The Explosion That Wasn't

A P Gore

There are many phenomena in nature where numbers increase fast as time progresses. Indian population has been increasing exponentially [1] roughly at a compound rate of about 2% per year. Insects can have a generation time of just a week or two and their numbers grow quickly so that in a short period a healthy looking crop can be wiped out by herbivorous insects, similarly in the fungi and viruses. But such populations can crash when they run out of food or shelter or light etc. This saves ecosystems from being swamped by just one or a few species. Malthus, the 18th century British economist clearly saw this and warned that in humans the population crash may take the form of famine, pestilence or war unless prudent leaders prevail upon the society to voluntarily slow down the population growth.

A curious instance of such a crash is seen in the so called pyramid schemes floated by conmen. A typical scheme may be something like this. The public is invited to join the scheme by paying a sum, say Rs. 100/-. On being a member one is authorised to recruit upto four members. As soon as their dues (Rs. 400/-) are received by the scheme coordinator the member responsible for this recruitment of 4 new cases, is paid back his contribution of Rs. 100/- with a bonus of Rs. 100/-. If you cannot recruit 4 members, your contribution is retained by the scheme. There may be many other details which are essentially irrelevant. This gives a person a chance to double his money very easily. Hence there is a frantic activity with everyone collecting bonuses and recruiting members. Finally the scheme crashes because there are no new cases to recruit. At n-th stage, the number of new recruits is 4^n which exceeds the pool of potential members sooner or later. All pyramid schemes are doomed from the start. When they fizzle out, the last generation of recruits loses its contributions. That sum is shared by previous recruits with the lion's share going to scheme coordinators.

The case of haemophilia [2] is also equally intriguing. At any point the number of haemophilic males is quite small. It is therefore very clear that many mothers failed to pass on the gene to their sons. In each generation bearers of the haemophilic gene are subjected to an adverse selection pressure and the number is expected to decline from one generation to next. Turning this logic upside down, the number of individuals carrying the haemophilic gene must increase as we go back in time. Indeed J B S Haldane argued on the basis of his calculations that if haemophilia in a generation is only due to inheritance, proportion of haemophilics in a society must be incredibly large, just a few centuries back. This of course is evidently not true. The effects of the gene are fairly dramatic and its widespread occurrence would have echoed in literary, historical and medical records. Hence the assumption behind Haldane's calculations must have been wrong. The error was the assumption that all patients

[1] If we start with a population of 1 crore, then at the above rate in 21 years it will become $(1.02)^{21}$ i.e. it will exceed 1.5 crores. This is exponential growth.

[2] This is a hereditary disease in humans in which bleeding from a wound does not stop soon because blood does not coagulate. It occurs mostly in males but is passed down the generations through females.

of haemophilia get the gene through their parents. The inevitable other possibility is that of new haemophilics or carriers being born of normal parents. This is mutation. If we assume that proportion of haemophilics is stable, then mutation must compensate for selection against the gene. Haldane exploited this logic to estimate the mutation rate.

Haemophilics are expected to decline in proportion under a model (without mutation) but persist. On the contrary sex ratio (number of females per 1000 males in human population) is expected to be around 1000 persistently but is not so. In India over the last century in which systematic population censuses have been conducted, the number of women has consistently declined to around 950 per 1000 men (in Kerala the number is above 1000). This anomaly is explained in terms of (a) neglect of female child and (b) female infanticide.

A recent article[3] by Ohno identifies yet another phenomenon of missing numbers and offers an explanation for it. The counting here is that of one's ancestors. Each human being has 2 parents and 4 grandparents and 8 great grandparents etc. If we go 15 generations back (300 years at 20 years per generation) the number becomes 32,768. At 20 generations (400 years) it crosses a million mark. At 40 generations (800 years) the number crosses 1000 billion mark. This is patently ridiculous since even today when the human population is supposed to have become too large for the resources of this planet, it is well below 10 billion. So something is obviously wrong. The catch is that all ancestors in the same generation do not have to be distinct. Suppose two cousins (their mothers are sisters) marry, then the offspring will have 2 parents, 4 grandparents but only 6 (and not 8) great grandparents. If the mothers of those cousins are stepsisters (i.e. have a common mother but different fathers) the ancestor number will be 7 instead of 6. At the extreme, if there is brother-sister mating in every generation (the custom among ancient Pharoah kings of Egypt) the number of ancestors can be just 2.

The reality lies somewhere between the two extremes of explosive billions on one hand and the miniscule 2 on the other. Ohno suggests that the number of ancestors in the preceding generation is the number in the current generation divided (discounted) by the average sibling size (ASZ), ASZ is 1 if ancestors are all unrelated. If as in the case of cross cousins above, of the four ancestors two are sisters, the average sibling size is $(1+1+2)/3=1.33$. The number of ancestors in generation $n+1$ (going backwards) is given by Ohno's formula $(N_n/\mathrm{ASZ})2=N_{n+1}$. So, as interrelatedness (or sibling interference as termed by Ohno) increases and exceeds 2, the number of ancestors goes on decreasing. The generation at which ASZ becomes exactly 2 is named ANSA (ANcestry SAturated).

We are all tempted to claim descent from some distant illustrious ancestor. Typically the number of ancestors can be quite large and the chance that a particular distinguished individual was one of that large set can be considerable. So the claim need not be dismissed as fantasy. The only caveat is that many others including ordinary men and women as well as deviant individuals such as criminals and conmen are also as much a part of our ancestry.

[3] Ohno Susumu. The Malthusian Parameter of Ascents: What Prevents the Exponential Increase of One's Ancestors? *Proc. Natl. Acad. Sci. USA*, 93. 15276–15278. 1996.

A P Gore, Department of Statistics, University of Pune, Pune 411 007, India.