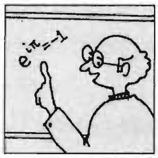

In this section of **Resonance**, *we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.*

1. One Two Three ... Infinity: A Critical Appraisal

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The Grand Scheme of the Book

Few popular science writers can present 'difficult' topics of modern science in a down-to-earth and humorous way as George Gamow could. Among his popular science books, *'One Two Three .... Infinity'* is notable as much for covering a large ground as for the masterly presentation. It discusses the most exciting recent developments of its time not only in the theory of relativity, quantum mechanics, atomic physics and nuclear physics, but also in genetics and cosmology. Explaining abstractions like four dimensions, curved spaces and the molecular basis of life at a popular level without losing rigour is not easy. For one thing, knowledge of imaginary numbers, topology, theory of probability, etc. is needed on the part of the reader. And, of course, a felicity with large numbers is necessary. The book covers all this mathematical ground in the typical Gamow style – that is, with the help of legends, anecdotes, games, treasure hunts and humour.

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It is quite a surprise for most readers that there can be a geometry without any measurement of lengths, angles, etc.

The author starts by giving hilarious examples of large numbers, including the famous one about wheat grains necessary to 'cover' the 64 squares of the chess-board, each square requiring twice the number of grains compared to the previous one. He goes on to talk about numbers which are really infinite, and even explains Georg Cantor's method of comparing infinities of various orders. The representation of imaginary and complex numbers on a two-dimensional plane is made lively by the story of a poor adventurer who missed a big treasure just because he could not locate a reference point given in the map, while a little complex algebra would have shown that the coordinates of that reference point were quite irrelevant because they cancelled out!

The section on relativity shows Gamow at his best. He starts with neither mechanics nor optics but topology and properties of space. It is quite a surprise for most readers that there can be a geometry without any measurement of lengths, angles, etc. Gamow introduces the notion of a curved space by drawing analogies with two dimensional curved spaces which are easier to imagine and 'construct'. By a series of fantastic mental exercises, he tries to enable the reader to visualise a four-dimensional space, and points out that the fourth dimension may be taken as a measure of time. He then explains how a rotation of the space – time axes amounts to a transformation to another frame of reference moving with a constant velocity with respect to the original one. The relativistic phenomena of Lorentz contraction and time-dilation are then explained on the basis of such a rotation of axes.

In the third section on 'Microcosm', the reader is led to successively finer units of the structure of matter: molecules, atoms, nuclei and elementary particles, also coming across the uncertainty principle and quantum concepts. The notions of thermal motion, probability theory and entropy are brought home through facts, games and humour. These notions are then related to genetics and an analysis of the distinction between 'living' and 'non-living' matter. The similarity of the structure of genes with that of viruses is pointed out and the possibility of synthesis of these units is explained.

In the last section on 'Macrocosm', Gamow describes how distances to the moon, the sun, other stars and galaxies are measured. Finally he gives a fascinating account of the birth of the planets of our solar system and life cycles of stars and galaxies.

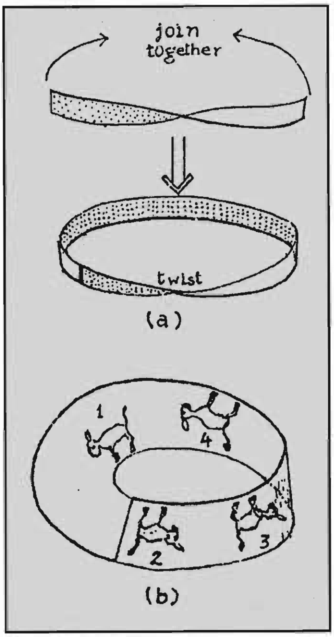
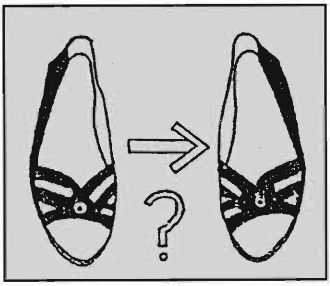


Figure 1. (a) Making a Möbius ring; (b) Gamow's donkey goes round a Möbius ring.

Turning Right Shoes into Left

One of the most interesting games played with curved spaces concerns the right–left inversion of objects like gloves and shoes. Can a right shoe be turned into a left one (without deforming it)? Or, for that matter, can a space traveller come back from his journey around the universe with his heart on the right side of his chest? To take a two-dimensional analogy first, Gamow describes in chapter IV the exotic properties of the *Möbius surface*, which can be made by taking a long strip of paper and gluing the two ends together into a ring *after giving the strip one twist* (*Figure* 1). Now, if a two-dimensional 'shadow-donkey', confined to such a surface, goes around this ring once, he will be converted from a 'left-looking' creature (position 1) into a 'right looking' one (position 4, turned upside down without lifting the donkey off the surface). This feat cannot be performed on an ordinary surface, whether open (say, a plane) or closed (say, a spherical surface). Now, Gamow says, "imagine that our three-dimensional space is also closed upon itself and, in addition, has a 'twist' in it". "In that case", he says, "manufacturers of shoes will have the dubious advantage of being able to simplify production by making only one kind of shoes (for one foot) and shipping one half of them around the universe to turn them into the kind needed for the other half of the world's feet ...." (p. 65) (illustrated in *Figure* 2).

Figure 2. Will a right shoe become left after going once round a curved and twisted universe?



The reader may be warned of a possible pitfall here. If one actually makes a Möbius ring with paper and attaches a paper -

"To be sure, if speeds that approached the velocity of light made time in a moving system run slower, a super-light velocity should turn time backward."

donkey to it (or, to make it easier, takes an iron strip and a magnetic animal), it will turn out that the donkey reaches the 'same point' but on the *other surface* of the strip. So, is it that the left shoes manufactured by Gamow Shoe company, after travelling all round the universe, will not be available for people's right feet in spite of reaching the correct (x,y,z) coordinates? It is interesting that Gamow has himself described two examples of such a 'double space' (pp. 55–56) and pointed out how "a friend of yours could be very close to you in spite of the fact that in order to see him, and to shake his hand, you would have to go a long way around".

Actually, what is meant here by Möbius surface is a pure surface, not a material strip embedded in a three-dimensional space and thus having *two* surfaces. Likewise, if we consider a three-dimensional space, curved and 'twisted' properly, then the left-right inversion promised by Gamow should indeed occur.

Relativity: Can Time Run Backwards?

Another slippery situation arises in chapter V on 'Relativity of Space and Time'. After vividly describing some highly imaginative effects of time-dilation in a situation where the velocity of a space-ship (v) approaches the velocity of light (c), Gamow asks what would happen if v were to be *greater than c*, and gives the following limerick:

"There was a young girl named Miss Bright,
who could travel faster than light.
She departed one day
In an Einsteinian way,
And came back on the previous night",

adding, "To be sure, if speeds that approached the velocity of light made time in a moving system run slower, a super-light velocity should turn time backward" (pp. 104). But, wait a minute! The factor involved in the time-dilation equation,

$(1-v^2/c^2)^{1/2}$, becomes an *imaginary* number, not a *negative* one, as v exceeds c. So, is Gamow wrong in saying that time could be turned backward?

Actually, he is right, although perhaps the point needs to be explained further than it has been in the book. When the spatial separation of two events is larger than the distance light can travel during the time interval between these events, we say their four-dimensional separation is 'space-like'. The time sequence of two such events is not absolute, i.e. a suitable frame of reference can be found in which the time sequence of the two events is inverted. Now, our Miss Bright, having a velocity $v > c$, is obviously travelling along a 'space like' curve. Therefore, it is possible for the event of her arrival at a point to be seen as *earlier* than the event of her departure from a 'previous' point (in a suitable reference frame).

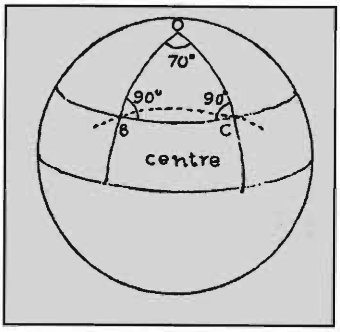


Figure 3 Wrong construction of a spherical triangle: BC is not a 'straight line', i.e. a geodesic. Dashed BC will correct this error, but then the sum of the three angles will not be so much greater than 180°.

Constructing a Spherical Triangle

Further on in the same chapter (p. 106–107), the problem of detecting any curvature in our three-dimensional space is dealt with. Gamow first gives a two-dimensional example: a spherical surface. If you construct a triangle on such a surface, the sum of its angles is greater than 180°! But the construction described by him is not exactly correct. He says, "Indeed, a spherical triangle formed by sections of two geographical meridians diverging from the pole and the section of a parallel (also in a geographical sense) cut by them, has two right angles at the base and can have any angle between 0° and 360° at the top...." (see *Figure* 3). Thus, it appears that the sum of the angles of such a triangle can be anywhere from two to six right angles, *depending solely on the angle at the pole, irrespective of the size of the triangle*. Since any point on a spherical surface can be called a pole, you should be able to construct triangles in your backyard where the sum of the three angles would be 200°, 240°, or 380°.......! Of course, this goes against everyday experience; so we run into a paradox.

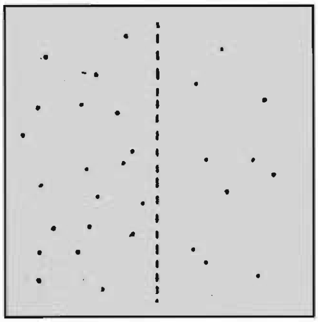


Figure 4. An uneven distribution (2:1) of air molecules in two halves of a room. With a room of usual dimensions (and normal pressure), such a distribution would occur extremely rarely, but with microscopic dimensions, it is expected to occur millions of times every second.

The key for resolving this paradox lies within the same chapter. On a curved surface, the counter-part of a straight line, i.e. the shortest distance between two given points, is called a geodesic. So, each of the sides of our triangle (*Figure* 3) must be geodesic, i.e part of a great circle. (Great circles are those circles on a spherical surface whose centres lie at the centre of the sphere. All meridians and the equator are great circles.) Looking at our 'triangle' OBC, we see that sides OB and OC, being parts of meridians, are indeed geodesics, but BC, which is described as a part of any parallel, is not a geodesic, *unless the parallel is the equator*. Therefore, Gamow's construction does not qualify as a triangle unless it extends from a pole to the equator. On the other hand, if a small triangle like OBC is to be drawn correctly, BC will have to be part of a great circle (shown as dashed line); in that case, angles B and C will not be right angles any more.

Density Fluctuations in Air

Later in the same chapter, another error creeps in when Gamow demonstrates how extremely improbable it is that all the molecules of air in a room gather in one half of the room, leaving the other half completely empty. (In fact, even a 2:1 imbalance is extremely unlikely.) He points out that the result would be very different at a much smaller scale. Replacing the room with a cube of size 10^{-6} cm, which will have just 30 molecules in it, he sets out to calculate how many times per second it will happen that 20 molecules gather in one half on this 'room', leaving only 10 in the other half (*Figure* 4). First he calculates (p. 229) that a reshuffling or randomization of position will take place $5\text{x}10^9$ times per second. Then he says, "... the distribution in which 20 molecules are at one end and 10 molecules at the other end (i.e. only 10 extra molecules collected at the one end) will occur with the frequency of $(1/2)^{10}$ x $5 \text{ x } 10^9$ =......".

Now, this is not quite correct. Gamow is apparently holding 20

Box 1

George Gamow (1904-68), made two of the most fundamental theoretical discoveries in physics – quantum tunneling and the hot dense early universe (big bang). He is also considered one of the most talented and creative popular science writers ever. Ironically (or typically?), his popular article on relativity (which was to become part of his now famous first book, *Mr Tompkins in Wonderland*) was rejected by many magazines. Less than 20 years later, in 1956, his popular writings won him UNESCO's Kalinga Prize and a lecture tour of India. Born and educated in Russia, Gamow left to do research in Europe and settled in the U.S.A. He used to humorously describe himself as on long leave from the Soviet Union. He made important contributions not only to theories of nuclear α and β decay, nuclear resonance, thermo-nuclear reactions in stars, fast contraction of dying stars and the origins of the universe, but also the role of DNA in protein synthesis. *One Two Three .... Infinity* is the most celebrated of his 30 popular science books.

molecules static, 10 in each half, and calculating the probability of the 'extra' 10 being concentrated in one half. This is an over-simplification: actually, all 30 molecules are full participants in this pick - and - choose game. The actual probability of the '20-and-10' distribution is not $(1/2)^{10} \approx 10^{-3}$, as given below, but ${}^{30}C_{10} \times (1/2)^{10} \times (1/2)^{20} \approx 4.5\times10^{-2}$, or 45 times higher. Nevertheless, the broad conclusion, that the above uneven distribution is expected to materialise hundreds of millions of times every second, remains true.

There are some other small errors involving nuclear recoil in beta decay (where momentum is treated as a scalar rather than as a vector), growth in living organisms (where a frog-like embryo in the gastrula stage is shown as developing into a human baby in *Figure* 95), etc. Some calculated results are off by one or more orders of magnitude. The chapter on nuclear and particle physics has inevitably grown outdated in many parts. But altogether, *One Two Three .... Infinity* remains a unique and wonderful book because it makes some of the most abstract notions in contemporary science exciting and accessible to lay readers and students or for that matter to researchers themselves.

Suggested Reading

G Gamow. *One Two Three ... Infinity* (Second Edition) Viking, 1961. Available in Bantam paperback edition.

G Gamow. *Paramanu Se Brahmanda Tak* (Updated Hindi version of the above, Tr. R Popli), Vigyan Prasar, New Delhi (in press).