Linear Algebra

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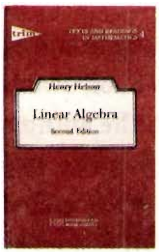

Linear Algebra
Henry Helson
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The book under review is comparatively slim, and yet by treading a carefully chosen path the author covers as much ground as many other standard books on linear algebra.

The book starts with solutions of systems of equations and by way of *Gaussian elimination* and *row reduction* quickly arrives at the *Fredholm alternative* (Theorem 3, page 13).

Vector spaces have been introduced in the second chapter along with the related concepts of *subspace, linear dependence, basis* and *dimension* of a vector space. The *cardinality invariance of bases* has been established using the results on systems of equations. Rightly, *inner products* have been introduced early and full use has been made of the standard inner product on $\mathbf{R}^n$ and on $\mathbf{C}^n$.

The author is of the view that the applications of linear algebra are not as obvious as those of calculus, for instance, and hence the subject should be learnt for its own sake first, before coming to meaningful applications. Therefore, the text has not been overloaded with applications which may otherwise have impeded the flow. The method of *least squares* in statistics is presented as the first application of the ideas involving *dot product*, *Gram–Schmidt orthonormalisation* and *linear equations*.

The chapter on *linear transformations* is short but contains all the essentials including *change of basis, self-adjoint, normal, unitary* and *positive operators*, etc.

The chapter on *determinants* is concise, sharp and in a very economic manner covers all the properties of determinants. The uniqueness of the determinant as a skew-symmetric n-linear functional has been established. This chapter should take the dread out of establishing the properties of determinants in detail in the class room. It concludes with a discussion of *volumes* in n-dimensions.

In the initial sections of the chapter on *reduction of matrices*, after discussing *eigenvalues* and *eigenvectors*, the author, with surprising ease, enters the heart of the subject matter of self-adjoint operators to establish the *spectral theorem*. The results of the first three sections are then used in the lucid section on *quadratic forms*. Next come the *upper triangular* and *Jordan canonical* forms of a matrix. This chapter contains a method of obtaining the largest or the least eigenvalue for a self-adjoint operator using the familiar methods of calculus. The *polar decomposition* of a linear operator on $\mathbf{C}^n$ as a product of a unitary and a positive semi-definite operator (similar to that of a complex z in the form $z = |z|e^{i\theta}$) is then established and the chapter concludes with an application to *factor analysis*, again in

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statistics!

The final chapter on *matrix factoring* ensures that a person studying linear algebra is familiar with the numerical techniques in the area such as the LU-factoring of a matrix into a product of a lower (L) and an upper triangular (U) matrix or the *Choleski decomposition* of a positive semi-definite matrix as LL^t where L is a lower triangular matrix with non-negative diagonal elements.

Of course, a book of such a size as the present one cannot contain everything that we feel the students should know. Two of the important topics that need more explicit attention are minimal polynomials of linear operators and simultaneous diagonalisation of commuting operators, although these are touched upon in an indirect way in the exercises. We hope that these topics will be included in a future edition. Also, those using the book for self-study would find it more helpful if the chapter on matrix factoring is enlarged with a few more examples and exercises.

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Reproducing 'Still photograph of a localized vibration. Exposure time was 1 sec' in colour for more clarity. See for details *Resonance* Vol.3, No.10, pp.58, 1998.

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