product of one function with translates of the other.)

After strolling through the first three chapters on topological groups, Haar measure, convolutions, Hilbert spaces and operators one is led to the Peter-Weyl Theorem in the last chapter. Slips and typographical errors are minor and easily rectifiable. Unfortunately, the proof of the last part of the Peter-Weyl Theorem (pp. 110–111) may bemuse an inexperienced wayfarer.

Doing mathematics is an art, though mathematics is often described as a science. An aspirant can develop this art only with direct, hands-on experience. So, anyone whose interests are whetted by this book should look at examples like the rotation group and the group of unitary matrices of determinant one. Supplemented with such examples, this pellucid little book can serve as an excellent text at the M.Sc 2nd year level and as a base camp for expeditions to higher reaches of harmonic analysis and representation theory.

Suggested Reading

- V S Varadarajan. *Harmonic analysis on semisimple Lie groups*. Cambridge, 1986. An enticing invitation to a part of the subject that is both grand and formidable. The first few chapters contain a lucid and illuminating overview of the various facets and sources of motivation for the subject.

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**Alan Turing – The Enigma of Intelligence**

*Priti Shankar*

For nearly thirty years after his death, Alan Turing was virtually unknown outside a small circle of mathematicians and computer scientists. This was partly due to Turing's own nature, and that of the scientific work he was involved in. His contributions to fundamental problems in logic, his crucial role in the cracking of the German Enigma code, and his pioneering work on the stored program digital computer, make him...
undoubtedly one of the most brilliant and original Englishmen of this century. However, Turing's fiercely independent spirit, his honesty and refusal to compromise his own nature – he was unapologetically homosexual – pitched him against the British establishment. His remarkable life was cut short by his suicide in June 1954 at the age of 42. Andrew Hodges' warm and scholarly biography of this versatile genius is written with extraordinary understanding and empathy. Himself a mathematician, Hodges gives an authoritative account of Turing's technical achievements, and the development of his unique personality.

The Turings had close connections with India. Alan Turing's father was a member of the Indian Civil Service. His mother and elder brother were born in India. Alan was conceived at Chatrapur in 1911 and was born on 23 June 1912 in Paddington. The Turings left their sons in the care of a retired Army couple in England, until their final return from India several years later.

Hodges paints a dreary picture of Turing's initial years at Sherborne, a public school which he joined in spring 1926. Many years later, while speaking to his friend Robin Gandy, Turing is reported to have said, "The great thing about a public school education is that afterwards, however miserable you are, you know it can never be quite so bad again". Turing's teachers placed him at the bottom of the class in Latin, Greek and English, found his writing illegible and work slipshod. In mathematics and science, however, the masters wrote approving reports. Turing had found on his own, the infinite series for the inverse tangent function. The mathematics teacher told Turing's form master, that he was "a genius". In Hodges' words, "... the news sank like a stone in the Sherborne pond. It merely saved Alan from a demotion...".

Turing joined Cambridge in 1931. Hodges, in his description of the academic scene at Cambridge at the time, points out that both G H Hardy, the distinguished British mathematician, and Sir Arthur Eddington, whose *Nature of the Physical World* Turing had pored through in 1929, were his teachers. In 1933, Turing attended Eddington's lectures on the methodology of science, and rediscovered the Central Limit Theorem on his own. This was submitted as his dissertation at King's College later on. But Alan Turing's most important discovery was yet to come.

In 1900, the German mathematician Hilbert had posed seventeen unsolved problems to the mathematical world. Then in an international congress in 1928, Hilbert put forth the following three questions. First, was mathematics *complete*, that is, could every statement be either proved or disproved? Second, was mathematics *consistent*, that is, was it true that statements that were obviously wrong could never be arrived at by a sequence of valid steps? Thirdly, was mathematics *decidable*, that is, did there exist a definite method which could, in principle, be applied to a mathematical statement, and which would come up with the *answer* as to whether it was provable? The Czech mathematician Kurt Gödel was able to provide answers to the first two of the three questions; it was left to Alan Turing to settle the third.
In 1936, at the age of twenty four, Turing proposed a machine (later called the *Turing Machine*), with a remarkably simple description (in Hodges' perception, much like the typewriter, that had fascinated Alan as a young boy), which would be able to perform *any* mechanical task, reading input from, and writing output on an infinite tape. A table describing the behaviour of such a machine would have a finite size, every distinct machine having a different table. Turing showed that he could construct one particular machine that could simulate the behaviour of any other machine. Supposing one interrogated this machine, restricting questions to the kind for which 'Yes' or 'No' was an appropriate answer, and one posed the question, "Consider the machine specified as follows ... Will this machine ever answer 'Yes' to any question?" The dots were to be replaced with the description of some arbitrary Turing machine. Turing showed that in general, this question could not be answered by the interrogated machine, thus pinning down a clearly specified example of an *unsolvable problem*. Turing had been stimulated by Hilbert's decision problem - or the *Entscheidungs problem* as it was called in German. He would entitle his famous 1936 paper *On Computable Numbers with an Application to the Entscheidungs problem.* In Hodges' words, "Alan had proved that there was no miraculous machine that could solve all mathematical problems, but in the process he had discovered something almost equally miraculous, the idea of a universal machine that could take on the work of *any* machine." This idea was to have profound implications for the design of the stored program computer to be developed later.

In September 1936 Turing left for Princeton on a visiting position. At Princeton were the elite of the European intelligentsia, among them Albert Einstein, J Von Neumann, Hermann Weyl, Richard Courant, G H Hardy, and S Lefshetz. However, to Turing's disappointment Gödel had returned to Czechoslovakia. Hodges remarks that not many mathematicians were aware of Turing's important paper. In Hodges' words, "He rather expected truth to prevail by magic, and found the business of advancing himself and putting his goods in the shop window, too sordid and trivial to bother with." While at Princeton, Turing wrote up a Ph.D thesis on a topic relating to the implications of Gödel's theorem. At the end of his second year Von Neumann offered him a job as his assistant. Turing however decided not to take it, and chose to return to King's College at Cambridge where he felt he really belonged.

In a chapter appropriately entitled 'The Relay Race' (as all cryptanalytic equipment was built using mechanical relays), Hodges gives a masterly description of Turing's work on the cryptanalysis of the German Enigma codes. Bletchley Park was the location of the Government Code and Cypher School. The German Enigma, used for enciphering radio messages during the second world war, consisted of a sequence of three rotors, each of which performed a substitution on the letters of the alphabet fed in at its input. There was also a plugboard attachment that performed an extra swapping of the letters, both before entering the rotors and after...
emerging from them. The Enigma had, as a result, an enormously large number of possible states. In 1939 Alan Turing was inducted into a small group of cryptanalysts, consisting mostly of mathematicians, physicists and chess champions. The British analysts under the leadership of Turing, came forth with a technique that in 1941 enabled them to decrypt German naval traffic fast enough for the British to take counter measures. Luckily for the British, the Germans ruled out the possibility that the enigma cipher had been broken, and gave the credit to the British secret service; else a double encipherment procedure would have rendered the British scheme ineffective!

In early 1943 Turing spent a few months at Bell Labs in New Jersey. It was here that he met Claude Shannon, with whom he found much in common. Both scientists were interested in the possibility of building a machine that would imitate the brain. In fact, Turing obsessively pursued this idea later on. He returned to Britain after a couple of months. In 1944 he was involved in the design of a speech encipherment system, christened Delilah (... the biblical 'deceiver of men'...). In 1945, Turing began working on the ACE – an electronic universal machine. He perceived this as a random access machine (unlike the Turing machine) with a stored program. The greatest challenge at that time was to build a large fast random access memory. He even envisaged the use of subroutines, which at that time was an entirely new idea. Construction of the ACE officially began in August 1947. The ACE was completed only in late 1949 by which time Turing had left for Manchester. Turing's brilliant essay, 'Can a Machine Think?' was published in 1950. In 1951 Turing was made a Fellow of the Royal Society. The citation referred to his work on computable numbers, completed fifteen years earlier, and Bertrand Russell was one of the sponsors. With his characteristic humour, Turing wrote 'I hope I am not described as "distinguished for work on unsolvable problems".'

By the early fifties Turing's curious and restless mind had moved into the sphere of biology, and was occupying itself with working out a theory of morphogenesis. However his emotional life was still far from satisfactory. Hodges spends a good deal of time in the last two chapters of the book describing the events that led to the trial of Turing on charges of homosexuality. Outwardly, Turing made light of the consequences and continued with his work. Thus, no one was prepared for his sudden suicide on June 7, 1954.

This book is a first rate contribution to history. Firstly, it places Turing in his rightful position as the presiding genius at Bletchley Park, and as the intellectual parent of the digital computer. Secondly it provides a readable account of Turing's technical achievements, for the intelligent lay-person. Finally, it offers an in-depth analysis of the dilemma of an individual who was by any reckoning a national asset but was constantly pitched against bureaucracy and the establishment, and finally became its victim.

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