Fermat and the Minimum Principle

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The inside back cover of the January 1996 issue of *Resonance* describes Pierre-Simon de Fermat (1601-1665) as "the discoverer of the principle of least time in optics, the earliest example of a minimum principle in physics."

Arguably, least action and minimum principles were offered or applied much earlier. This (or these) principle(s) is/are among the fundamental, basic, unifying or organizing ones used to describe a variety of natural phenomena. It considers the amount of energy expended in performing a given action to be the least required for its execution. Again, where motion is concerned, a particle or wave chooses the shortest possible path in moving from one point to another (as in the law of reflection, which goes back to Hero), or tries to complete the motion in the shortest possible time (as in the law of refraction, which Fermat established).

Engineers believe that Archimedes, who lived during the 200's BC (nineteen centuries before Fermat), derived the principle of the lever using an approach that is essentially a variation of the least action principle. This technique, now known as the virtual work principle, is used as the basis for structural mechanics, the science underlying structural engineering.

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Even in optics, as already alluded to earlier, we find a famous application of this law of economy of physical behaviour to optics being made by Hero (or Heron) of Alexandria who lived during the first century AD. He discovered the law of reflection which every school child is familiar with. This law states that angles of incidence and reflection formed by a light ray incident on a plane mirror are determined by the condition that the ray travel from its source to its reflected position along the shortest possible path. This was some sixteen centuries before Fermat demonstrated that the law of refraction of light also follows from the minimum principle.

Suggested Reading

J R Newman. The World of Mathematics, Simon and Schuster, 1956.

R P Feynman, R B Leighton, M Sands. The Feynman Lectures on Physics, Vol.1 & 2. Addison-Wesley, Reading, Mass. 1963 and Narosa, New Delhi, 1987, pp.315-316 and 956-974.