Game Theory

2. Conventions and Knowledge [1]

P G Babu

This article tries to outline what game theory is all about. Part II explores concepts such as repeated games, social norms and common knowledge.

The author is an assistant professor at Indira Gandhi Institute of Development Research, Mumbai.

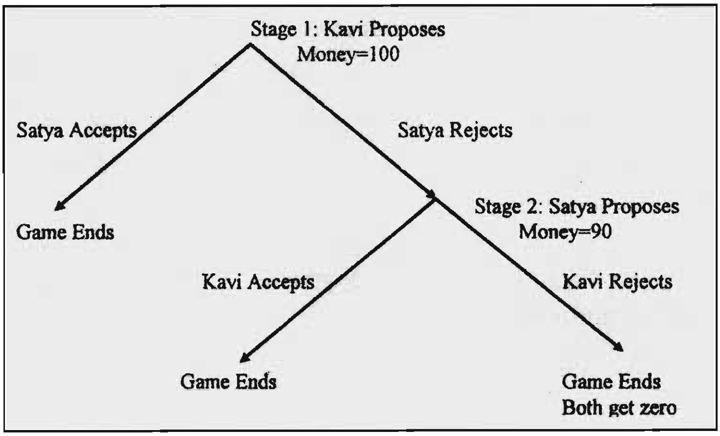
[1] This paper is based in part on a lecture given at the Centre for Theoretical Studies, Indian Institute of Science,Bangalore as part of a lecture series on 'learning and evolution'. I am grateful to Vivek Borkar and Sanjay Jain for this opportunity. I thank the editor-in-charge of the paper for valuable comments. This is an expository article; no originality, other than that of the organization of material, is claimed.

[2] See *Resonance*, Vol.3, No.7, 1998.

Sequential Moves

In part 1[2] of this article, we examined various types of simultaneous move games and solved for Nash equilibria. Let us move further and introduce temporal structures into our games. Our next story is that of two compulsive bargainers.

Kavi and Satya try to divide Rs.100 which their father has agreed to give them, provided they mutually agree on a division. Kavi begins the bargain by proposing the share of the money which he wants for himself. If this is agreeable to Satya, the game ends then and there. But, if Satya disagrees, the game moves on to the next round with the proviso that the money now shrinks to Rs.90. In the second round it is Satya's turn to make the offer. If Kavi agrees the game ends; but, if he disagrees neither of them get any money and the father walks away. This temporal structure, or the game tree is common knowledge to Kavi and Satya. How



much does Kavi keep for himself in the first round?

The above structure of the game (game tree) is known in technical terms as an extensive form game. In such a game, the sequential nature of the players' moves is explicitly modelled. All the information related to who moves when with what information and what are the action choices available to them and when it is their turn to move are laid out explicitly. Such a game modelling approximates the reality better than a normal form game.

To solve such games, one uses backward induction logic. Start from the bottom of the game tree. In stage 2, if Satya proposes to keep all of Rs.90 for himself, Kavi will reject the offer and the game will end with no money for both. Going back to stage 1, Kavi will know for sure that if the game goes to stage 2, Satya will propose Rs.90 and he would end up getting nothing. So, the best he can do is to propose Rs.10 for himself in the first stage. Satya will immediately accept this offer as he can get only that much (viz., Rs.90) if the game moves to the second stage. This game will end in the first time period itself and the first mover has a disadvantage.

Now, assume that the game can go to a third stage where Kavi gets the last move. The similar logic will tell us that Kavi will now possess the last mover advantage. If the game ends in even time periods, the first mover loses; on the contrary, if it ends in odd time periods, the first player to move gains. The solution procedure we have used is known in the technical parlance as 'subgame perfect equilibrium' due to Reinhard Selten for which he won a Nobel Prize in Economics. Note that this solution concept is basically a further refinement of Nash equilibrium. Let us take a small detour in the next section to take a quick look into the notion of common knowledge.

Subgame perfect equilibrium is basically a further refinement of Nash equilibrium.

Common Knowledge

All along I have been dropping this term 'common knowledge' often without saying much about it. Let us look at the following story to grasp this concept which forms the foundation of game theory. The story I am going to recount goes back to Littlewood's *Mathematical Miscellany* (1953). Imagine three girls sitting in a circle, each wearing either a red hat or a white hat. Suppose that all the hats are red and that the students were sleeping when the hats were put on their head by a naughty teacher. As a result, the students do not know which colour hat they are wearing. Now, the teacher enters the class and disturbs their cosy nap and on top of it asks each student to identify the colour of her hat. The students are of course confused as to what the teacher is talking about; but, they can see that the other students are indeed wearing a hat and feel their head to realize that there is a hat on their head too. They are not allowed to remove their hat to see the exact colour. They all cut a sorry figure and admit that they do not know the colour of their respective hats.

Suppose now that the teacher takes pity on the poor kids and decides to give a hint. The hint is given through a remark which the teacher makes in the class: 'there is at least one red hat in the room'. This is a fact well known to all three students as they can very well see at least two red hats in the room. Even though the hint sounds innocuous, it changes the situation dramatically. The teacher then repeats the same question and allows the students to answer one after the another. The first student still cannot tell the colour of her hat and says so; nor can the second student. But, the third student after hearing the answers of the first two can tell that she is wearing a red hat. How can the third student suddenly change her answer? What is at work here? At this stage, I will only utter a magic word, 'common knowledge'.

To solve it we need to get a little more formal. The set of all possible states of the world (for our story) are given by $\Omega = \{a, b, c, d, e, f, g, h\}$; the possible states are enumerated below for your convenience.

$$a = RRR,\ b = RRW,\ c = RWR,\ d = RWW,\ e = WRR,\ f = WRW,\ g = WWR,\ h = WWW.$$

Given the teacher's remark, if the first girl looks around and finds the second and third girls wearing the red hats, then she will conclude that the true state has to be either a or e. But, she will not be able to distinguish between the two states. Hence, if the first girl sees two red hats around her then she will only know that either state a or e has occurred. Similarly, if she finds one red hat and one white hat, then again she will know that either b or f has occurred. By similar reasoning, we get the other possibilities.

This kind of (set theoretic) classification of the various possible states of the world according to what one knows is defined as *information partition* in the technical parlance. If one does not have much information to begin with, then the information partition will be coarser[3]. As we get more and better information, the partition gets finer. Let us see how this happens. The initial information partitions for the three girls (using similar reasoning process as that of the first girl) in the class will be:

[3] Imagine $P_1 = \{a, b, c, d, e, f, g, h\}$. In this scenario, the first girl knows almost nothing probably because she is so dazed she cannot even make out the hat colour of the other girls.

$$P_1^{\,1} = \{\{a, e\}, \{b, f\}, \{c, g\}, \{d, h\}\}$$
$$P_2^{\,1} = \{\{a, c\}, \{b, d\}, \{e, g\}, \{f, h\}\}$$
$$P_3^{\,1} = \{\{a, b\}, \{c, d\}, \{e, f\}, \{g, h\}\}.$$

Here, the subscript stands for the students and the superscript stands for the level of information. For example, at level 1 only the teacher's remark is known to all. But at the next level, in addition to the teacher's remark, they will also know student 1's answer.

By our assumption, the true state is a. The event that the player 1's hat colour is red is given by, $E = \{a, b, c, d\}$. Then, agent 1 is informed of $P_1(a) = \{a, e\}$ and thus knows that the true state is either $a = RRR$ or $e = WRR$. But, she cannot conclude about her hat colour as $P_1(a) \not\subset E$ and declares so.

Now, the second girl reasons as follows: the fact that the first girl is not able to tell her hat colour means that state h is not possible. Why? Because if the first girl had seen two white hats on the heads of the rest, then this fact in combination with the teacher's remark would have convinced her that she is indeed wearing a red hat. The same logic holds good for state d too. As a result of the first student's answer, the second student's information partition becomes finer:

$$P_2^2 = \{\{a, c\}, \{b\}, \{e, g\}, \{f\}\}$$

Does this enhanced information set help student 2 to arrive at her answer? She cannot find the answer as there is no way she can identify her hat colour using any of these information partitions. For example, by our initial assumption of all red hats, f cannot happen. At state b she sees at least one red hat; but that in conjunction with the teacher's remark reveals nothing new. At the information partition $\{a, c\}$ again she is not sure whether she is in state a or state c. Note that $\{e, g\}$ cannot happen either.

Let us see whether the negative replies of the first two students in conjunction with the teacher's remark reveals any new insight to student 3. The states b, d, h, f are not possible. That would result in 3's new information partition:

$$P_3^3 = \{\{a\}, \{c\}. \{e\}, \{g\}\}.$$

You can now easily replicate the reasoning process which leads student 3 to an affirmative answer.

Notice that in our earlier games we have implicitly assumed that the rationality of players (viz., they are interested in maximizing their own payoffs), the structure of the game, and the action sets are common knowledge. Without this assumption, the kind of reasoning which the players in our earlier games have used to arrive at the optimal strategy would fail; check this out in the case of location game.

If you are reluctant to reason out, consider the following age old story. There are two generals belonging to the same country who are waiting on opposite hill tops with their armies. In the valley between these two hills, the enemy is stationed. The war situation is such that the generals can win only if they coordinate their attack on the enemy in the valley simultaneously. As our story takes place at a time when there were no telecommunications, our generals are forced to send a messenger to agree on the time of their attack. The messenger has to cross the valley to reach the other general. But, he might be caught by the enemy in the process in which case the general would not know if the other got the message or not. Hence, to know that the message has been read by the other general, it is not enough if the messenger reaches the other hill top safely and delivers the message; he has to come back and report that he has handed over the message. How can the other general be sure that the messenger has gone back safely? Now then, the same logic keeps unfolding and the generals will never be able to coordinate their attacks.

Some of you might object to this outcome saying that it might be due to archaic communication methods. To those of you who think so, I strongly recommend the modern version of the above story told by Ariel Rubinstein (1989) where the messenger has been replaced by an unreliable electronic mail. If you delve deep into the reasons for this outcome, you will find that the problem is due to the fact that messages sent through either a messenger or electronic mail can never become common knowledge.

With this additional knowledge about common knowledge, let us get back to our first story (of part 1).

Repeated Game

Our first story, viz., that of two unfortunate prisoners is famous for just reasons. We said earlier that the only possible solution for the prisoners is to plead guilty. Now, that solution raises an interesting question. Why cannot these two prisoners coordinate on the mutually best outcome, viz., not to plead guilty. In other words, why can not they cooperate?

Consider repeating this game a few times. That is, these two prisoners are engaged in such joint crimes quite a few times and are caught every time. Use the backward induction logic similar to that of our 'compulsive bargainers' story. Start at the last stage. Because this is the last time, even if they had agreed before hand to cooperate, there is nothing which prevents them from breaking that pact and making a one-time gain; in other words, there is no punishment possible for reneging on their earlier oral commitment to cooperate. Hence, it is not in their interest to cooperate with each other in the last stage. Go back to the last but one stage. The same logic unfolds convincing us that any finite repetition of the prisoners' dilemma game will always yield the dismal strategy, viz., to plead guilty.

What happens if there is uncertainty about when the repetition will stop? In other words, we are considering an infinite repetition of the game. Now, we need to introduce individual's time preference into our model. The value of Re.1 today may be quite different from that of tomorrow or the year after; we all talk about inflation. Hence, we need to devalue the payoffs we receive in the future. This is what is known as the discount factor.

We can now easily see that for certain discount factor values, the strategy which stipulates 'cooperation till the opponent does it and punishes the opponent ever after if he deviates from it' produces cooperative solution in the prisoners' dilemma game. But, unfortunately one can sustain whole lot of other strategies as equilibria as well. For example, consider Axelrod's 'Tit for Tat' strategy. According to this, I will do what you do. If you cooperate, I will cooperate; if you do not, then, I also do not choose to cooperate. If one can get tit for tat as equilibrium, one can also get 'tit for two tats' as an equilibrium. This depressing result (depressing because it has multiple Nash equilibria) is known as 'Folk theorem' as it was discovered independently by many game theorists in sixties and seventies.

How can one select one of these many Nash equilibria? Any reasonable theory should tell you how to choose from among many competing solutions. For example, we have been talking in terms of people playing Nash equilibrium strategies in all our decision situations narrated earlier. Why or using what process do, people come to learn to play such strategies? This deep question has led the game theorists to inquire into a variety of interesting issues such as formation of conventions and in general, the process of learning. We turn now to this important question.

Why does a human society get into one of many types of conventions?

Norms and Conventions

The above is not a purely abstract question. For example, any human society can get into one of many types of conventions. But, why in reality do we find the societies sticking to one and not the other. Why do we Indians not accept torn rupee notes? How do rumours spread so fast across a vast geographical distance? Why do we have stereotypes such as South Indians and North Indians? These are a few of many examples of one of the most striking regularities of human life, viz., *localized conformity*.

The crux is that 'learning by observing others' can explain the conformity, idiosyncrasy and fragility of human behaviour. When people can observe other's behaviour, they often end up making same choices and hence *conformity* results. If the early movers err, followers are likely to imitate too, hence we get to see *idiosyncratic* choices. If later on, a few people start behaving differently (for example, from observing the mistakes of others), a sudden phase change occurs whereby the old convention is swept away by a new one yielding *fragility*.

Consider Gary Becker's analysis of restaurant pricing (Becker, 1991). Two sea food restaurants in Palo Alto serve the same kind of food with similar prices, comparable service and amenities. Yet one is half empty. Becker being the epitome of rationality, wonders why the popular restaurant cannot increase its prices so as to maximize profits and reduce the long queues. Becker's reasoning leads him to believe that people would like to dine in an 'in' restaurant. By raising prices, one might reduce the long queues and inadvertently lower the 'in'ness of the restaurant. If the price is raised a little too high, this process would feed on itself and the conspicuously large clientele will soon vanish into the thin air (probably into the neighbouring restaurant).

The crux is that 'learning by observing others' can explain the conformity, idiosyncrasy and fragility of human behaviour.

Why should it happen so? Those ahead of you in the queue give you some information. Perhaps the food is good here. And perhaps those ahead of you know the quality of the food better than you do. So, you follow them, even if your own limited information suggests a different course of food. Thanks to this *self-reinforcing* tendency, the seller has a strong incentive to induce early movers to buy (or eat). Hence, one sees the practice of introductory offers for new products in real life.

I will narrate one personal experience which confirms this hypothesis. During my visit to Jerusalem, I (along with some others) was hunting for a Falaffel shop. Falaffel is the most popular vegetarian food in Israel. We were walking all over the city centre and finally chose one shop, called 'King of Falaffel' shop at the King George Street. There was a Falaffel shop right next to this one selling for nearly the same price. But, we did not go there. Now, reasoning backwards, I think we chose this shop just because it had lots of crowd. Much later we found out to our amazement that this is the only Falaffel shop in entire Jerusalem to sell Falaffel for three shekels (Israeli currency): *Localized conformity*!

If you are not convinced by this example, consider a more sobering experience of submitting a scholarly manuscript to an equally scholarly journal for a second time (which I presume most of you would be doing soon). If the second set of referees know that the paper was rejected by some other journal, he or she would certainly form a preconceived notion about the quality of the paper and chances are that the paper would be rejected. If it becomes widely known, even a good paper might never get published. You can also think about the possible havoc played by the gaps in one's resume; even a strong applicant might get frozen in the job market.

An informational cascade occurs whenever the information implicit in the predecessors' actions is so conclusive that a rational follower will unconditionally imitate them, without regard to the information from other sources.

We categorize such situations under the heading, '*informational cascades*'. An informational cascade occurs whenever the information implicit in the predecessors' actions is so conclusive that a rational follower will unconditionally imitate them, without regard to the information from other sources. Such cascades, as they are usually based on very little information, can easily be reversed. The system bounces around randomly until it reaches a point of precarious stability.

One can think of other alternative theories to characterize similar situations. A *theory of conformity based on payoff interactions* could explain situations where the choice of actions of one might increase the benefit to the other person if he or she chooses to do the same thing as the first person. Essentially, one evaluates the benefits and costs of being different from others and concludes that it is best to conform.

Becker's restaurant example which we used earlier might suggest another feasible theory, viz., *conformity preference*, where people directly prefer to do the same things that others are doing. There could be *parallel reasoning*, where every one is wise enough to reach the same conclusions independently[4]. One could think of *direct communication* as another option wherein those who could figure out the best choice are good enough to explain the benefits to other lesser mortals. But, note that by construction, the last two theories cannot explain why mass behaviour is error prone.

To set the stage starkly, transcend now to an English countryside in eighteenth century. Two horse drawn carriages are approaching each other from opposite directions. Since roads are bad, both are hogging the middle. The coachmen could now decide to either pass on the right or left. There is no convention to help them make an easy decision. This decision problem could be depicted as an one-shot game similar to that of depicted in *Figure 4* of part 1. The strategies here will be right and left instead of movie and saree and players names will be coach 1 and coach 2 respectively.

There are three possible Nash equilibria for this game. Both can go left, both can go right or both can randomize between right and left with equal probability 1/2. The last strategy, that of randomizing, is what is known as a 'mixed strategy'.

Classical game theory gives no clues as to what the coachmen would do in such a situation. How does one decide? In part 3, we will try to figure a way out.

Suggested Reading

[1] Becker G S. A Note on Restaurant Pricing and Other Examples of Social Influences on Prices. *Journal of Political Economy*. 99. 1109–1116,1991.

[2] Rubinstein A .The Electronic Mail Game: Strategic Behavior under Almost Common Knowledge. *American Economic Review*. 79. 385–391,1989.

[4] But, note that even when the entire population is supposedly wise, phenomena such as 'group think' might arise. Recall the McCarthyism in the United States of fifties.

Address for Correspondence
P G Babu
Indira Gandhi Institute of Development Research
Gen. A K Vaidya Marg
Goregaon(East)
Mumbai 400 065, India.