Origin (?) of the Universe

3. The Big Bang

Jayant V Narlikar

In this part of the series we look at the simplest cosmological models based on the simplifying assumptions of the Weyl postulate and the cosmological principle. These models were discovered independently by Friedmann, Lemaitre and Robertson in the 1920s. They led to the striking conclusion that the universe started in an enormous explosion often called the Big Bang.

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This six-part series will cover: 1. Historical Background. 2. The Expanding Universe. 3. The Big Bang. 4. The First Three Minutes. 5. Observational Cosmology and 6. Present Challenges in Cosmology.

Relativistic Cosmology

The simplifying postulates described in part 2 of the series (***Resonance***, Vol. 1, No. 2) allow the theoretician to construct mathematical models of the expanding universe. In 1922 the Russian physicist Alexander Friedmann made such attempts and arrived at what are today known as the *Friedmann models*. During 1922-24 when Friedmann was constructing these models he needed a theory of gravity to determine the large scale dynamics of the universe. Why a theory of gravity?

Physicists today know of essentially four basic interactions in nature: the strong and the weak interactions which describe the microscopic behaviour of subatomic particles, the electromagnetic interaction which describes the force between electric charges, at rest or in motion and the gravitational interaction which is none other than that first quantified by Isaac Newton three centuries ago.

In the 1920s the knowledge of the first two of these four interactions was rudimentary. But it was clear that these microscopic forces were of very short range, being confined to atomic nuclei and would not be of much relevance to the large scale structure of the universe today. So also the electromagnetic interaction which by then was well understood at the classical level. Although it is of long range (i.e. its influence can be felt over arbitrarily large distances albeit with decreasing intensity), in order for it to be effective in cosmology, the major constituents of the universe would have to be electrically charged. The observations indicate (as they did in the 1920s) that matter on the large scale is electrically neutral, and so this force is also ruled out of contention.

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This left gravity — another long range force which though very weak at the atomic level, comes into its own at the cosmological scales of large masses as in the case of galaxies and clusters of galaxies. However, it is here that the Newtonian picture becomes suspect. For, it involves the concept of instantaneous gravitational action at a distance which is inconsistent with special relativity. Newtonian dynamics also needs to be revised so as to be consistent with special relativity. By 1922, Einstein's general theory of relativity was already being established as a theory that was free from the above two conceptual difficulties and in better agreement with the solar system observations. Thus it was natural for Friedmann to use the framework of general relativity to describe the simplest cosmological models.

The Einstein Universe

Relativistic cosmology, as this subject is now known, started even earlier than Friedmann's pioneering work. In 1917, Einstein himself had attempted to obtain a simple relativistic model of the universe; but he had made the assumption that the universe was a static system. At that time there was no conclusive evidence for the expanding universe (Hubble's result came in 1929) and so, in looking for a simple model, Einstein was perfectly right in assuming that there is no large scale motion in the universe.

However, Einstein soon discovered that there is no static solution to his relativistic equations! Rather than look for a dynamical solution like Friedmann did five years later, Einstein sought to

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modify his field equations so that he could get a *static* solution. For this he had to introduce a 'cosmological term' describing an extra cosmic force of repulsion. The rationale for Einstein's approach can be understood by the following simple argument.

Imagine two masses m and M separated by a distance r in the Newtonian framework. If the two masses are left to themselves they cannot remain at rest but will move towards each other with an attractive force GmM/r^2. To have a static situation we can introduce a repulsive force λr which will balance the above force if the distance r is adjusted to be

$$r = (GmM / \lambda)^{1/3}.$$

Here λ is a constant. In Einstein's general relativistic formulation the constant λ appears in the same spirit but the mathematical details are different. Einstein called it the *cosmological constant*. Its effect on the whole universe was to adjust its radius so that the repulsive force exactly balances the gravitational contraction of the universe. The radius of the universe is then given by the formula

$$R = [2GM/(\pi c^2 \lambda)]^{1/3}$$

where M is the mass of the universe. (Note: the dimension of λ is different from that in the Newtonian example.)

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What do we mean by the 'mass' of the universe? How can an infinite system have a finite mass? The answer to this query is that the universe is unbounded but has a finite volume. How can space be unbounded but finite? The answer is that in Einstein's model, the universe is the surface of a hypersphere in four dimensions. Just as the two dimensional surface of an ordinary sphere of radius R is finite and has an area equal to $4\pi R^2$, the three dimensional surface of a hypersphere is finite and has a finite volume equal to $2\pi^2 R^3$. In both the examples, there is no boundary to the space, i.e., it is unbounded. Thus it is perfectly legitimate to talk of an unbounded but finite system.

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Hubble's discovery became well established. Einstein himself withdrew it along with the cosmological term saying that it was his greatest blunder! Others may disagree. For example, in 1917 W de Sitter wrote a paper in which he solved Einstein's equations with the λ-term, and obtained a model in which the universe expands with a scale factor that increases exponentially with time. The *Hubble constant* for this universe is given by

$$H = [(1/3)\,\lambda c^2]^{1/2}.$$

Thus this model is apparently closer to reality than Einstein's because it expands. But it isn't, because this universe is empty! Nevertheless, as we shall see, the de Sitter universe has played an important role in cosmology in different contexts. We will return to the λ-term later in the series.

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The Simplest Friedmann Models

During the early twenties, however, the notion of the expanding universe was not established and thus Friedmann's models remained of academic interest and were not widely known; even Einstein did not show any enthusiasm for them. Later in 1927, G Lemaitre (a priest!) and H P Robertson independently worked out similar models. And in the mid-1930's Robertson and A G Walker independently produced a mathematically rigorous derivation of the geometrical features of space and time starting from the symmetries of the Weyl Postulate and the Cosmological Principle (see part 2 of the series). For this reason these spacetimes are called the *Robertson-Walker* (R-W) spacetimes.

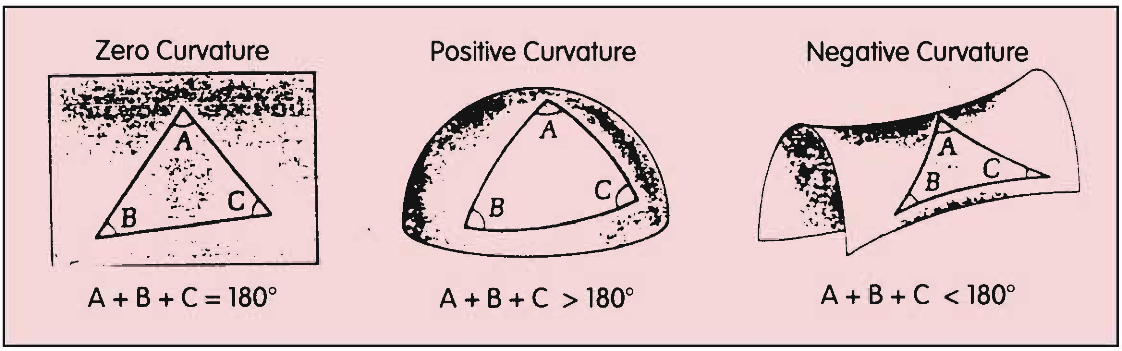

***Figure 1** Surfaces of different curvatures: The figure illustrates the idea of curvature for two dimensional surfaces. The flat surface has* zero *curvature, the sphere has* positive *curvature while the saddle shaped surface has* negative *curvature. If triangles are drawn on the three surfaces their internal angles will add up to 180°, more than 180° and less than 180° respectively.*

The three-dimensional spaces described by the sections at cosmic time t = constant in the R-W spacetimes can be characterized by a single parameter k which specifies what type of curvature these spaces have. For $k=0$ we have spaces of zero curvature which have the *flat* Euclidean geometry that we are so familiar with in everyday life. For $k=1$, the spaces have positive curvature and these are similar to the space in Einstein's universe. These spaces are called *closed* and they have the feature that if we continue in a straight line in any direction, we would return eventually to the starting point. (Analogy: going in a 'straight' line on the earth would bring us back to where we started). The spaces given by $k=-1$, likewise have negative curvature and describe an *open* universe. (Analogy: the surface of a horse saddle.) The parameter k is therefore called the *curvature parameter*.

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The simplest solutions obtained by Friedmann had all three possibilities for the curvature parameter. Einstein's equations relate this to the contents of the universe. But what did the universe contain by way of matter? The simplest models had matter in the form of 'dust', that is, pressureless fluid. This is an idealization of the real universe which has small random motions of galaxies and hence very small cosmic pressure. We will return to this aspect in part 4 of this series.

The formidable Einstein equations reduce under these conditions to just two relations which describe the linear scale factor $S(t)$ as a function of time in terms of the curvature parameter k and the density ρ of the universe. These relations are given below:

$$S^3 \rho = \text{constant}$$

$$[(dS/dt)^2 + kc^2] / S^2 = 8\pi G\rho / 3.$$

The first relation is simply the law of conservation of matter. It tells us that as the linear size of space expands in proportion to S, its density falls as the inverse cube of S. The second relation is a dynamical one which tells us that the rate of expansion is made slower by the gravitational attraction within the universe.

If we solve these relations together, we get $S(t)$ as a known function which, in the special case of the flat universe would correspond to $t^{2/3}$. The following features are noticed in general. All three types of solutions have similar beginnings. The universe 'begins' its existence at the stage when $S=0$. It expands in an explosive fashion ($dS/dt \longrightarrow \infty$ as $S \longrightarrow 0$) at an epoch a finite time ago. Thereafter it either continues to expand till S becomes infinite ($k = 0,-1$) or its expansion comes to a halt and it contracts back to $S = 0$ (for $k = 1$).

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Some Cosmological Parameters

How do these models relate to Hubble's law? A simple analogy with a laboratory system will illustrate how the Hubble constant can be obtained from the expanding Robertson-Walker models.

Suppose you heat a metal ruler. It will expand. Let the function $S(t)$ describe how the original length l of the ruler grows with time. Keeping $S(t)=1$ at $t=0$ we say that the expanding length of the ruler is $lS(t)$. Thus, when seen from one end of the ruler, the other end appears to move with velocity $l\,dS/dt$, whereas the distance between the two ends is $lS(t)$. Hence velocity divided by distance is $(dS/dt)/S$, which is the 'Hubble constant'.

Apply this analogy to the cosmic ruler between two galaxies and

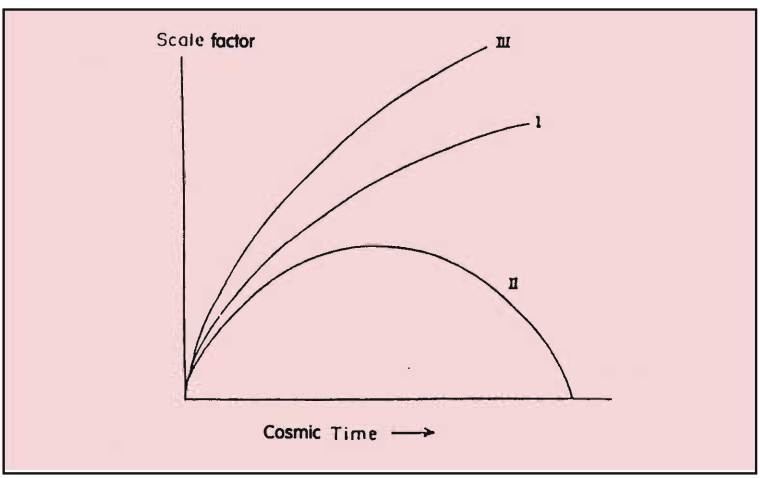

Figure 2 Three simple Friedmann models: The figure shows schematically how the scale factor changes with cosmic time in the three different Friedmann models. Curve I corresponds to the flat case (k=0), curve II to the closed case (k=1) and curve III to the open case (k=−1). All three curves meet where the axes intersect. This point, identified with the Big Bang event has the scale factor equal to zero. We may consider the cosmic clock to start ticking from this epoch.

The past epoch when the scale factor $S(t)$ was equal to zero is called the epoch of Big Bang.

you get the observed Hubble constant as

$$H=(\mathrm{d}\,S/\mathrm{d}\,t)\,/\,S.$$

A rigorous analysis based on general relativity confirms this simple minded derivation. The analysis also gives the *redshift* z as

$$1+z = S(t_0)\,/\,S(t_1).$$

Here the redshift z is the fractional increase in the wavelength of light from the observed galaxy. It is assumed that light left the galaxy at the earlier epoch t_1 in order to reach us at the present epoch t_0. Since the observations show that $z > 0$, we conclude that the scale factor of the universe has increased between t_1 and t_0. That is, the universe is expanding.

The three types of Friedmann models are also distinguished by the *density parameter* which is usually denoted by Ω. If we take the flat ($k=0$) model, then calculations tell us that the cosmic density ρ_c is related to the Hubble constant by a simple formula:

$$\rho_c = 3H^2\,/\,(8\pi G).$$

We then define the density of any other model by the formula

$$\rho = \Omega\,\rho_c$$

Again, detailed calculations show that for the closed ($k=1$) models $\Omega > 1$, whereas for the open ($k=-1$) models we have $\Omega < 1$, We will refer to ρ_c as the *critical density* or the *closure density*.

The Big Bang

The past epoch when S was equal to zero is called the epoch of the Big Bang. At this epoch the density is infinite and so is the curvature of spacetime. In fact, standard equations of physics break down and physicists encounter what they refer to as a *singularity*. Clearly, there is no justification for pushing our mathematical model further back in time beyond the Big Bang epoch. Cosmologists like to identify this epoch with the origin of the universe. In the next part of the series we will examine the state of the universe at epochs very close to the Big Bang.

Suggested reading

J V Narlikar. The Primeval Universe. Oxford University Press. 1988.

H Bondi. Cosmology. Cambridge University Press. 1960.

S Weinberg. Gravitation and Cosmology. Wiley. 1972. Very technical but a classic.

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