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Investigation of optimal energy deposition of the aluminium ion beam in pre-compressed DT fuel

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Abstract. Fast ignition (FI) by the laser-accelerated ion beams is an advanced possible remedy to produce high energy in inertial confinement fusion. Low-divergence high-power beams can now be produced due to new technological advances in laser–plasma accelerators. Using the Deira-4 simulation code, conditions optimal for the ignition of deuterium–tritium (D-T) pellets by the aluminium heavy ion beam were investigated in this work. The results show that an aluminium ignitor beam with 1.4 GeV energy can provide hot-spot optimal ignition conditions.

Keywords. Stopping power; heavy-ion beam; Deira-4 simulation code; laser–plasma accelerator.

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1. Introduction

The high-energy experiments have been developed in the laboratory by expanding laser technology, especially high-power laser pulse. The chirped pulse amplification (CPA) method allows it conventionally to be compressed in time and raised by factors of 1000 or more. This approach has become available with small devices [1]. The interplay pulses with 10^{18}–10^{21} W cm^{-2} intensities and hollow conical targets generate relativistic laser–plasma, ion beams and multi-MeV electrons [2]. They also open a new series of nuclear physics, such as fast ignition (FI) and ion beam production.

Tabak *et al* (1994) suggested igniting pre-compressed inertial confinement fusion (ICF) fuel with CPA laser pulses. Fast ignition (FI) has continually been suggested for about three decades as an alternative and novel approach to ICF, which enhances the gain at lower driver energy with separate ignition and compression drivers of the primary fuel [3]. First, the fuel is compressed in the isochoric ignition model of Kidder [4] and Meyer-ter-Vehn [5] to a high density while the temperature is kept below the ignition temperature. Then, to produce a hot spot, a small amount of the pre-compressed fuel from the other beam rapidly reaches the combustion temperature. The fast ignition facilitates the generation of hot spots for compression and explosion [6]. There are challenges with electron FI, such as electron beam divergence [7,8], plasma cone filling [9], appropriate capsule implosions by a re-entrant cone [10] and the capability to bore a direct channel to the centre of the fuel [11,12].

A new design was introduced to ensure the definitive progress and success of FI using CPA lasers with the conversion of laser energy to protons [13,14]. This mechanism is called target normal sheath acceleration (TNSA) [15–19]. In the TNSA method, a nanometre-thick foil impregnated with specific materials is irradiated by a laser with a power of more than 10^{18} W/cm^2. When a laser with a 10^{20} W/cm^2 intensity radiated on a foil covering the outside of the fuel capsule, a population of electrons is generated and eventually, ions with high energy are produced [20]. The ion beam has a strong coupling to the fuel and deposits its energy at the end of its path (Bragg peak) [21]. Roth and his team demonstrated that the TNSA accelerator method could generate a proton beam with a 15–23 MeV energy, which could ignite the pre-compressed hot-spot DT fuel. In this case, the energy threshold is estimated to be around 10–100 kJ. The ions produced through the TNSA mechanism are relatively dispersed MeV protons. There have been significant challenges to proton fast ignition (PFI), such as the production of low-energy protons with Maxwellian spectra and the transfer of laser energy to protons with minimal energy dissipation [22,23]. The high intensity of the laser pulse required in this method is also one of its disadvantages.

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The energy threshold is reduced when heavier ions are used instead of protons. The heavy-ion beams are simply transported and centralised and have much higher stopping power in fuel, which would permit a much more exact deposition of the beam energy at the end of their range and hot-spot volume. Heavy-ion beams are considered to be the most helpful ion drivers for power reactor foundations on ICF [24–29].

Ions with higher energies and low divergence cannot be produced using the TNSA method. It is important to use other accelerators that can accelerate ions to higher energies and at the same time more cost-effective. The radiation pressure acceleration (RPA) method is an optimised idea for this design. The idea is based on the acceleration of the ions up to multi- MeV energies with low divergence [30–33]. Using a circular polarisation laser beam, Esirkopov *et al* could attain 1.5 GeV energy by a PIC simulator [34]. The method in which acceleration of ions happens in a very thin target at the end of the pulse is the called light sail (LS) method. This method has high relative efficiency and is easily available. Relative ions were obtained, which can be used in the LS method [35–37].

In 2013, Domensky achieved 10 GeV energy for carbon and 1 GeV energy for protons by thinning the incident laser beam and using the highest intensity. A simulated circular polarisation laser beam yields a proton beam with 7.7 GeV energy and a carbon beam with 72 GeV energy [38]. Fernandez *et al* proposed using heavier ions like lithium, carbon and beryllium in 2014 [35]. Because of the limitation in the number of beams which are produced, ion beams which are heavier, appear to be better for ignition because within relativistic energy limits their infusion of energy is better [39]. Due to the limitations of high-energy flux production, it is more desirable to use high-energy heavy ions for higher ignition to satisfy the energy required for ignition because heavier ions transmit more energy. For this reason, the use of heavy ions has attracted much attention because of its greater stopping power and ability to produce hot spots with fewer ions. In this study, the use of heavy aluminium ions for spark ignition in hot spots and ignition in deuterium–tritium fuel has been investigated.

The rest of this paper is organised as follows: In §2, the suggested fuel is introduced. In §3, the fast ion's stopping power model is discussed for the proposed plasma medium. In §4, the simulation model is presented for calculating the energy deposit conditions for heavy aluminium-ion beam, optimal laser power and aluminium beam energy for the suggested fusion process through the Deira-4 simulation code. An analysis of the simulation results is presented in §5 and the concluding remarks are given in §6.

2. Suggested type of fuel

The most important criteria for selecting a fusion reaction in future fusion plants are the high fusion cross-section to increase the fusion rate and the abundance of ions used. Selected ions must be abundant and inexpensive. Lighter ions are preferred because their repulsive force and temperatures are lower than that of heavier ions. Therefore, the most convenient fusion reactions appear to be related to the hydrogen isotopes, deuterium and tritium, which have the lowest threshold energy for conducting nuclear heat reactions and the highest fusion cross-section.

Equimolar hydrogen isotopes consisting of a mixture of deuterium and tritium (DT) is a fuel with the lowest ignition point temperature. It has the greatest gain and use. Also, they have recently been considered as fusion fuel for ICF reactors. The neutron resulting from this reaction is electrically charged and can easily exit the plasma environment. Due to their high permeability, neutrons can produce radioactive materials by hitting the reactor wall. However, it should be noted that the half-life of the material resulting from this reaction is shorter than that of the fission fragments. In contrast, the α-particle, due to its heaviness and being pregnant, can deposit all its energy into the plasma and cause ambient heat. Deuterium can be produced from seawater (about one in every 6700 hydrogen atoms is a deuterium atom) and tritium, an unstable element, which has an approximate half-life of 12.36 years, and its sources are extremely rare in nature. It can also be produced directly in a nuclear fission reactor with the reaction of lithium and neutron; therefore, it needs to be produced close to the reactor building in ICF reactors within the DT fuel cycle. Tritium breeding is possible according to the following reactions:

$$^{6}\mathrm{Li} + n(\mathrm{Slow}) \rightarrow {}^{3}_{1}\mathrm{T} + {}^{4}\mathrm{He} + 4.8\,\mathrm{MeV} \tag{1}$$

$$^{7}\mathrm{Li} + n(\mathrm{Fast}) \rightarrow {}^{3}_{1}\mathrm{T} + {}^{4}\mathrm{He} + n - 2.5\,\mathrm{MeV}. \tag{2}$$

The $n/^{6}$Li reaction is exothermal and will chip into power production in an ICF reactor. In an optimally designed ICF reactor, it can increase the power release by 25% [40–43].

3. The stopping power model

The stopping power of the high-energy ion beam in hot plasma is very significant in the ICF since it makes provisions for the ignition and then burns fuels. Because of this, the depositional energy of plasma-energetic ions during ionisation and stimulation proceedings of

Coulomb forces are presented using suitable mathematical equations. Here, 3-temperature, 1-dimensional hydrodynamic Deira-4 code written by Basko in Fortran language 77 is used for the simulations. The Deira-4 code is a physical–mathematical model used to simulate IF-targets ignited by an ion beam driver. The Deira-4 code is a 3-temperature code that works with a finite-difference solving method. Using this code, we can create fusion targets with flat, cylindrical or spherical geometries [44]. In the Basko model, the loss of Coulomb energy is computed for the ions within the temperature range of $0 < T < 300$ keV, energy range of $0.1\,\mathrm{MeV/amu} \le E_1 \le 1\,\mathrm{GeV/amu}$ and density of $0 < \rho < 10^6\,\mathrm{g/cm^3}$. According to this theory, we can write the stopping power as [45]

$$-\frac{1}{\rho}\frac{\mathrm{d}E_1}{\mathrm{d}x} \equiv S = S_{be} + S_{fe} + S_{fi} + S_{nu}, \tag{3}$$

where S_{fe}, S_{be}, S_{nu} and S_{fi} describe the stopping power of free and bound electrons, bare nuclei and plasma free ions, respectively. The stopping power of the fast ions by the bound electrons is calculated by the Basko and Sov using eq. (4) [45]:

$$S_{be} = \frac{4\pi e^4 Z_{1ef}^2}{m_e v_1^2}\left(\frac{Z_2 - y}{A_2 m_A}\right)\left[L_{be} - Ln\left(1 - \frac{v_1^2}{c^2}\right) - \frac{v_1^2}{c^2}\right], \tag{4}$$

where eZ_{1ef} is the average charge of the fast ions that are moving with a velocity v_1, $(Z_2 - y)$ is the number of electrons bound by the atom, m_A is the atomic mass and L_{be} is the Coulomb logarithm. Within the limit $L_{be} \gg 1$, the Coulomb logarithm is described by the following equation:

$$L_{be} = \ln\left(\frac{P_{\max}}{P_{\min}}\right), \tag{5}$$

where $P_{\min}$ and $P_{\max}$ are the maximum and minimum of the momentum of a fast ion in collision with field particles, as defined by the following equations:

$$P_{\max} = 2m_e v_1 \tag{6}$$

$$P_{\min} = [(\gamma Z_{1ef} e^2 \bar{\omega}/v_1^2)^2 + (\hbar\bar{\omega}/v_1)^2]^{1/2}. \tag{7}$$

In eq. (7), the excitation frequency average of the atomic electron is $\overline{\omega}$. The Eulerian constant is equal to $\gamma = 0.577$. No theory has yet been formulated for $\overline{\omega}$ heavy atoms. We need an approximate method to estimate $\overline{\omega}$. Basko set:

$$\hbar\bar{\omega} = g(y)\varepsilon_{n,l,j}(y) \tag{8}$$

the electron separation energy in the sublayer n, l, j is defined as

$$\varepsilon_{n,l,j} = \varepsilon_{n,l,j}(y).$$

Using $g_0 = g(y_0)$ and $g(Z_2 - 1) = g_H = 1.105$, we have the following equations for $g(y)$:

$$g(y) = \begin{cases} g_0 & 0 < y \le y_c \\ g_0 + (g_H - g_0) & \\ (y - y_c)(Z_2 - 1 - y_c) & y_c < y < Z_1 - 1 \\ g_H & Z_2 - 1 \le y < Z_2 \end{cases} \tag{9}$$

The following equation defines the stopping power of the fast ions by the free electrons in plasma:

$$S_{fe} = \frac{4\pi e^4 Z_{1ef}^2}{m_e v_1^2}\frac{yG(x_e)}{A_2 m_A}\left[L_{ef} - \ln\left(1 - \frac{v_1^2}{c^2}\right) - \frac{v_1^2}{c^2}\right]. \tag{10}$$

The function $G(x_e)$ is determined by the following equation:

$$G(x_e) = \frac{2}{\sqrt{\pi}}\left[\int_0^{x_e} \exp(-t^2)\mathrm{d}t - x_e \exp(-x_e^2)\right] \approx \left(1 + 1.33x_e^3\right)^{-1}. \tag{11}$$

Also, we have these equations:

$$x_e = \left(\frac{m_e v_1^2}{2kT_{ef}}\right)^{\frac{1}{2}}, \quad x_i = \left(\frac{m_A A_2 v_1^2}{2kT_i}\right)^{\frac{1}{2}} \tag{12}$$

$$kT_{ef} = [(kT_e)^2 + (2^{1/3}\pi\hbar^2 n_e^{2/3}/m_e)^2]^{1/2} \tag{13}$$

$$L_{fe} = \ln[1 + \Lambda_{fe}/(1 + 0.5/\Lambda_{fe}^{\frac{1}{2}})] \tag{14}$$

$$m_e v_{ef,e}^2 = 2kT_{ef}\eta(x_e), \qquad m_A A_2 v_{ef,i}^2 = 2kT_i\eta(x_i). \tag{15}$$

The value $n_e = \rho y/A_2 m_A$ is the free electrons volume density. When dealing with fast ions, stopping power of the plasma ions is critical at the end of the penetration depth in a hot medium with $xe \ll 1$ or in a cool medium with $E_1 \le 0.5\,\mathrm{MeV/amu}$, $Z_1 \gg 1$ and $Z_2 \gg 1$. The collision of the ions can be segregated into distant and close collisions. Collision-effective parameter, $r = r_s$, is regarded as the boundary between the two kinds of collisions. The corresponding contribution of their co-ownership is denoted by S_{fi} and S_{nu} for the total stopping power. When $r \gg r_s$, the beam and the target ions act as a point charge (Z_{1eF} and ey) in dealing with each other. When $r \ll r_s$, the nucleus of the colliding ions will be activated in a cloud of electrons. They are considered as point charges eZ_1 and eZ_2. Here, the Debye parameter,

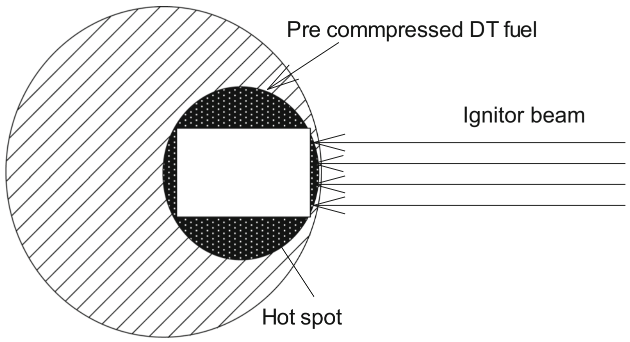

Figure 1. The formation of hot spots and the ignition of the energy-beam deposition scheme.

$\Lambda_{fi} = P_{\max}/P_{\min}$, can be obtained by the following expression for free-electron collisions:

$$\Lambda_{fi} = \min\left\{\frac{2M_0 v_{ef,i}^2 r_{ef}^*}{(\hbar v_{ef,i}^2 + \gamma Z_{1ef}^2 y^2 e^4)^{1/2}}; \frac{r_{ef}^*}{r_s}\right\}, \tag{16}$$

where $M_0 = m_A A_1 A_2/(A_1 + A_2)$ is the mass reduction of the collider ions and $v_{ef,i}^2$ is the test ion's effective speed. The stopping power of the plasma-free ions, S_{fi}, is calculated as follows: presuming the range is the same as the heavy metal ion shift along its initial line:

$$S_{fi} = \frac{4\pi e^4 Z_{1ef} y^2}{m_A A_2 v_1^2} \frac{(1 + A_2/A_1)^{\frac{1}{2}}}{m_A A_2} G(x_i) L_{fi}. \tag{17}$$

Heavy ions move in an almost direct path, except at the end of their path where nuclear collisions dominate. The following equation can be used to calculate the heavy-ion penetration depth:

$$R = \int_0^E \frac{\mathrm{d}E}{(\frac{\mathrm{d}E}{\mathrm{d}x})_e + (\frac{\mathrm{d}E}{\mathrm{d}x})_i}. \tag{18}$$

In eq. (18), $(\frac{\mathrm{d}E}{\mathrm{d}x})_i$ and $(\frac{\mathrm{d}E}{\mathrm{d}x})_e$ are the contribution of ion and electron energy, respectively.

4. The simulation model

The equimolar DT fusion fuel is pre-compressed in the fast fusion mode before being exposed to carbon ion beam radiation. Ion beam creates a hot spot during the ignition stage, and burning wave is transferred to the cool volume around it (figure 1). The one-dimensional Deira-4 code examines the ignition stage. When ignition occurs with the smallest amount of energy input, it is

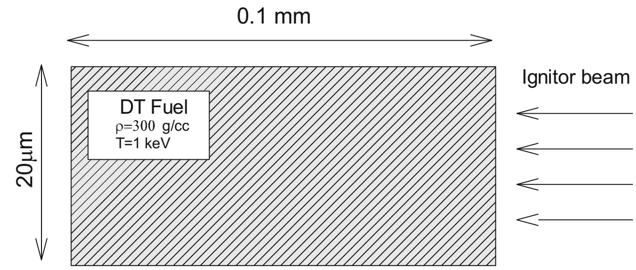

Figure 2. Two-dimensional depiction of a fuel pellet irradiated with an ion beam.

said to be in the optimal mode. In the ignition stage, an equimolar DT fusion fuel with 300 g/cm^3 density and 1 keV temperature is considered. The area of the spherical fuel pellet is considered flat to better explain changes in fuel parameters, according to figure 2. The ignitor beam pulse must be irradiated to the fuel in less time than the ignition stop time. Or else, the fuel will not have the density required to ignite with the expansion of the fuel after this time. There is no chance of ignition in this situation. For optimal energy extraction for the aluminium ion, the aluminium ion beam in the energy range 1–1.5 GeV is radiated to the pre-compressed DT target at 1 keV and intensity of 300 gr/cm^3. For the proposed fuel pellet, the optimal energy required is then presented by the summary of the simulation's findings carried out by the Deira-4 code.

5. Simulation results

Numerical simulations using Deira-4, a 3-temperature, one-dimensional hydrodynamic code used to simulate ion beam-induced ICF, gave the relative threshold energy of the aluminum beam that is suitable for fast ignition. The results are presented in figure 3. In

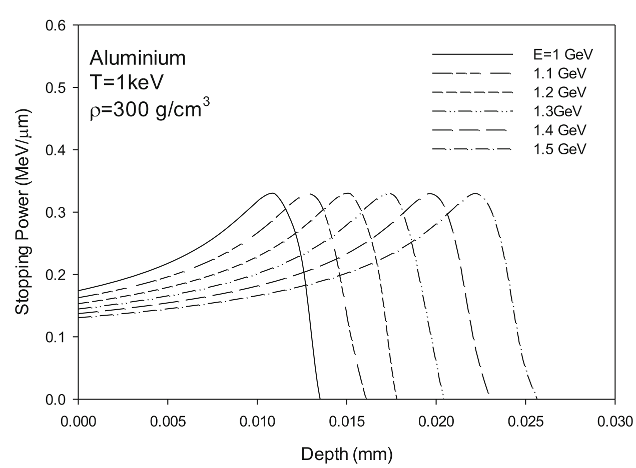

Figure 3. Bragg peak location.

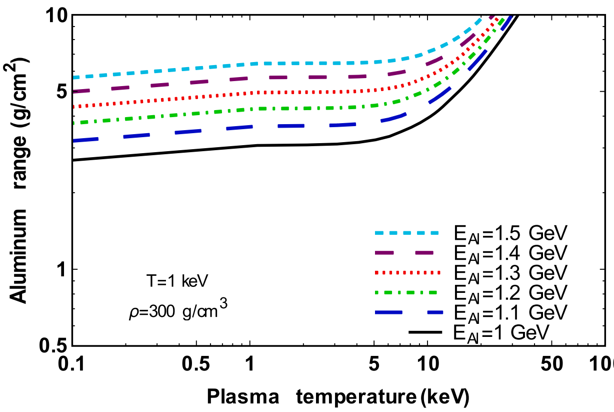

Figure 4. Variation of density-radius of aluminium ion beam in the DT fuel vs. the temperature of plasma for different energies with a density of 300 g/cm^3 and background temperature of 1 keV.

this figure, we consider the spherical target of pre-compressed fuel with a density of 300 g/cm^3 and an initial temperature of 1 keV, which contains the equimolar composition of D-T fuel and its depth-to-resistance distribution curve for energy values of 1–1.5 GeV. The results of the numerical calculations show a relationship between the radius of the hot spot and density of the compressed fuel, with the radius of the hot spot equal to 20.6 μm and the compressed fuel density equal to 300 g/cm^3. Accordingly, it is expected that the optimum radius of the hot spot is set to be equal to the range of α-alpha particles so that the ideal heating of the hot spot by α-particles resulting from DT fusion is provided [46].

As can be seen from figure 4, for an aluminium beam energy equivalent to 1.4 GeV, effective energy at a depth of 21 μm (Bragg peak location) is realised (figure 4). The results of the Atzeni's calculations show that an ignitor beam can form a hot spot. Its mass range must be above 1.2 g/cm^2. As shown in figure 4, these conditions are initially provided with high energies for the aluminium beam. There is a practical advantage to use relatively heavy ion beams in fast ion ignition. Fast ignition of ICF targets needs a small part of pre-compressed DT fuel about 1000 times the solid density with temperatures above 10 keV and a surface density of about $\rho R \approx 0.3$ g cm^{-2}. The hot spot is heated by an external ignitor beam to a temperature above 10 keV to investigate the hot-spot ignition using data obtained using the Deira-4 code.

The temporal evolution of the ion and electron temperature with the 1.4-GeV energy aluminium ion beam and irradiation time of 0.02 ns is shown in figure 5. It is shown that the calculated temperature increases to 50 ps; then the electron energy is deposited into the ions. The maximum ionic heating occurs at 150 keV and 100 ps, which provides the necessary condition for combustion; it then decreases with time for the electron and ionic temperatures to reach thermal equilibrium. Using Deira-4 code simulation, we investigated DT fuel blast using a heavy aluminium beam. For this purpose, we irradiated a beam of 1.4-GeV aluminium ions with a pre-pressurised target with an initial 1-keV temperature condition of 0.02 ns and a flash pulse power of 2.5 TW/mm^2. Radiation of the heavy and energetic aluminium ion at 1.4 GeV will heat the fuel and increase its temperature to provide explosive conditions. With the ignition, a steady ignition wave, as shown in figure 6, is formed and spread along with the target. An ion temperature of 10 keV, as an ignition temperature condition, can be provided with a heavy and energetic aluminium ion at 1.4 GeV.

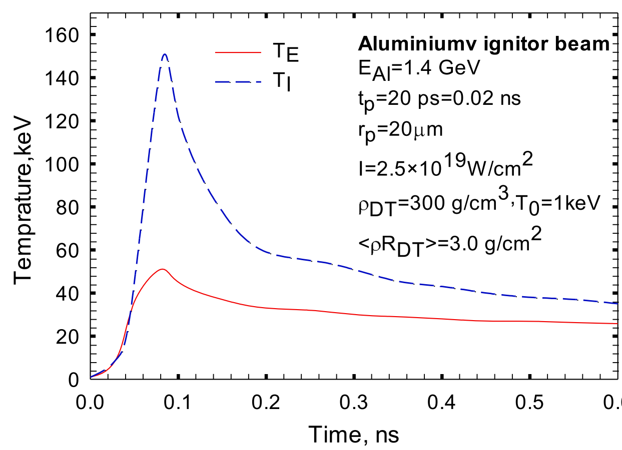

Figure 5. Variation of electron and ion temperature based on time for an aluminium ion beam ignitor.

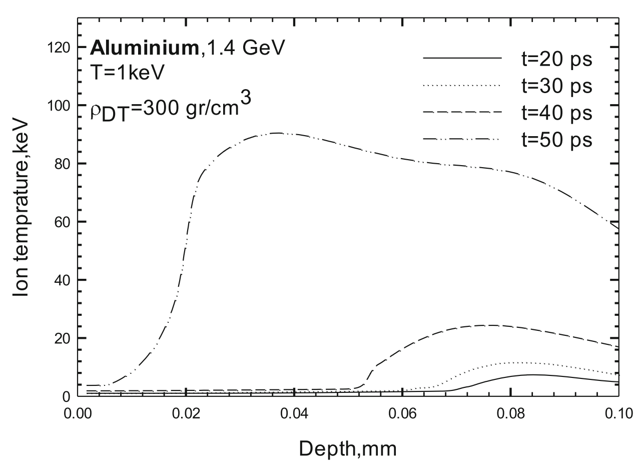

Figure 6. Ion temperature vs. the penetration depth.

6. Conclusion

In this study, the appropriate conditions of the aluminium beam and the DT target are investigated in the fast ignition. For this purpose, a planar target is considered by Deira-4 code as the 2D pre-compressed fuel. This method provides the possibility of fast ignition by calculating the ion and electron temperature. The results showed that ignition will occur if the aluminium beam energy is deposited at the hot spot. For this purpose, the range of aluminium beam in pre-compressed DT fuel is calculated with the physical conditions governing it. Also, the energy of the aluminium beam was selected as the optimal energy for generating an ignition.

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