Why is an Ant's Trail Straight?

Problems of Pursuit

G S Ranganath

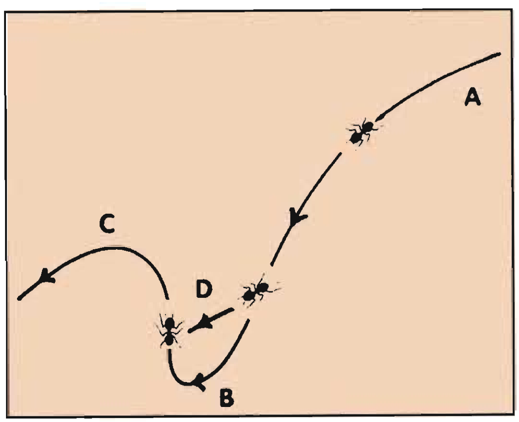
This interesting question puzzled the eminent theoretical physicist Richard Feynman when he was young. As a boy he did many ingenious and interesting experiments. One of them concerned ants. One day while taking a bath he placed a lump of sugar at one end of the bath tub and waited for an ant to locate it. Feynman used a colour pencil to mark the trail of his ant. He noticed that the first ant that located the food took a random wiggly route. But the successive ants did not exactly follow the trail. Instead, each ant straightened the trail of its predecessor a little bit. Thus after some time the trail became a near straight path. Many years later Feynman described this process beautifully in the book *Surely you're joking, Mr. Feynman!*. He concluded from his observations that: "It is something like sketching. You draw a lousy line at first, then you go over it a few times and it makes a nice line after a while." We might argue from this that ants understand geometry. Feynman did not overlook this possibility. He said: "Yet the experiments that I did to try to demonstrate their sense of geometry did not work."

Feynman had recognized the importance of this problem of one ant pursuing another. Historically, pursuit problems are pretty old, dating back even to Leonardo da Vinci. In recent times this question has become all the more relevant in robotics.

This problem has now been readdressed by Alfred Bruckstein, a computer scientist at Technion, Haifa, Israel. His answer in the case of ant trails will surely be of interest to biologists and others. Incidentally he did this thought provoking work on ants when Haifa was being bombed during the Iraq-Kuwait war.

Since ants have no sense of global geometry, Bruckstein proposed a simple local interaction between them. In this model each of the 'mathematical ants' goes directly towards the one ahead of it. In other words, at any instant of time an ant's velocity vector always points towards the ant immediately ahead. Hence the distance between any two neighbouring ants either remains the same or decreases, of course avoiding collisions. If the leading ant and its immediate follower move in a straight path their distance of separation remains the

***Figure 1** The pioneer ant takes the curved path ABC. The immediate follower avoids the bend B and takes the route ADC.*



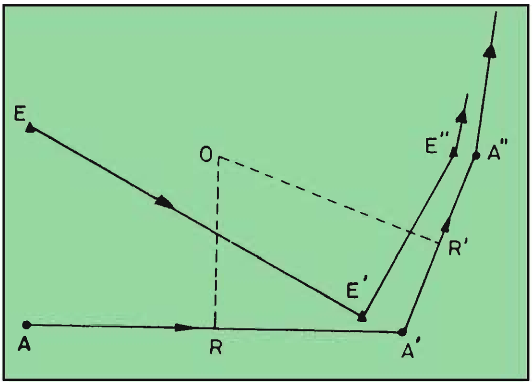
same. However, when the ant ahead is going round a bend the follower heads straight towards it avoiding the bend (see *Figure 1*). This process continues with every successive ant avoiding the kinks in the path of its predecessor. In course of time, the random path continuously shrinks in length and is finally transformed into one of minimum length, which is a straight line.

Mathematically this process is governed by a non-linear differential equation. Solving it for the pursuit path of the follower given the trail of the leader is a very difficult task. Although attempts to solve such problems began in the eighteenth century, to date we have solutions only in two cases! The first one concerns pursuit on a straight line at constant speed and the second one relates to pursuit on a circle. Bruckstein has worked out only the limiting behaviour of the solutions of the differential equation. He finds that the solutions finally converge to a straight line. This study has unravelled a possible interaction that can exist in the world of ants. But a word of caution is in order. In principle one can try different trail following models based on other local interactions. These models may also have solutions that finally converge to a straight line. Thus only one of the possible interactions between neighbouring ants has been suggested by Bruckstein.

It is important to mention here that there is one case where the differential equation has a neat solution. Let us say that four ants are located at the vertices of a square, at the centre of which is a lump of sugar. Can all the four ants reach this lump of sugar simultaneously? Yes, they can. If each ant starts moving at the same speed but follows the ant to its right (or left), then the differential equation yields the answer that all the ants will spiral towards the centre at the same time. Interestingly, in this case each ant covers a distance equal to the edge of the square. This problem has been generalised to the case of different numbers, of ants at arbitrary starting positions with variations of the speed and local pursuit laws.

In this context we recall Littlewood's lovely problem of a lion catching a gladiator. We can recast this problem to our present theme. Let us say an ant A is threatened by an enemy ant E in a closed area. This is not impossible in the world of ants. Further, let us say that the two ants are equally energetic and hence can always move with the same speed. Then surprisingly our ant A can avoid being captured by its enemy E if it adopts the following strategy: The ant A begins by moving at right angles to

***Figure 2** The paths taken by ant A and its enemy, ant E, according to the strategy described in the text.*



the line of sight AE i.e, the line joining it and the enemy ant (see *Figure 2*). It moves in a straight line along this direction until its path intersects the radial line OR, emanating from the centre O of the closed area, which is parallel to the initial line of sight AE. From here it continues in the same direction AR through a distance equal to it and reaches the new position A'. By this time the enemy ant would have moved to a different position E'. Now A locates the new line of sight A'E' and repeats the whole procedure. By successive applications of this method, the ant A can avoid the enemy ant E eternally. In the process A's own path will be a squiral, i.e, a spiral with successive line segments. More on this problem can be found in Ian Stewart's article in *Scientific American.*

Bruckstein's work not only sheds light on what is going on in the world of ants but is also useful in the world of robotics. We conclude from his work that globally optimal solutions for navigation problems can be obtained as a result of near neighbour co-operation between simple agents or robots. It is very expensive and technically difficult to make a single robot that can find the shortest path around obstacles. Instead of making a single sophisticated robot we gain considerably by making many simple robots. These can find the best path through a mere pairwise nearest-neighbour interaction.

Next time you seen an ant, approach it in all humility. It is not for nothing that the Bible says

Go to the ant, thou sluggard; consider her ways, and be wise.

(Proverbs 6,6.)

Long live the members of Formicidae.

Suggested Reading

R P Feynman. Surely you are joking Mr Feynman. Bantam Books 1985.

A Bruckstein. *The Mathematical Intelligencer* Vol.15, No.2, pp 59. 1993.

I Stewart. *Scientific American* pp 113. April 1992.

G S Ranganath is with Raman Research Institute, Bangalore 560080.

Hickory Dickory Dock

Molecular Clues to the Control of Circadian Rhythms

T R Raghunand

Most time keeping systems are based on the sun, reflecting age old patterns of human activity. For most practical purposes, according to our social contract, a day starts when the sun rises and ends when it sets. But the organisation of activity into day and night cycles is not merely an arbitrary agreement for setting clocks; it is also a biological imperative. (Recall Geetha's experiences in a timeless environment: ***Resonance*** Vol.1, No.3, 1996.) Most organisms - animals, plants and even microbes, have internal clocks that dictate daily or *circadian* (from the Latin *circa*, about, and *dies*, day) rhythms of a myriad life