unruly Sun Emerges from Solar Space Observatory, SOHO, *Resonance*, Vol.2, No.9, 1997 and Ashok Ambastha, Probing the Solar Interior, *Resonance*, Vol.3, No.3, 1998.

Neutrino Oscillations

Let us calculate the probability for oscillation where a neutrino of one type (flavour) changes into a neutrino of another type. One essential condition for the oscillation to take place is that different types of neutrinos have different masses. As of now, we know three types of neutrinos – electron, muon and tau neutrinos, along with their distinct anti particles. Existence of electron and muon neutrino has been established experimentally but the existence of tau neutrino is surmised from the kinematical considerations in the reaction:

$$e^+ + e^- \rightarrow \tau^+ + \tau^-$$

$$\tau^- \rightarrow \mu^- + \bar{\upsilon}_\mu + \upsilon_\tau$$

$$\tau^+ \rightarrow \mu^+ + \upsilon_\mu + \bar{\upsilon}_\tau$$

If lepton flavour is not conserved and neutrinos have finite masses, then mixing may occur between the neutrinos of different flavour. For simplicity we consider only two of them, viz., υ_e and υ_μ. Let $|\upsilon_e>$ and $|\upsilon_\mu>$ indicate states which have flavour but do not have definite masses. And let $|\upsilon_1>$ and $|\upsilon_2>$ represent mass eigenstates which do not have definite flavour. We can write the flavour state as a combination of the two mass eigenstates through the unitary transformation

$$\begin{pmatrix} \upsilon_e \\ \upsilon_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \upsilon_1 \\ \upsilon_2 \end{pmatrix} \quad (3)$$

where θ is the mixing angle with maximal value as $\pi/4$. We start with a beam of $|\upsilon_e>$ at time $t = 0$ propagating with momentum **p**. We can write

$$|\upsilon_e, t = 0> = |\upsilon_1, t = 0> \cos\theta + |\upsilon_2, t = 0> \sin\theta$$

Let the state evolve in vacuum up to time t when we will have (using natural units where $c = h = 1$)

$$|\upsilon_e, t = t> = e^{-iE_1t}|\upsilon_1, t = 0> \cos\theta + e^{-iE_2t}|\upsilon_2, t = 0> \sin\theta \quad (4)$$

where

$$E_1 = \sqrt{m_1^2 + p_1^2} \; ; \; E_2 = \sqrt{m_2^2 + p_2^2}$$

Since we are considering spatial coherence, the states $|\upsilon_1(t)>$ and $|\upsilon_2(t)>$ will have same momentum $\mathbf{p} = \mathbf{p}_1 = \mathbf{p}_2$. The states $|\upsilon_e(t)>$ and $|\upsilon_\mu(t)>$ are eigenstates of the momentum but not of the mass. Using the inverse of (3) the R.H.S. of (4) can be written as:

$$\begin{aligned} & e^{-iE_1t}(|\upsilon_e> \cos\theta - |\upsilon_\mu> \sin\theta)\cos\theta \\ & + e^{-iE_2t}(|\upsilon_e> \sin\theta + |\upsilon_\mu> \cos\theta)\sin\theta \\ & = |\upsilon_e> [e^{-iE_1t}\cos^2\theta + e^{-iE_2t}\sin^2\theta] \\ & + |\upsilon_\mu> [-e^{-iE_1t} + e^{-iE_2t}]\sin\theta\cos\theta. \end{aligned}$$

Hence the amplitude for $|\upsilon_e> \rightarrow |\upsilon_\mu>$ will be given by $\sin\theta\cos\theta(e^{-iE_1t} - e^{-iE_2t})$. We can, therefore, write the probability for the state $|\upsilon_e>$ at time $t = 0$ to be in the state $|\upsilon_\mu>$ at $t = t$ as

$$\underset{\upsilon_e \rightarrow \upsilon_\mu}{P(t)} = \sin^2\theta \cos^2\theta \, |e^{-iE_1 t} - e^{-iE_2 t}|^2$$

$$= 1/4 \sin^2(2\theta)[1+1-2Re\,(e^{+iE_1 t} \quad e^{-iE_2 t})]$$
$$= 1/2 \sin^2(2\theta)\,[1-\cos(E_1 - E_2)\,t].$$

Now $E_1 - E_2 = (m^2_1 + p^2)^{1/2} - (m^2_2 + p^2)^{1/2}$.

Assuming $E \cong p >> m_1, m_2$, we get

$$E_1 - E_2 \cong (m^2_1 - m^2_2)/2E = \Delta m^2/2E$$

We can then write

$$\underset{\upsilon_e \rightarrow \upsilon_\mu}{P(t)} = 1/2 \sin^2 2\theta .\ 2\sin^2(t\,\Delta m^2/4E).$$

The expression inside the bracket for $\sin^2$ should be a dimensionless quantity for numerical calculations. Note that the terms exp. $(-iE_1 t)$ and exp. $(-iE_2 t)$ in (4) should have been exp. $(-iE_1 t/\hbar)$ and exp. $(-iE_2 t/\hbar)$; similarly, the terms m_1 and m_2 should have been $m_1 c_2$ and $m_2 c_2$ in erg units. Using natural units we had put $c = \hbar = 1$. Hence we get $\sin^2(t\,\Delta m^2 / 4E\hbar)$ where Δm^2 is in eV^2 and E is in eV. We also write $\hbar$ in eV and then the expression becomes

$$\sin^2(t\,\Delta m^2 \times 1.602 \text{ x } 10^{-12}) / (4E \times 1.054 \times 10^{-27})$$

where Δm^2 is in eV^2, E is in eV.

We now write $t = L/c$ where L is the distance travelled by υ_e with energy E and velocity c. Further, let us express L in metres and E in MeV and then the expression becomes

$$\sin^2(1.267\Delta m^2\, L/E).$$

We then write the probability $P(L)$ for a υ_e with energy E MeV to change into υ_μ due to oscillation in travelling distance L

$\Delta m^2 = m^2_1 - m^2_2$	Neutrino energy E			
	1MeV	10MeV	1GeV $L_{osc.}$	100 GeV
1 $(eV)^2$	10 m	100 m	10 km	10^3km
$10^{-3}(eV)^2$	10 km	100 km	10^4 km	10^6km
$2.5.10^{-7}(eV)^2$		d_{eff}		
$1.6.10^{-10}(eV)^2$		1 AU		

Table 1.

metres as

$$\underset{\upsilon_e \to \upsilon_\mu}{P(L)} = \sin^2 2\theta \, \sin^2(1.267 \Delta m^2 \, L/E)$$

where Δm^2 is in eV^2, E is in MeV and L is in metres. For convenience we introduce $L_{osc.} = 4\pi E/\Delta m^2$ as the oscillation length. Envisaged values for $L_{osc.}$ for different E and Δm^2 are taken from Mossbauer [5] and given in *Table 1* where $d_{eff} \cong 10^8$m $\cong$ effective diameter of the solar fusion core; one astronomical unit = 1 AU = mean distance between sun and earth. In experiments to detect neutrino oscillation, values of E and L are usually predetermined. One then plots the estimated values for the probability $P(L)$ in the parametric space covered by Δm^2 and $\sin^2 2\theta$.

Experimental Observations

Experiments to look for neutrino oscillations have been in progress for quite some time using (i) sun (ii) cosmic rays and (iii) reactors and accelerators in the laboratory as neutrino sources. Range for the mass parameter, Δm^2, for different neutrino sources [5] is given in the *Figure 1*.

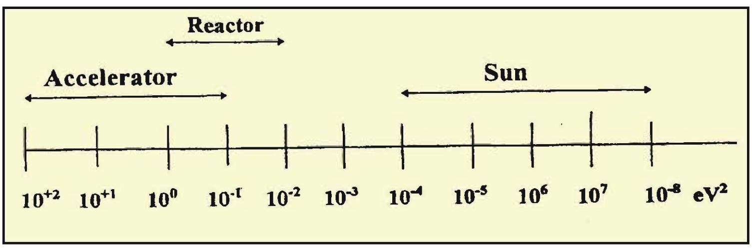

Figure 1. Mass parameter range Δm^2 of neutrino oscillation experiments for different type of neutrino sources.

(i) **Solar neutrinos:** As mentioned earlier fusion reactions through *p-p* and CNO chains in the sun produce υ_e. There are three on going experiments, which detect solar neutrinos through the radiochemical technique. The flux of solar neutrinos detected in the experiments is compared with the expected flux on the basis of SSM. The deficit in the flux is attributed to disappearance of solar υ_e into other flavours due to oscillation during their journey from the sun to the detectors. In the Homestake Gold Mines in South Dakota, USA, the detector consisting of 615 tons of perchloroethylene is kept 1500 m below ground. The υ_e interacts with the chlorine nuclei to produce radioactive Ar, which is collected and counted and from this data flux of υ_e emitted from the sun is estimated. The idea of accuracy involved can be imagined from the fact that only one of the incident solar neutrinos reacted every few days with a single nucleus of chlorine to produce Ar through the reaction $^{37}Cl + \upsilon_e \rightarrow {}^{37}Ar + e^-$ The threshold energy $E_\upsilon \approx 0.8$ MeV. The GALLEX experiment in the GRANSASSO mountain range in Italy has 30 tons of gallium dissolved as a chloride solution. The υ_e from the sun interacts via $^{71}Ga + \upsilon_e \rightarrow {}^{71}Ge + e^-$ and has a threshold energy $E_\upsilon \approx$ 0.233 MeV. Here the counting of activated ^{71}Ge nuclei involves further chemical treatment before its decay into ^{71}Ga can be counted in a special low-activity proportional counter. Finally, the flux of ^{71}Ge so determined gives the flux of solar neutrinos. The SAGE experiment (Soviet American Gallium Experiment) is being done in the Caucasian Mountains in Russia where the detector consists of 57 tons of metallic gallium. The background in SAGE is lower than in GALLEX.

The Super-Kamiokande experiment is a collaboration between USA and Japan. Here the detector is 50,000 tons of ultrapure water in a tank kept 1000 metres below ground in Central Japan. The neutrinos produce charged particles like μ/e, which in turn produce Cerenkov radiation, which is detected. The Cerenkov radiation also indicates the direction of the incoming particle. In their latest result [6] the authors claim evidence for neutrino oscillations and hence for nonzero masses for neutrinos. This might have bearing on the 'missing mass' in the universe. Complete details of the results are not yet available. The Sudbury Neutrino Observatory (SNO) is a collaboration between Canada, UK and USA and is located 2070m below the ground in a nickel mine in Ontario. This experiment uses heavy water and is expected to give results by 1998. Heavy water as a detector is expected to tell the difference between υ_e and the other two flavours (υ_μ and υ_τ) through the break-up reactions of the deuterium nuclei in the heavy water. Two types of reactions are possible by the υ_e in heavy water,

(i) $d + \upsilon_e \rightarrow p + p + e^-$
(ii) $d + \upsilon \rightarrow p + n + \upsilon$

The first reaction is a charged current interaction and can be caused only by the υ_e. The second is through a neutral current and can be caused by neutrinos of all flavours. From the products detected, one can get υ_e flux and the total υ flux. This can give definite information about neutrino oscillation.

Krastev and Petcov [7] had analysed the data available upto 1992 and found that a small region of values of the two parameters characterising the oscillations, $\Delta m^2 \cong (0.55 - 1.1) \times 10^{-10}$ eV^2 and $\sin^2 2\theta \cong 0.75 - 1.0$ was allowed. Later on they have analysed [8] the data from Homestake, GALLEX and Kamiokande and find that their earlier results on the allowed regions of the parameters Δm^2 and $\sin^2 2\theta$ are ruled out as a solution to the solar neutrino problem in terms of two neutrino oscillations in vacuum.

(ii) **Cosmic ray neutrinos**: Strongly interacting particles of cosmic rays, in their interactions with the atmospheric nuclei, produce pions, muons and kaons among other particles. These, in their decays, produce υ_e and υ_μ. The expected ratio of $(\upsilon_\mu + \bar{\upsilon}_\mu)/(\upsilon_e + \bar{\upsilon}_e)$ is about 2. However, observations show this ratio to be nearly 1. Recent data by the Kamiokande group [9] reports zenith angle dependence of this ratio which makes neutrino oscillations a strong candidate to understand this anomaly because the change in the flux can be related to different travel lengths for different zenith angles for the atmospheric neutrinos before they reach the ground based detectors. These results again give a possible range of values for the pair of parameters viz., $\sin^2 2\theta$ and Δm^2 to fit the observed data.

(iii) **Reactor/Accelerator neutrinos:** Nuclear fission reactors produce $\bar{\nu}_e$ (antielectron neutrinos) with energy up to about 8 MeV. Even if oscillation takes place, the energy available is not sufficient to produce charged leptons, which can be observed. Hence in such experiments one only measures the decrease in the intensity of the neutrinos of a given flavour. A beam of $\bar{\nu}_e$ of known flux from the reactors is allowed to travel a certain distance and then their intensity is measured. These $\bar{\nu}_e$ are detected through the reaction. $\bar{\nu}_e + p \rightarrow n + e^-$ An anomalous behaviour in the spectrum of the neutrinos as a function of distance L and energy E would be indicative of neutrino oscillations. So far no experiment has reported a positive evidence for neutrino oscillations from reactor experiments.

The neutrinos available from the accelerators are of much higher energy and hence capable of producing charged leptons, which can be detected. The liquid scintillation neutrino detector (LSND) uses the Los Alamos meson physics facility (LAMPF). Here accelerated protons produce π^+ mesons which decay via the reaction $\pi^+ \rightarrow \mu^+ \nu_\mu$ followed by $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$. This supplies $\bar{\nu}_\mu$ with a maximum energy of 52.8 MeV which are incident on a tank filled with 167 metric tons of dilute liquid scintillator located 30m from the neutrino source. If the $\bar{\nu}_\mu$ produce ν_e due to oscillations, the $\bar{\nu}_e$ will produce e^+ and n through the reaction $\nu_e p \rightarrow e^+ n$. The neutron is captured producing a photon of 2.2 MeV through $np \rightarrow d\gamma$. Thus the signature for an oscillation event is the detection of e^+ followed by a 2.2 MeV photon correlated with e^+ signal in space as well as time. The background events originated by ν_e are suppressed by kinematical constraints. The LSND experiment [10] is consistent with oscillations at $\sin^2 2\,\theta = 10^{-2}$ and $\Delta m^2 \geq 1\,\mathrm{eV}^2$. The authors have given a value

Suggested Reading

[1] (i)F Reines and C L Cowan. *Phys. Rev.* 113. 273, 1958. (ii)R Davis. *Phys. Rev.* 97. 766, 1956.

[2] B Pontecorvo. *Sov. Phys. JETP*. 7. 172, 1958.

[3] G Danby et al. *Phys. Rev. Lett.* 9. 36, 1962.

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[7] P I Krastev and S T Petcov. *Phys. Rev. Lett.* B285. 85, 1992. and 299. 99, 1993.

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[9] Y Fukuda et al. *Phys. Lett.* B.335. 237, 1994.

[10] C Athanassopoulos et al. *Phys. Rev. Lett.* 77. 3082, 1996.

[11] A Romosan et al. *Phys. Rev. Lett.* 78. 2912, 1997.

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of (0.31 ± 0.12 + 0.05%) for the probability for $\bar{\upsilon}_\mu - \bar{\upsilon}_e$ oscillation with the energy of $\bar{\upsilon}_\mu$ being up to 52.8 MeV in a path length of 30 m.

Romosan et al [11] have done a detailed statistical analysis of the data collected by the CCFR collaboration experiment done with Fermi lab, Tevatron. The CCFR detector consists of 18 m long 690 ton total absorption target calorimeter with a mean density of 4.2 gm/cm^3 followed by a 10m long toroidal spectrometer. The neutrino source is located 1.4Km. before the neutrino detector and produces υ_μ, $\bar{\upsilon}_\mu$, υ_e and $\bar{\upsilon}_e$ with energy between 30 – 600 GeV with a mean value of 140 GeV. Flight length for υ_μ varies between 0.9 and 1.4 km. The results of this analysis exclude oscillations in the region of $\sin^2 2\theta > 1.8 \times 10^{-3}$ with $\Delta m^2 > 1000$ eV^2 and $\sin^2 2\theta = 1$ with $\Delta m^2 > 1.6$ eV^2. The authors claim that their result is the most stringent limit to date for $\Delta m^2 > 25$ eV^2 and excludes the high Δm^2 oscillation region favoured by the LSND experiment.

One may, therefore, say that so far there is no confirmed evidence for neutrino oscillations using reactor and accelerator experiments. Cosmic ray neutrinos do indicate possibility of neutrino oscillations. Results from some other experiments including KARMEN at the Rutherford Laboratory in UK are expected in the near future. The two experiments planned with accelerators, CERN and Fermi Laboratory, allow the neutrinos to travel hundreds of kms of path length before hitting the detector. One therefore, hopes that by 1999 we may have more exciting results. We have considered neutrino oscillations in vacuum only. One may also anticipate neutrino oscillations in matter originating from MSW (Mikheyev–Smirnov–Wolfenstein) effect. This becomes relevant when one considers that neutrinos produced inside the sun have to travel through dense matter before they reach the surface of the sun and escape. But discussion of MSW effect is beyond the scope of this article.

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