



In this section of Resonance, we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

Question raised by
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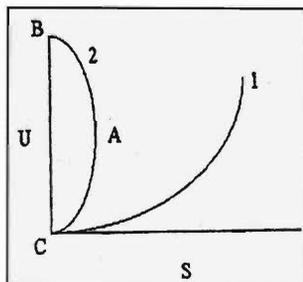
What are Negative Absolute Temperatures?

In Thermodynamics temperature is defined as the rate of increase of internal energy U of a system with its entropy S , keeping other independent variables, like volume, constant i.e.

$$(\partial U / \partial S)_{\alpha, \beta, \dots} = T; \alpha, \beta, \dots = \text{independent variables.}$$

Take a system like a gas. The internal energy arises mainly from the kinetic energy of the gas molecules which can increase without an upper bound. Both the internal energy and entropy increase as the gas becomes warmer and the variation of U with S at constant volume is shown by curve 1 of *Figure 1*.

Figure 1. The slope of this graph $(\partial U / \partial S)_{\alpha, \beta, \dots}$ is positive everywhere and the system will only have positive temperatures T . Most physical systems show such a behaviour.



The slope $(\partial U / \partial S)_{\alpha, \beta, \dots}$ of this graph is positive everywhere and the system will only have positive temperatures T . Most physical systems show such a behaviour. In paramagnetic salts we have atoms with magnetic moments arising from electron spins. The electron spins interact among themselves with a spin-spin relaxation time τ_s . They also interact with the lattice with a spin-lattice relaxation time τ_l . The relaxation time τ_s is a measure of how rapidly the spins exchange energy with one another and the time τ_l is a measure of how rapidly the spins exchange energy with the lattice. Since τ_s becomes very long at low temperatures while



τ_s remains still reasonably short, at low temperatures we may consider the spins and lattice as two independent thermodynamic systems very weakly coupled to each other. This is not possible in a gas of paramagnetic atoms in which the relaxation time τ_l is very short.

Suppose the spin system consists of n atomic spins which are capable of two orientations in a magnetic field with energies ϵ_1 and ϵ_2 ($\epsilon_2 > \epsilon_1$). Such discrete orientations is a quantum mechanical behaviour of spins. The energy is a minimum when the magnetic moment of the atom points in the direction of the applied magnetic field and maximum when it points in a direction opposite to the magnetic field. Then a general state of the system will have n_1 spins in the lower energy state and n_2 spins in the higher energy state such that

$$\begin{aligned} n_1 + n_2 &= n \\ U &= n_1 \epsilon_1 + n_2 \epsilon_2 \end{aligned}$$

The minimum value of U will be when $n_1 = n$ and the maximum value when $n_2 = n$. The entropy of the state with energy U is calculated from the number of ways W one can choose n_1 spins out of a total of n spins (ie.)

$$W = {}_n C_{n_1}$$

which takes a value 1 for both U_{\min} and U_{\max} . The entropy is related to W through the Boltzmann relation

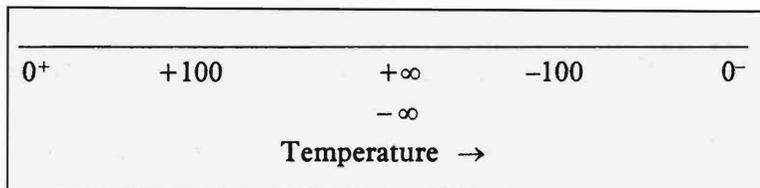
$$S = k \ln W$$

and is zero when U has a minimum or a maximum value. For other values of U , S is positive. The variation of U as a function of S for this system is sketched as curve 2 of *Figure 1*. From C to A , U is an increasing function of S and so the system has a positive temperature. From A to B , U is a decreasing function of S and the system has a negative temperature. At A , T can be both $\pm \infty$.

Thus for a system with the internal energy being bounded, there



Figure 2.



is a possibility of a negative temperature. If a body with a higher internal energy is deemed to be warmer than a body with a lower internal energy then it follows that the temperatures have to be ordered on an increasing scale of hotness as shown in *Figure 2*.

Is a body at a negative temperature really warmer than a body at a positive temperature? To answer this question we put the bodies in contact. Let the body at a positive temperature T^+ acquire a quantity of heat (dQ^+) and the body at negative temperature T^- acquire a corresponding quantity (dQ^-). The first law of thermodynamics states that

$$dQ^+ + dQ^- = 0$$

The second law states that the total change in entropy in this process should be positive

$$\begin{aligned} (dQ^+ / T^+) + (dQ^- / T^-) &> 0 \\ \text{or } dQ^+ (1/T^+ - 1/T^-) &> 0 \end{aligned}$$

Since the quantity within the bracket is positive it follows that dQ^+ is positive and dQ^- is negative. Thus the body at temperature T^+ gains heat at the expense of the body at the temperature T^- . The body at temperature T^- is therefore warmer than a body at temperature T^+ . As heat flows the body at temperature T^- gradually cools down to $T^{-\infty}$ and then to a positive temperature T_f while the body at temperature T^+ warms up to T_f , at which point the two will be in thermal equilibrium.

One can apply a sufficiently large magnetic field at low temperature to orient most of the magnetic moments parallel to the field so that the system has a total energy slightly greater than U_{\min} . The system has a positive temperature. If the magnetic field is suddenly reversed without giving time for the spins to reorient

themselves, the energy of the majority of atoms is now ϵ_2 and the system has an energy a little less than U_{\max} . The spins reach equilibrium among themselves in a time τ_s with the same energy. This is a state with a negative temperature. At a negative temperature the higher energy level is more populated than the lower energy level. This is called population inversion.

Since the spin system is coupled to the crystal lattice weakly, one may temporarily have a spin system with a negative temperature and a lattice with a positive temperature. As time progresses the spin temperature changes from negative to positive as stated above and after a long time both spin and lattice will reach the same positive temperature. At this positive temperature the higher energy spin level will be less populated than the lower energy spin level.

Answered by
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The concept of negative temperature does not negate the statement that $T = 0^+$ is the lowest temperature.



Whose Name Goes on the Paper?

Under Hugh Richard's direction I had done a couple of experiments on boron-10 which demonstrated that some very beautiful work done by a Caltech group headed by T Lauritsen and W A Fowler was wrong. The sequence of states I then determined is still correct, and there was considerable theoretical interest in our results at the time. I say "our" results advisedly since Hugh had more to do with their interpretation than I did. But he insisted, over my strong objections, in not putting his name on the resulting papers. He had then, and continued to have throughout his career, the notion that the graduate students should get all the credit. He is a very unusual professor!

Fay Ajzenberg-Selove

A Matter of Choices – Memoirs of a Female Physicist

(Rutgers University Press, New Brunswick, New Jersey, 1994), pp.72-73, 317)