Editorial

N Mukunda, Chief Editor

Fourier series, Fourier integrals, Fourier transforms – you see them wherever you turn, whether in physics or chemistry or electrical or communication engineering. It is amazing that Fourier's mathematical idea – born in the course of solving the problem of heat conduction – has been so successful and applies to so many practical situations. We give you a lot of Fourier in this issue – an Article-in-a-box by Sitaram and Thangavelu tracing the transition from Fourier series to Fourier integrals; the first of several articles by Vijay Chandru and M R Rao on linear programming, to which too our friend had made crucial early contributions; the magic of the Fast Fourier Transform, captured by V U Reddy; and a brief life sketch of Jean Baptiste Joseph Fourier by Sitaram on the inside back cover.

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Thinking about Fourier brings to mind many deep ideas in mathematics and in physics linked to his name. Within mathematics, his work can be said to mark the beginning of orthogonal function expansions. In 'recent' times we have the work of Wiener and of Harald Bohr on almost periodic functions; and the growth of the theory of distributions (like Dirac's delta function) or generalised functions. Reading some of the books of the Russian school on this last, one gets the impression that the attitude is: if the Fourier transform of an object is something we can reasonably well contemplate, then by hook or by crook we will show you how to make sense of the object itself! In probability theory we have the beautiful concept of the characteristic function, Bochner's theorem and so on. All this by no means exhausts all that we can say about Fourier within mathematics.

Turning to physics, we all know how much Fourier's methods are used in wave propagation problems, classical electromagnetic

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theory, diffraction theory and problems of signal processing and communication. But the deepest of all is in quantum mechanics: how could Fourier have guessed that a century after his time, physics would find that the position and the momentum of a particle are mutually complementary, cannot be simultaneously measured, and obey an uncertainty principle because they are Fourier conjugate to one another? And the same for energy and time? Behind the formulae of Planck, Einstein and de Broglie linking the particle and wave aspects of matter and radiation lies the magic of Fourier! One cannot avoid the eerie feeling that Nature knew Fourier theory all along!

This brings me to the notion of rigour in mathematics. It is good to realize – again – that rigour and precision are matters of mathematical evolution. In Fourier's day and in his own work, their levels were not particularly high. But sad experiences have taught mathematicians that one must be careful! To quote from history, here are passages from an account by I M Yaglom of the work of Sophus Lie, the Norwegian mathematician who created the theory of continuous groups a century ago:

> *"Today Lie's books and articles may seem archaic in some ways, and not always up to the standards of rigour achieved in the same questions by modern mathematicians. ... In Cartan's view, Lie's memoir fell so short of the standards of rigour prevailing in Cartan's time, that he regarded it as a stimulus for the imagination rather than a predecessor's research work to be continued."*

But probably Lie understood the situation best of all, for "he believed, quite sensibly, that any 'natural' mathematical theory should be transparent, and ... difficulties in mathematics usually arise not from the essence of the problem but from badly conceived definitions at the base."

So there you are – do you feel transformed?