An Introduction to Measure and Integration

A Good Text Book for Demanding Students

K Viswanath

A good understanding of measure theory is essential for mathematics students at the post-graduate level. The book under review is a good addition to the standard texts on the subject. A special feature is the extensive historical and motivational discussion of the drawbacks of the Riemann integral and the method of overcoming them. At every step, whenever a new concept is introduced, the author takes pains to explain how the concept can be seen to arise naturally from a careful analysis of the situation. Exercises are interwoven with the text and those needed later are clearly marked with a bullet symbol.

The book attempts to be comprehensive and largely succeeds. It includes a proof of Ulam's theorem on the impossibility of extending the Lebesgue measure to the class of all subsets of the real line, a discussion of the cardinalities of the sigma-algebras of Lebesgue sets and Borel sets, a treatment of improper Riemann integrals, the Lebesgue version of the fundamental theorem of integral calculus and the Jacobian theorem on the change of variables in Euclidean spaces. An appendix even contains a proof of the inverse function theorem. The only topic not here which the reviewer would like to see included is the concept of conditional expectation as an application the Radon–Nikodym theorem and its connection to projections on a Hilbert space.

There are 10 chapters in this book. The first two examine the drawbacks of the Riemann integral and the methods of extending it. Chapter 3 gives a good exposition of the extension theory while Chapter 4 discusses Lebesgue Measure. Chapter 5 deals with integration. The remaining chapters cover product measures, L^p spaces, the Radon–Nikodym theorem and signed measures. The text can be used for either a one-semester or a one-year course at the MSc level.

The book is clearly a labour of love. The exuberance of detail, the wealth of examples and exercises and the evident delight in discussing variations and counter examples all attest to that. The proofs are carefully detailed. (The only mistake which came to notice is in the proof of property 5 on page 3.) Two negative points of this book are the number of minor grammatical errors throughout the text and the fact that the cross-referencing numbers are frequently wrong. But these can be easily rectified and should not detract from the many merits of this book.

All in all, the book is highly recommended to serious and demanding students of mathematics. But then such students seem, alas, to be an endangered species these days.

K Viswanath, Department of Mathematics, University of Hyderabad, Hyderabad, India