Game Theory

1. Nash Equilibrium

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This article tries to outline what game theory is all about. It illustrates game theory's fundamental solution concept viz., Nash equilibrium, using various examples.

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The Genesis

In the late thirties, the mathematician John von Neumann turned his prodigious innovative talents towards economics. This brief encounter of his with the day's economic theory convinced him that it was in need of a new mathematical tool. In the years that followed, he along with Morgenstern went about creating a brand new mathematical tool (anticipated by Borel and before him by Cournot). This new tool was offered to the profession in their now classic book '*Theory of Games and Economic Behavior*'. In this book, they developed two person zero sum games and other cooperative game theoretic concepts. But, soon economists found out that the phenomenon of 'one person's gain is the other person's loss' was too restrictive in many applications. After some initial euphoria, the interest in this new tool died down except for a small hard core group of mathematicians who continued to work on these concepts. Princeton was the epicentre for most of them. Hence, it is not a surprise to see young Nash Jr taking the next giant step in the Fine Hall of Princeton towards what we now know as 'modern non-cooperative game theory'.

Game theory can be viewed as an interactive decision theory. It deals with the situations where people with different (mostly competing) goals try to take into account others' actions in deciding on the optimal course of action. Take for instance chess. When you decide what move to make, you also take into account the likely response of the opponent and your next response, his reply and so on. The fact that your opponent has

Box 1. John Nash Jr.

John F Nash Jr came to Princeton for his graduation in mathematics with a one line recommendation which said "This man is a genius" Within two years of his arrival, when he was just twenty, Nash completed his 21 page doctoral dissertation under Al Tucker; this dissertation lays down much of what we now know as modern game theory, with the famous equilibrium concept (which stands in his name) to analyse n-person, non-zero sum situations as the centre piece. Even before his arrival at Princeton, Nash Jr, as an undergraduate student, wrote a term paper for a course on international economics (his only encounter with economics). This term paper later on became one of the all time great papers in game/economic theory which laid the foundation stone for bargaining theory. Soon afterwards, Nash Jr was struck by schizophrenia and lost his next thirty years to this disorder and recovered miraculously in 1989 to win the Nobel Prize in Economics for the year 1994 along with John Harsanyi and Reinhard Selten.

equal intelligence and self interest enables you to duplicate his reasoning process. For example, consider two companies producing the same product and competing in the same market. Their aim would be to capture as much market share as they can using various ploys such as price cutting, gift schemes, and advertising. Which one of these ploys would be successful given the opponent company's ploy? Game theorists are interested in exploring such situations.

Eternal Dilemma

The best way to learn game theory is through playing games. Let us begin with one of the most popular games, viz., *prisoners' dilemma*. This game has been attributed to Al Tucker of Princeton University. In this game there are two thieves who have been caught by the police and brought before the magistrate. As there is only circumstantial evidence to their crime, the magistrate comes up with the following clever scheme. He locks them up in separate cells in such a way that they cannot communicate with each other. If they both plead guilty, they get 10 years imprisonment each. On the other hand, if both plead not guilty they get away with 1 year prison terms. But, if one pleads guilty

		PRISONER 1	
		CONFESS	DON'T CONFESS
PRISONER 2	CONFESS	-10, -10	-0.5, -15
	DON'T CONFESS	-15, -0.5	-1, -1

Figure 1. Prisoners' dilemma.

and the other not guilty, then the one who pleads guilty gets 0.5 years and the other gets 15 years imprisonment. The question now is: did the magistrate do the right thing by offering this scheme to the prisoners?

The actual game structure of this decision situation is given in *Figure 1*. This structure is known as the normal form game where the sequences of moves and countermoves (in other words, the temporal structure) are suppressed. Players are assumed to move simultaneously and choose one of the two actions: to plead guilty or not guilty. The actions of the two players result in an outcome. An outcome is generally marked by the payoffs. These payoffs are given by the numbers appearing in each cell corresponding to their actions. The first element stands for the row player's payoff and the second corresponds to that of the column player. Players are assumed to be rational economic agents who are interested in maximising their payoffs. The following features of the game are common knowledge ('everyone knows it, everyone knows that everyone knows it, ...'): the rationality of the players, the action choices, and the payoffs.

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Let us see if the magistrate did the right thing or not. Look at the row player. His best payoff is 1 year prison term but to get that he needs to plead 'not guilty'. But, his co-conspirator now has the incentive to plead guilty as she can get away with 6 months imprisonment. Given this fact along with the knowledge that she is a rational player leads the row player to believe that if he were in her position he would choose to plead guilty. With this reasoning at the back of his mind, he prefers to plead guilty. The column player also reasons along the same lines and opts to plead guilty. Note that pleading guilty is the only possible solution in this game. Hence, both end up pleading guilty vindicating the magistrate's scheme.

This game brings about the contradiction between the individually rational outcome (both pleading guilty) and what is collectively good for all (both pleading not guilty). It also shows that the resulting solution to the game could be Pareto inefficient (See *Box 2*).

Box 2. Pareto Efficiency

Pareto efficient solution is one where the players cannot in any way improve their current payoffs through a different action choice without reducing others' payoffs. In the above game, the outcome where both the prisoners are 'pleading guilty' can be improved upon by both deciding to plead 'not guilty' without hurting the other player's payoff. Hence, we conclude that the unique Nash equilibrium of the game is Pareto inefficient.

Note that conjectures about the opponent's play has no role in picking the final solution in this game. The equilibrium concept we have used to solve the game is what is known as the Nash equilibrium: choose the best action from among your action set given that the opponent will choose her best action. Of course, we assume that the action sets, payoffs and the game structure are common knowledge. This game can be used to study various issues such as: two firms competing to sell the same product (say, a toothpaste), two nations erecting trade barriers and the exploitation of common property resources [1] such as fisheries.

[1] These are the resources for which no one has exclusive property rights and hence none have any incentive to protect the resource or optimally use it.

Notice that the key to arriving at the Nash equilibrium outcome in the above game is the individual's ability to duplicate other's reasoning process. But if you do not know the other person's characteristics, his tastes, ability, ideology, etc., then you can not reason what he will do in a given situation. In real world, such lack of knowledge about one's opponent is the norm. For a long time, game theorists did not know how to formulate such a situation. This proved fatal to the applicability of game theory to real world problems. In this situation, John Harsanyi provided a breakthrough. In a three part paper written in early 1960s, he showed how one can formulate games where people do not have much information about each other. It would be apt to say that Harsanyi was the one who resurrected game theory's applicability. This formulation of Harsanyi is however beyond the scope of this paper.

Box 3. John Harsanyi

Harsanyi, born in a pharmaceutical family in Hungary (1920) proved his early mathematical prowess by winning the first prize in high school mathematics in the whole of Hungary. Later on he vacillated between various subjects including botany and finally went on to earn a doctorate in philosophy from the University of Budapest. In the aftermath of the second world war, Harsanyi migrated to Australia where he worked as a factory worker and later as a clerk for the association of the coal mining industry. To improve his career opportunities, he learnt economics during his spare time and wrote a few papers in international economics to win a job as a lecturer at the University of Queensland. From there he went to Stanford University in the USA on a Rockefeller fellowship and wrote his second doctoral thesis, this time in economics, in 1959. After a few years in Australia, he went back to the USA to join the faculty of University of California, Berkeley from where he retired in 1990. Harsanyi's academic career offers a complete contrast to the fairy tale career of John Nash Jr along with whom he shared a Nobel Prize in 1994.

Rational Pigs

Let us move on to the next game situation (*Figure 2*). This is a story of two rational pigs. Though both the pigs are rational, one is weak (called subordinate pig) while the other one is strong (christened as dominant pig). These two pigs are put in a cage. There is a lever at one end of this cage which when pressed delivers (for quantification) exactly six grains of maize at the other end. Now, if the pigs want food, they need to learn to press the lever and run to the other end. If the subordinate pig presses the lever, the dominant pig can eat all the six grains. On the contrary, if the dominant pig is the one to press the lever, the subordinate pig can eat up five grains

Figure 2. The rational pigs.

		DOMINANT PIG	
		PRESS	DON'T PRESS
SUBORDINATE PIG	PRESS	1.5, 3.5	-0.5, 6
	DON'T PRESS	5, 0.5	0, 0

before the dominant pig arrives. It is observed that once the dominant pig reaches the grain dispenser, it can get everything that is remaining. In the unlikely event of both the pigs pulling the lever together, the subordinate pig (given its lean constitution) can run faster and eat two grains. Pressing the lever and running to the grain dispenser involves expenditure of some calories and for the sake of quantification, we assume that it can be measured in grain units, viz., 0.5 grains. Who will learn to press the lever?

The above normal form game corresponds to the decision problem facing our two rational pigs. For the subordinate pig, whatever the dominant pig does, not pressing the lever is the best strategy (in the sense that its payoff is higher than the other strategy 'pressing the lever'). Now, what would the dominant pig do? It has to form conjectures about the subordinate pig's behaviour. If it believes that the subordinate pig would press the lever, then, the best strategy available is to wait near the dispenser. But, putting itself in the subordinate pig's shoes, the dominant pig would realise that the subordinate pig will not press the lever. Given this logical conclusion, it is optimal for the dominant pig to press the lever and get 0.5 grains net compared to starvation. Hence, the dominant pig will learn to press the lever. At times, weakness could be strength[2].

[2] Remember what the first husband of Mia Farrow had to say about her in Woody Allen's movie 'Husbands and Wives'.

In this game there is no conflict between individual rationality and collective rationality unlike the prisoners' dilemma game.

Setting the Shop

Let us move on to the third story. Here we face two pav bhaji vendors on Juhu beach trying to find proper locations for their shops. They have to charge the same price to survive but, they can locate anywhere they want on a straight line beach. Their customers are spread out all through the beach and are in general averse to walking. Hence, they would prefer to buy from a shop which is closest to them. Where should the pav bhaji vendors locate their shops?

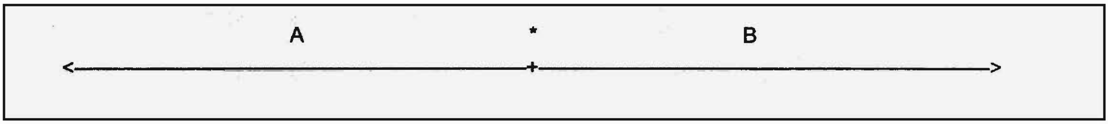

Figure 3. Location game.

Unlike our two earlier games, here players are forced to form dynamic conjectures. Given the nature of the problem, all the customers to the left of, say, pav bhaji vendor A will buy from him and those to the right of B would buy from her. Of those in between them, half will go to A and the rest to B. Now, A reasons as follows: if I can shift my shop one yard closer to B then I can get that many extra customers; but, B being a rational seller will reason the same way and hence she would also move a yard closer to my location. Given that, I need to go two yards closer to B and she would also do the same reasoning and move two yards closer.Hence, I need to go three yards closer... . Ultimately, this logic will stop when A decides to locate the shop right next to that of B and B also does the same thing and it is not difficult to figure out that this location will be exactly in the middle of the beach.

Unlike the previous games, here the logic is fairly subtle. In the previous games, either both or at least one player had a unique best response to whatever the other player did. In this location game, both the players have to simultaneously form conjectures about their rival's choice.

There are many real world situations which bear out the logic inherent in this game. You would have noticed that petrol pumps are located close to each other. The same is true for shops selling foreign goods: witness the Burma Bazaars of Madras, Bangalore and Trichy. Also, next time when you fly notice how different airlines schedule their flights close to each other on a given route.

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One too Many

You might have noticed that in some sense the previous location game involved dynamic reasoning. In all the games we have seen

		WIFE	
		MOVIE	SAREE
HUSBAND	MOVIE	1, 1	0, 0
	SAREE	0, 0	1, 1

Figure 4. Battle of the sexes.

so far, there have been more or less unique solutions. Could it be that Nash equilibrium always yields an unique solution? Alas! How I wish it were true! But, unfortunately the answer to that question is 'no'. See for example the following game called 'battle of the sexes'. There is a husband and his wife who have to decide where to go for the evening outing. The husband wants to go to a movie while the wife wants to go to the saree shop. Being newly married, wherever they go, they go together. Their problem can be formalised as follows: A careful perusal of this game will tell us that there are two Nash equilibria (to be precise, two pure strategy Nash equilibria). One is when both the husband and wife coordinate on movies and the other when they decide to go to a saree shop. We cannot a priori predict which one of these two solutions will occur. This example alerts us to the problem of multiplicity of Nash equilibria which is a rule rather than an exception in real life decision situations.

So far we have not explicitly introduced the sequential (or temporal) nature of moves by players. In the next part we will address this in a more realistic structure.

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Suggested Reading

[1] Nash J F. Equilibrium Points in n-person Games. *Proceedings of the National Academy of Sciences (USA)* 36. 48–49, 1950.
[2] Nash J F. Non-co-operative Games, *Annals of Mathematics*. 54. 286–295, 1951.
[3] von Neumann J and Oskar Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press. Princeton. NJ, 1944.

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