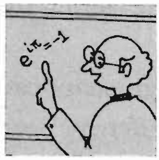
Classroom



In this section of* Resonance, *we invite readers to pose questions likely to be raised in a classroom situation. We may suggest strategies for dealing with them, or invite responses, or both. "Classroom" is equally a forum for raising broader issues and sharing personal experiences and viewpoints on matters related to teaching and learning science.

! Getting the Facts Right Through Puzzles

From J Chandrasekhar, Indian Institute of Science.

The pleasure in science generally comes from the recognition of underlying patterns and unifying principles in diverse phenomena. But before one can appreciate these features, it is necessary to know a large amount of factual information. This is specially so in subjects like chemistry. This feature represents one of the major challenges in the teaching and learning of chemistry, especially at the undergraduate level. Excessive emphasis on memory would turn away students seeking intellectual content in the subject. But the logic of the subject cannot be fully appreciated with a superficial knowledge of chemical facts. Clearly, one needs to strike a balance. There is also plenty of scope for innovation in the classroom.

G M Kulkarni from Karnatak University, Dharwar, has written to us about an interesting method that students can use to become familiar with chemical symbols. The idea is simple. Take a word like CHEMISTRY. How many elements can be represented using the letters in the word? One can readily spot the elements with one-letter symbols, viz., C, H, I, S and Y. But there are several other elements with two-letter symbols which are contained in the word: Ce, Cm, Cs, Cr, He, Es, Ir, Sc, Se, Sm, Si, Sr, Tc, Th, Te,

Take a word like CHEMISTRY. How many elements can be represented using the letters in the word?

Tm, Ti, Rh, and Re. There are 24 elements (one more, if we accept the name Meitnerium and symbol Mt for element 109) which are contained in the word CHEMISTRY!

The game (assignment?) can have endless variations. We just have to try out a different word. Kulkarni says this type of puzzle worked wonders in her class. The students mastered the symbols of the periodic table and enjoyed the process.

I would like to suggest another puzzle along the same lines for learning about proteins. The naturally occurring amino acids are now represented by one-letter codes (see the poster in ***Resonance***, Vol 1, No.1, 1996). Students may be asked to work out the sequence of residues from a word. For example, in this code CHEMISTRY stands for a peptide with 9 residues in the specific sequence: cysteine-histidine-glutamic acid-methionine-isoleucine-serine-threonine-arginine-tyrosine.

It must be noted that the aminoacid code is a restricted alphabet with only 20 letters. Some words can be made, but others not. It is amusing that peptides with the sequence CHEMISTRY, PHYSICS and MATHEMATICS are possible, but there is no peptide called BIOLOGY (There is no amino acid with the code B or O).

From C S Yogananda, Indian Institute of Science

! McKay's Proof of Cauchy's Theorem on Finite Groups

Cauchy's theorem on finite groups states that if a prime p divides the order of a finite group G then there is an element of order p in G

The purpose of this note is to present a beautiful proof of Cauchy's theorem on finite groups by J M McKay (*American Mathematical Monthly,* Vol. 66 (1959), page 119). Cauchy's theorem states that if a prime p divides the order of a finite group G then there is an element of order p in G, In fact, McKay's proof shows that there are at least p of them.

To begin with, consider the case of $p = 2$, i.e., when G is of even order. The number of elements in G other than the identity element is odd. Pair off each element other than the identity with its inverse. Since the inverse of an element is unique and there are an odd number of elements to be paired off, it follows that some elements will have to be paired with themselves; in other words there are elements of order 2.

McKay's proof is a generalisation of this simple proof for the prime 2. Consider the set

$$P = \{ g_1, g_2, \ldots, g_p) \mid g_i \varepsilon G, \Pi g_i = e\}$$

where e is the identity element of G. For example, $(e, e, \ldots, e) \varepsilon P$. The theorem will be proved if we show the existence of a non-trivial *diagonal* element of P, i.e., a p-tuple all of whose entries are the same, say $g \neq e$; g will then be an element of order p. We show the existence of such elements by counting the number of elements of P in two ways. On the one hand, note that we can choose any $p-1$ elements of G and then take the inverse of the product of these $p-1$ elements for the p-th entry. Therefore we have $|P| = n^{p-1}$ where n is the order of G.

We now do the counting by defining an equivalence relation on P. Declare that two p-tuples are related if one of them is a cyclic permutation of the other; in other words, if you think of the elements of each p-tuple to be arranged on a circle then a suitable rotation of one will give the other. It is easy to verify that this is an equivalence relation and hence will give rise to a partition of P.

> *Exercise:* If a p-tuple has at least two of its entries distinct, then all its cyclic permutations, p in number, are distinct. (This is where the fact that p is a prime is used.)

Using the above partition to do the counting we get $|P| = A + B$, where $|P|$ denotes the number of elements in P, A is the number of diagonal elements in P and B is the number of non-diagonal elements in P (p-tuples at least two of whose entries are distinct). We have $|P| = n^{p-1}$ and, by the exercise above, B is a multiple of p. Since p divides n and p divides B it follows that p divides A as well. Thus, to finish the proof, we only have to know that A is not zero; but this follows since $(e, e, \ldots, e) \varepsilon P$.