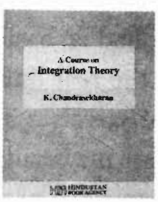
and the role of earthworms. Some of these are placed after the more interesting chapters and this makes them a little boring to read. But they demonstrate an important fact that Darwin was not only a naturalist and a theorist but also an experimental biologist.

Is there any cost associated with being such a genius and original thinker? May be there is. Darwin probably paid it thus "My mind seems to have become a kind of machine, for grinding out general laws, out of large collection of facts.... For many years, I cannot endure to read a line of poetry: I have tried lately to read Shakespeare, I found it so intolerably dull that it nauseates me..... It is a horrid bore to feel, as I constantly do, that I am a withered leaf for every subject, except science."

Shailesh Deshpande and Milind Watve, Life Research Foundation, Pune.

A Course on Integration Theory

B J Venkatachala



A Course on Integration Theory
K Chandrasekharan
Texts and Readings in Mathematics trim(8)
Hindustan Book Agency, Delhi, 1996, pp.118.

A mathematician normally associates the word integration with the names of Riemann and Lebesgue. Lebesgue's theory still remains an esoteric subject at the undergraduate level though the theory of Riemann integral is studied in detail.

The first evidence of integration can be traced back to early Greek geometers. They employed the 'method of division' to give meaning to and calculate areas of plane figures with circular or parabolic boundaries. The first definitive formulation of differential and integral calculus was put forth by Newton and Leibnitz. However they primarily focused on the inverse nature of differentiation and integration. While Gauss and Euler confined themselves to more practical matters of evaluation of integrals, it was Dirichlet, Riemann and Cauchy who gave a conceptual framework for the definite integral as the limit of a sum.

Stieltjes, at the close of the 19th century, showed that the integral is a linear functional and hence was able to generalize Riemann's theory to introduce *Riemann-Stieltjes integrals*. However, there were many shortcomings in Riemann's theory and it was felt that an integration theory which could encompass a larger class of functions was needed. This paved the way for the concept of measure, which was introduced by Borel and Lebesgue at the beginning of 20th century. It was the brilliant idea of Lebesgue of partitioning the range of a function rather than the domain which led to the *Lebesgue* *integral*. This overcame a host of deficiencies in the Riemann integral. The limit theorems like monotone convergence theorem and dominated convergence theorem stand out as the striking examples of the superiority of Lebesgue's theory over Riemann's.

There are two ways of studying any mathematical theory. One can first look at concrete examples and understand in-depth the concepts that go into the making of a theory. Indeed, any generalization comes only from examples. The other way is to first study the most general theory and then specialize it to particular cases. It is a continuing pedagogic debate as to which course should be followed.

K Chandrashekaran follows the second path here and you can see a master at work throughout the book. Indeed, the stress is on conciseness, clarity and accuracy. The measure spaces and integration on a measure space occupy the first chapter and convergence theorems are proved. The novelty lies in presentation and details. The second chapter gives various integral inequalities and the completeness of the Lebesgue space L^1 is proved.

The third chapter deals with outer measures and the classical Hahn- Kolmogoroff theorem on the extension of measures from a ring to the σ–ring is proved. The Lebesgue outer measure is defined and it is shown how one can get the Lebesgue measure from it.

Product measure has been introduced in the fourth chapter and it is shown how multiple integrals can be evaluated using Fubini's theorem. The concept of product measure would have been much appreciated by a student if only it had been shown that Lebesgue measure on $\mathbf{R}^n$ via outer measure is identical with the product measure on $\mathbf{R}^n$

Set functions and their derivatives occupy the last chapter which culminates with the Radon-Nikodym theorem and the Lebesgue decomposition of a measure. Absolutely continuous functions have also been dealt with here. More information on such functions and their characterization would have delighted the reader.

This book is ideally suited for students at the post-graduate level who have had a basic course on Lebesgue theory on the real line. A major drawback is the lack of exercises in the book. A student can appreciate the beauty of a theorem if she can use it to solve problems. A student therefore should read this book along with a good source of problems such as *Real and Abstract Analysis*, Hewitt and Stromberg, Springer-Verlag Publishers.

In conclusion, this book is a good addition to any library.

B J Venkatachala, Department of Mathematics, Indian Institute of Science, Bangalore 560 012, India