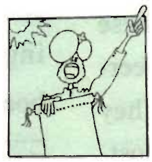

India Wins 1 Gold, 3 Silver and 1 Bronze Medals at the 37th IMO

The 37th International Mathematical Olympiad recently concluded at Mumbai, India. 424 students from 75 countries participated in the event. Romania, USA and Hungary were ranked at the first 3 positions. India was ranked 14 in its overall performance.

Ciprian Manolescu of Romania performed brilliantly. He was the only student who secured the full point 42. The following are the results of the Indian participants:

1	Ajay C Ramdoss, Bangalore	Gold
2	Kaustubh P Deshmukh, Pune	Silver
3	Ashish Mishra, Ranchi	Silver
4	Rishi Raj, Ranchi	Silver
5	K Gopalakrishnan, Madras	Bronze
6	Ashish Kumar Singh, Kanpur	No medal

Ajay is a student of National Public School, Rajajinagar, Bangalore. Rishi Raj, the youngest participant from India is a student of the 10th standard.

The six problems which appeared in the 37th IMO are reproduced below. (The name in the brackets refers to the country which proposed the problem.) Problem 5 turned out to be the toughest; only six of the 424 participants solved it fully.

First day (10 July, 1996)

1. Let $ABCD$ be a rectangular board with $|AB| = 20$, $|BC| = 12$. The board is divided into 20×12 unit squares. Let r be a given positive integer. A coin can be moved from one square to another if and only if the distance between the centres of the two squares is $\sqrt{r}$. The task is to find a sequence of moves taking the coin from the square

which has A as a vertex to the square which has B as a vertex.

(a) Show that the task cannot be done if r is divisible by 2 or 3.

(b) Prove that the task can be done if $r = 73$.

(c) Can the task be done when $r = 97$? (Finland)

2. Let P be a point inside triangle ABC such that

$$\angle APB - \angle ACB = \angle APC - \angle ABC.$$

Let D, E be the incentres of triangles APB, APC respectively. Show that AP, BD and CE meet at a point.

(Canada)

3. Let $S = \{0, 1, 2, 3, \ldots\}$ be the set of non-negative integers. Find all functions f defined on S and taking their values in S such that

$$f(m + f(n)) = f(f(m)) + f(n),$$

for all $m, n \in S$.

(Romania)

Second day (11 July, 1996)

4. The positive integers a and b are such that the numbers $15a + 16b$ and $16a - 15b$ are both squares of positive integers. Find the least possible value that can be taken by the minimum of these two squares. (Russia)

5. Let $ABCDEF$ be a convex hexagon such that AB is parallel to ED, BC is parallel to FE and CD is parallel to AF. Let R_A, R_C, R_E denote the circumradii of triangles FAB, BCD, DEF respectively, and let p denote the perimeter of the hexagon. Prove that

$$R_A + R_C + R_E \geq \frac{p}{2}.$$

(Armenia)

6. Let n, p, q be positive integers with $n > p + q$. Let $x_0, x_1, \ldots, x_n$ be integers satisfying the following conditions:

(a) $x_0 = x_n = 0$;

(b) for each integer i with $1 \leq i \leq n$, either $x_i - x_{i-1} = p$ or $x_i - x_{i-1} = -q$. Show that there exists a pair (i, j) of indices with $i < j$ and $(i, j) \neq (0, n)$ such that $x_i = x_j$.

(France)