



Convective radiative moving fin with temperature-dependent thermal conductivity, internal heat generation and heat transfer coefficient

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Abstract. This paper deals with temperature distribution in a moving fin. Practically, we know that thermal conductivity changes with temperature. So in our study, we consider thermal conductivity as temperature-dependent which is constant, linear, quadratic and exponential. The heat transfer coefficient is taken as a power-law type form in the present work. Internal heat generation has been taken as temperature-dependent. For solving the problem, we used numerical methods such as Legendre wavelet collocation method (LWCM), least square method (LSM) and moment method (MM). An exact solution is computed in a particular case. The percentage error is calculated to find out the most suitable method for solving the problem, which is given in the tabular form. The effect of different parameters on temperature distribution is studied in detail. The whole paper is presented in dimensionless form.

Keywords. Collocation; conductivity; heat; numerical; temperature; thermal.

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1. Introduction

Fins are used to increase the rate of heat transfer in a device to prevent failure in system devices. Kraus *et al* [1] gave the concept of finned surfaces, fin performance and designs considering heat transfer functions. Yunus [2] wrote a book on heat transfer indicating the basics of heat transfer. This book explains the concept of convection, radiation, heat exchangers, cooling of electronic equipment, mass transfer, etc. Yunus [2] also explained the sense of mastering practical problems and underlying physical mechanisms. Basavarajappa *et al* [3] pointed out that heat exchangers are extensively used in industrial applications like refrigeration, power stations, air conditioning, petroleum refineries, aerospace industry, petrochemical plants, chemical plants, sewage treatment, natural-gas processing, space heating, etc. Fins are widely used in a variety of heat exchangers to increase the rate of heat transfer between a solid surface and the surrounding fluid. Bergman *et al* [4] described the fundamentals of heat and mass transfer in their book and explained the relationship between thermodynamics and heat transfer. Dongonchi and Ganji [5]

studied heat transfer in a longitudinal rectangular moving fin with heat transfer coefficient, heat generation and thermal conductivity using differential transformation method (DTM). Unal [6] studied the temperature distribution in one-dimensional rectangular and straight fins. Shateri and Salahshour [7] explained the heat performance and temperature distribution in porous fins with a longitudinal profile. Hatami *et al* [8] applied DTM, least square method (LSM) and moment method (MM) to predict temperature in a porous fin in which internal heat generation is temperature-dependent. Zerroukat *et al* [9] used radial basis functions and collocation method to solve the heat transfer problem. Sobamowo *et al* [10] studied heat transfer in radiative–convective fin when Lorentz force is present in the material. Thermal conductivity is taken in three different cases and power-law form. Torabi and Zhang [11] explained the efficiency and thermal behaviour in straight fins with different profiles. Temperature distribution is studied for internal heat generation, surface emissivity, thermal conductivity and heat transfer coefficient which changes with temperature. It has been demonstrated that the heat transfer coefficient might not be uniform, with

significant variations found along the length of the fin and from the base to the tip. Roy *et al* [12] solved the heat equation for the moving fin with a triangular profile using the modified decomposition method and compare the results with numerical solutions. Effects of various parameters on temperature in the fin are also studied. Santos *et al* [13] applied approximations of second order for solving one-dimensional heat transfer equation in spherical and cylindrical coordinates which is in steady-state. Error analysis was also computed for the problem. Singh *et al* [14] solved the fin problem in non-linear form with temperature variant heat transfer coefficient and thermal conductivity which is taken as a linear, constant and quadratic form of temperature. Aziz [15] considered the optimisation of triangular and rectangular fins for convective boundary conditions and studied heat transfer in fins. Shouman [16] studied a steady-state one-dimensional fin in which heat exchanges by convection–radiation between the fin and the environment. The physical properties of the fins are temperature- and displacement-dependent. Mosayebidorcheh *et al* [17] studied fin geometry with optimum design to obtain the high value in constant fin volume. Effects of several parameters and fin efficiency were evaluated. They considered different shapes of the fin profile in which thickness of the fin with longitudinal profile changes with length. Lindstedt *et al* [18] studied single rectangular, trapezoidal and triangular fins. Results showed that the fin with the triangle profile is the best as single fins. Jayesimi *et al* [19] applied Legendre wavelet collocation method (LWCM) to study the straight fin along with thermal conductivity which is temperature-dependent and also showed the effect of convective–radiative and magnetic parameters on thermal performance in the fin. Torabi *et al* [20] applied DTM to study heat transfer within convective–radiative moving fin in which thermal conductivity is temperature-dependent. Singh *et al* [21] studied the model of Fourier and non-Fourier heat conduction under periodic boundary conditions. Laplace transform and its inversion technique are used to find an analytical solution. Girgin and Cuneyt [22] used finite difference method (FDM) to study circular fins with a rectangular profile and compared the results with analytical solutions. Chiu and Chen [23] applied ADM to find temperature in longitudinal fins with temperature-dependent thermal conductivity and found analytic solution for the problem. Roy *et al* [24] studied the effect of heat generation and convection–radiation sink temperature on temperature distribution and fin efficiency by using Adomian decomposition method (ADM). Jayesimi and Oguntala [25] predicted the temperature distribution in a fin with longitudinal rectangular profile with internal heat generation and thermal conductivity which are

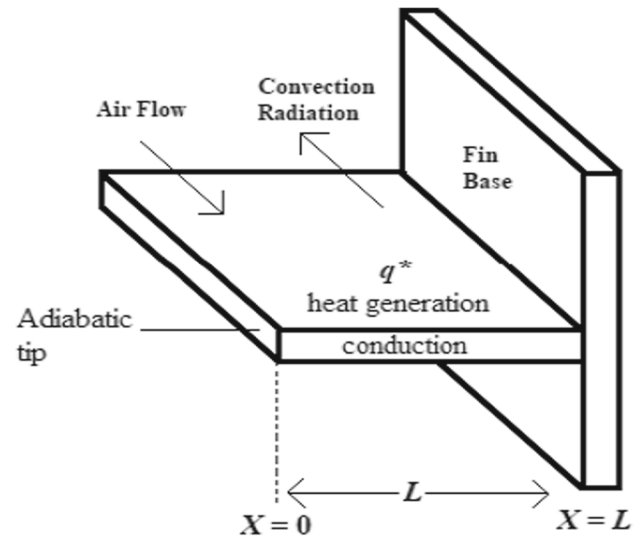


Figure 1. Schematic diagram of a longitudinal rectangular moving fin.

temperature-dependent using LWCM. Kezzar *et al* [26] applied a new technique called the Duan–Rach method to solve the non-linear problem of the longitudinal fin with temperature-dependent heat generation and thermal conductivity.

The objective of this study is to determine how thermal conductivity, internal heat generation and heat transfer coefficient affect the temperature distribution in a moving fin. Thermal conductivity is taken in four different cases and the effect of parameters is also shown here. LWCM, LSM and MM are applied to solve the problem and to find the exact solution to validate the results calculated by the methods mentioned above.

2. Problem description

A one-dimensional longitudinal rectangular moving fin with a cross-sectional area of A_c is considered for the study (see figure 1). The fin moves horizontally with constant velocity U . P is the periphery and L is the length of the fin and surface of the fin is radiative. T_b is the convective base temperature and T_a is the temperature of the ambient fluid. Surface of the moving fin is supposed to be gray and expanded with emissivity ε which is constant. The radiation factor could be rational if force convection is absent or weak. When the material experiences a variation in temperature during the thermal process, then thermal conductivity cannot be constant. For many materials, thermal conductivity changes linearly with temperature. The equation in steady-state is written as

$$\begin{aligned} & \frac{d}{dx} \left(K(T) \frac{dT}{dx} \right) \\ & - \frac{P}{A_c} H(T)(T - T_a) - \frac{\varepsilon \sigma P}{A_c} (T^4 - T_a^4) \\ & - \rho c_p U \frac{dT}{dx} + q^* = 0, \quad 0 \leq x \leq L, \end{aligned} \tag{1}$$

where q^* , $H(T)$ and $K(T)$ are heat generation, heat transfer coefficient and non-uniform thermal conductivity respectively which are temperature-dependent. The parameters ε is the emissivity, ρ is the density of the material, c_p is the specific heat, σ is the Boltzmann constant, x is the spatial variable and T is the temperature distribution. Also, eq. (1) contains the terms conductive heat flux, convective heat flux, radiative heat flux, advection term and internal heat generation. By [1] the boundary conditions are applied on an insulated fin at one end and base temperature on the other end, which is expressed as

$$T(L) = T_b, \quad \left. \frac{dT}{dx} \right|_{x=0} = 0. \tag{2}$$

When the heat generation changes with temperature [5], we have

$$q^* = q_a^*(1 + \varepsilon_g(T - T_a)). \tag{3}$$

To simplify the above equation, dimensionless parameters are introduced follows [5]:

$$\begin{aligned} X &= \frac{x}{L}, \quad \theta = \frac{T}{T_b}, \quad \theta_a = \frac{T_a}{T_b}, \\ k &= \frac{K}{k_a}, \quad h = \frac{H}{h_b}, \quad Nr = \frac{\varepsilon \sigma P L^2 T_b^3}{A_c k_a}, \\ M^2 &= \frac{P h_b L^2}{A_c k_a}, \\ G &= \frac{q_a^* A_c}{h_b P T_b}, \quad Pe = \frac{U L \rho c_p}{k_a}, \quad \varepsilon_G = \varepsilon_g T_b. \end{aligned} \tag{4}$$

By applying these dimensionless parameters, eq. (1) reduces to

$$\begin{aligned} & \frac{d}{dX} \left(f(\theta) \frac{d\theta}{dX} \right) \\ & - M^2 h(\theta)(\theta - \theta_a) - Nr(\theta^4 - \theta_a^4) \\ & - Pe \frac{d\theta}{dX} + M^2 G(1 + \varepsilon_G(\theta - \theta_a)) = 0, \\ & 0 \leq x \leq 1. \end{aligned} \tag{5}$$

and the boundary conditions become

$$\theta(1) = 1, \quad \left. \frac{d\theta}{dX} \right|_{X=0} = 0. \tag{6}$$

Here we consider $f(\theta)$ as the general function of thermal conductivity, θ is the non-dimensional temperature, X is the non-dimensional spatial variable, k is the

non-dimensional thermal conductivity and M is the thermogeometric fin parameter. At ambient temperature, k_a is the thermal conductivity, at fin base h_b is the heat transfer coefficient and Nr is the radiation–conduction parameter. In a moving fin, Pe is the Peclet number which shows the non-dimensional speed and $Pe = 0$ defines a stationary fin, q_a^* is the generated heat at temperature T_a internally. Unal [6] stated that for most of the industrial applications, the heat transfer coefficient may be given as a power law.

$$h(\theta) = h_b \left(\frac{\theta - \theta_a}{1 - \theta_a} \right)^n,$$

where h_b and n are constants. The constant n may vary between -6.6 and 5 . But in many practical problems, it lies between -3 and 3 given by [6]. Exponent n represents condensation or laminar film boiling at $n = -1/4$, laminar natural convection at $n = 1/4$, turbulent natural convection at $n = 1/3$, constant heat transfer coefficient at $n = 0$, nucleate boiling at $n = 2$ and radiation at $n = 3$ as given by [6]. Now eq. (5) becomes

$$\begin{aligned} f(\theta) \frac{d^2\theta}{dX^2} + f'(\theta) \left(\frac{d\theta}{dX} \right)^2 - M^2 \frac{(\theta - \theta_a)^{n+1}}{(1 - \theta_a)^n} \\ - Nr(\theta^4 - \theta_a^4) - Pe \frac{d\theta}{dX} \\ + M^2 G(1 + \varepsilon_G(\theta - \theta_a)) = 0. \end{aligned} \tag{7}$$

Boundary conditions are

$$\theta(1) = 1, \quad \left. \frac{d\theta}{dX} \right|_{X=0} = 0. \tag{8}$$

Thermal conductivity is described in the following cases:

By applying these cases in eq. (7), we get

Case I: When thermal conductivity is constant, $f(\theta) = 1$,

$$\begin{aligned} \frac{d^2\theta}{dX^2} - M^2 \frac{(\theta - \theta_a)^{n+1}}{(1 - \theta_a)^n} - Nr(\theta^4 - \theta_a^4) \\ - Pe \frac{d\theta}{dX} + M^2 G(1 + \varepsilon_G(\theta - \theta_a)) = 0. \end{aligned} \tag{9}$$

Case II: When thermal conductivity is linear, $f(\theta) = 1 + \beta\theta$,

$$\begin{aligned} (1 + \beta\theta) \frac{d^2\theta}{dX^2} + \beta \left(\frac{d\theta}{dX} \right)^2 - M^2 \frac{(\theta - \theta_a)^{n+1}}{(1 - \theta_a)^n} \\ - Nr(\theta^4 - \theta_a^4) - Pe \frac{d\theta}{dX} \\ + M^2 G(1 + \varepsilon_G(\theta - \theta_a)) = 0. \end{aligned} \tag{10}$$

Case III: When thermal conductivity is a quadratic function of temperature, $f(\theta) = 1 + \beta\theta^2$,

$$(1 + \beta\theta^2) \frac{d^2\theta}{dX^2} + 2\beta\theta \left(\frac{d\theta}{dX}\right)^2 - M^2 \frac{(\theta - \theta_a)^{n+1}}{(1 - \theta_a)^n} - Nr(\theta^4 - \theta_a^4) - Pe \frac{d\theta}{dX} + M^2G(1 + \varepsilon_G(\theta - \theta_a)) = 0. \tag{11}$$

Case IV: When thermal conductivity is an exponential function of temperature, $f(\theta) = e^{\beta\theta}$,

$$(e^{\beta\theta}) \frac{d^2\theta}{dX^2} + (\beta e^{\beta\theta}) \left(\frac{d\theta}{dX}\right)^2 - M^2 \frac{(\theta - \theta_a)^{n+1}}{(1 - \theta_a)^n} - Nr(\theta^4 - \theta_a^4) - Pe \frac{d\theta}{dX} + M^2G(1 + \varepsilon_G(\theta - \theta_a)) = 0. \tag{12}$$

3. Computational methods

The concepts of the matrix of integration and wavelet collocation method are given below:

3.1 Operational matrix of integration

The integration of the Legendre wavelets is

$$\psi_{m,n}(x) = \begin{cases} \sqrt{(n+1/2)} 2^{k/2} P_n(2^k x - \hat{m}), & \frac{\hat{m}-1}{2^k} \leq x \leq \frac{\hat{m}+1}{2^k} \\ 0, & \text{otherwise,} \end{cases} \tag{13}$$

where $n = 0, 1, \dots, S - 1$ and $m = 1, 2, \dots, 2^{k-1}$. Here $P_n(x)$ represents Legendre polynomials of order n and $\psi(x)$ defined in eq. (13) is obtained as

$$\int_0^x \psi(t) dt = P\psi(x), \quad \epsilon \in [0, 1),$$

where P is $2^{k-1}M \times 2^{k-1}M$, $k = 1$ is the operational matrix of integration which is given by Razzaghi and Yousefi [27].

3.2 Legendre wavelet collocation method

Let

$$\theta''(X) = c^T \psi(X). \tag{14}$$

Using boundary conditions, we get

$$\theta'(X) = c^T P\psi(X) \tag{15}$$

and

$$\theta(X) = 1 - c^T P^2\psi(1) + c^T P^2\psi(X). \tag{16}$$

Substitute the values of $\theta''(X)$, $\theta'(X)$ and $\theta(X)$ in eqs (9)–(12). Here, $\theta(X)$ is an approximate solution of these equations. Take n collocation points $X_r, r = 1, 2, 3, \dots, n$ where residual $R(X, c_1, c_2, c_3, \dots, c_n)$ becomes zero. Number of these collocation points and number of coefficients must be equal. Thus, we get the residuals for different thermal conductivity cases.

3.3 Least square method

The LSM is the residual weighted based method that reduces residuals of the test functions applied to solve nonlinear differential equation [7]. If the continuous summation of all the squared residuals is reduced [8], then

$$S = \int_x R(x)R(x)dx = \int_x R^2(x)dx. \tag{17}$$

Derivatives of S with respect to all unknown coefficients must be zero to reduce this scalar function [8], i.e.

$$\frac{\delta S}{\delta c_i} = 2 \int_x R(x) \frac{\delta R}{\delta c_i} dx = 0, \tag{18}$$

where the weight function is

$$W_i = 2 \frac{\delta R}{\delta c_i}, \tag{19}$$

where the coefficient 2 will be expelled in the equation. Then the weight function for the LSM becomes just residual derivative with respect to unknown coefficients, i.e.

$$W_i = \frac{\delta R}{\delta c_i}. \tag{20}$$

3.4 Moment method

In this method, weight function is chosen from the family of polynomials, i.e.

$$W_i(x) = x^i, \quad i = 0, 1, 2, \dots, n. \tag{21}$$

Equation for the moment method is written as

$$\int_x W_i(x)R(x)dx = 0, \quad i = 0, 1, 2, \dots, n. \tag{22}$$

Table 1. Comparison of exact solution, LWCM, LSM and FDM.

X	Exact result	LWCM	LSM	MM
0	0.60736577	0.60736561	0.60736577	0.60736577
0.1	0.60996007	0.60995991	0.6099605	0.60996053
0.2	0.61813073	0.61813058	0.61813085	0.61813089
0.3	0.63254936	0.63254923	0.63254871	0.63254872
0.4	0.65402333	0.65402322	0.65402256	0.65402252
0.5	0.68351841	0.68351832	0.68351836	0.68351831
0.6	0.72218556	0.72218548	0.72218629	0.72218626
0.7	0.77139233	0.77139227	0.77139303	0.77139305
0.8	0.83275995	0.8327599	0.83275985	0.83275991
0.9	0.90820686	0.90820684	0.90820638	0.90820642
1	1	1	1	1

Table 2. Percentage error computation.

X	Percentage error for LWCM	Percentage error for LSM	Percentage error for MM
0	2.63E-05	0	0
0.1	2.62E-05	7.05E-05	7.54E-05
0.2	2.43E-05	1.94E-05	2.59E-05
0.3	2.06E-05	0.0001028	0.000101178
0.4	1.68E-05	0.0001177	0.000123849
0.5	1.32E-05	7.32E-06	1.46E-05
0.6	1.11E-05	0.0001011	9.69E-05
0.7	7.78E-06	9.08E-05	9.33E-05
0.8	6.00E-06	1.20E-05	4.80E-06
0.9	2.20E-06	5.29E-05	4.84E-05
1	0	0	0

Using (21), eq. (22) becomes

$$\int_x x^i R(x)dx = 0, \quad i = 0, 1, 2, \dots, n. \tag{23}$$

Now we calculate the residual using MATLAB software and find the temperature distribution for the fin.

4. Exact solution

Exact solution is required for the validation of results. Here, we assume thermal conductivity gradient $f(\theta) = 1$, constant $n = 0$ and radiation–conduction parameter $Nr = 0$ in eq. (7). Then equation is reduced to the following form:

$$\frac{d^2\theta}{dX^2} - M^2(\theta - \theta_a) - Pe \frac{d\theta}{dX} + M^2G(1 + \epsilon_G(\theta - \theta_a)) = 0. \tag{24}$$

The boundary conditions are

$$\theta(1) = 1, \quad \left. \frac{d\theta}{dX} \right|_{X=0} = 0.$$

After solving the above equation using the boundary conditions, we get

$$\theta = c_1 e^{m_1 X} + c_2 e^{m_2 X} + \frac{M^2 G}{Q}, \tag{25}$$

where

$$c_1 = -\frac{m_2(1 - \frac{M^2 G}{Q})}{m_1 e^{m_2} - m_2 e^{m_1}},$$

$$c_2 = \frac{m_1(1 - \frac{M^2 G}{Q})}{m_1 e^{m_2} - m_2 e^{m_1}}$$

and

$$Q = M^2 - M^2 G \epsilon_G.$$

5. Results and discussion

We studied the moving fin with a rectangular profile. Effects of parameters such as thermogeometric (M), thermal conductivity (β), Peclet number (Pe), radiation–conduction (Nr), heat generation (ϵ_G), generation number (G), ambient temperature (θ_a) and n on temperature are studied in detail. In table 1, the exact

Table 3. Effect of n on temperature distribution in the fin for Cases I and II.

X	Case I	Case I	Case I	Case II	Case II	Case II
	$n = -1/4$	$n = 1/4$	$n = 3$	$n = -1/4$	$n = 1/4$	$n = 3$
0	0.55386929	0.6000896	0.70166774	0.70175569	0.72294255	0.78132918
0.1	0.55660423	0.60245387	0.70330688	0.70415941	0.72511888	0.78293678
0.2	0.56523455	0.60992392	0.70849648	0.71157384	0.73183718	0.78790737
0.3	0.58052259	0.62318995	0.71775088	0.72433125	0.74341841	0.79651047
0.4	0.60343214	0.64314549	0.73176458	0.74280029	0.76023472	0.80908612
0.5	0.63517325	0.6709364	0.75145924	0.76738743	0.78271311	0.8260559
0.6	0.67728084	0.70804473	0.77807492	0.79854146	0.81134168	0.8479391
0.7	0.73172569	0.7564068	0.81330122	0.8367625	0.84667965	0.87537541
0.8	0.80108909	0.81859585	0.85948854	0.88261773	0.88937302	0.90915743
0.9	0.88886671	0.8981304	0.92002382	0.93676671	0.94017861	0.95027808
1	1	1	1	1	1	1

Table 4. Effect of n on temperature distribution in the fin for Cases III and IV.

X	Case III	Case III	Case III	Case IV	Case IV	Case IV
	$n = -1/4$	$n = 1/4$	$n = 3$	$n = -1/4$	$n = 1/4$	$n = 3$
0	0.68326926	0.70827521	0.77361853	0.75210071	0.76705093	0.81194649
0.1	0.68592699	0.71064905	0.77532528	0.75430359	0.76907536	0.81347941
0.2	0.69414493	0.71799418	0.78061227	0.76104924	0.77527886	0.81818439
0.3	0.70829649	0.73066805	0.78977168	0.77254072	0.78586446	0.82624624
0.4	0.72875043	0.74904932	0.80315364	0.78897268	0.80104175	0.83788441
0.5	0.75585394	0.77352604	0.82116892	0.81052556	0.82102202	0.85335427
0.6	0.78991525	0.80448208	0.84429274	0.83736084	0.84601384	0.87294929
0.7	0.83118988	0.84228398	0.87306985	0.86961812	0.87621928	0.89700414
0.8	0.87987413	0.88727067	0.90812217	0.90741467	0.91183141	0.92589928
0.9	0.93610933	0.93974925	0.95016014	0.95084777	0.95303346	0.96006706
1	1	1	1	1	1	1

result, LWCM, LSM and MM results are presented. It has been observed that results computed by these methods and exact results are in good agreement. So to find the validated method for further computation, we calculated the percentage error which is presented in table 2. From the percentage error computation, we found that LWCM has the least error. So, for further computation, we used LWCM.

In tables 3 and 4, the effect of n on temperature is presented. It has been observed that as n increases, the temperature in the fin also increases. Case I has the lowest temperature while Case IV has the highest temperature.

Figure 2a shows the effect of thermal conductivity which is taken as (i) constant, (ii) linear, (iii) quadratic and (iv) exponential. From the figure, we can see that the temperature is maximum when thermal conductivity is an exponential function of temperature whereas temperature is minimum in Case I when thermal conductivity is constant.

Effect of thermogeometric parameter on temperature is shown in figure 2b. It has been observed that by increasing M , the temperature in the

moving fin decreases. Case I have the lowest temperature and Case IV has the highest temperature.

Figure 2c shows the effect of thermal conductivity. We observed that if the thermal conductivity in the fin increases, the temperature in the fin also increases. Case IV has the highest and Case III has the lowest temperature.

Effect of Peclet number is presented in figure 3a. By increasing the value of the Peclet number, fin temperature decreases. Case I has the lowest and Case IV has the highest temperature.

Effect of radiation–conduction is presented in figure 3b, where we can see that the temperature decreases by increasing the value of radiation–conduction. The lowest temperature occurs in Case I and the highest in Case IV.

In figure 3c, the effect of heat generation is shown. It shows that temperature in the fin increases as ε_G increases. Case IV has the highest and Case I has the lowest temperature.

Effect of G on temperature is presented in figure 4a. We can see that by increasing G , fin temperature also increases. The lowest temperature occurs in Case I and highest in Case IV.

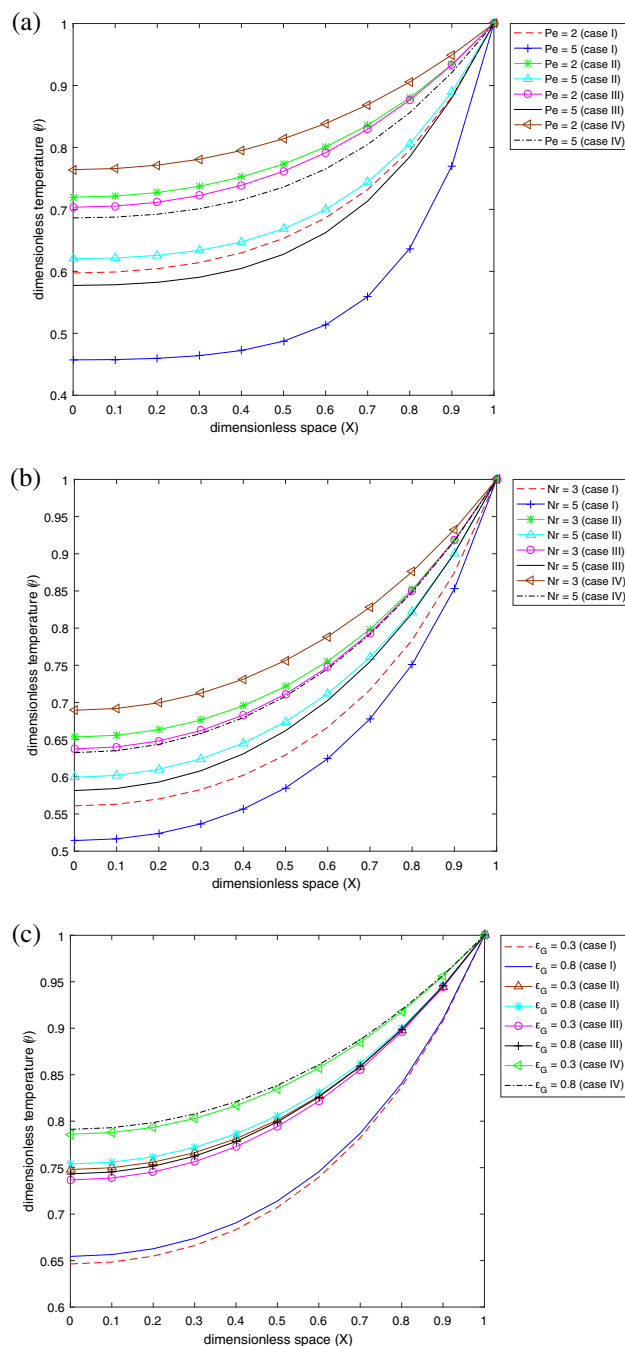
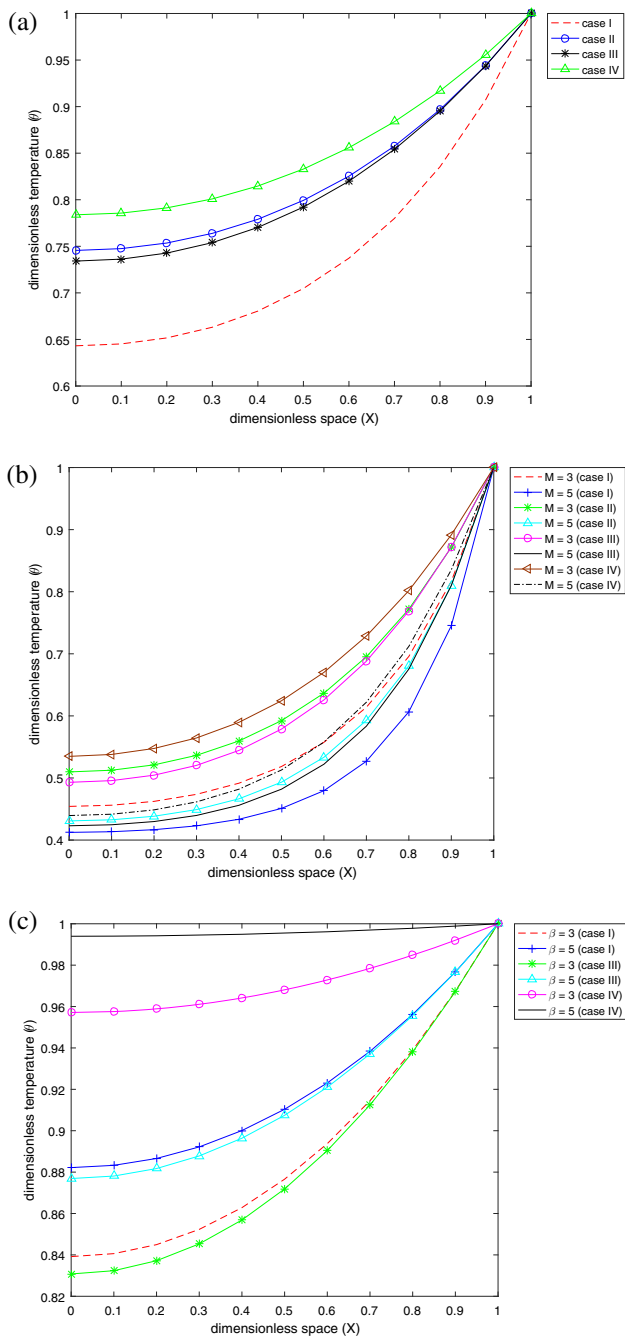


Figure 2. (a) Temperature distribution in the fin for thermal conductivity in Cases I, II, III and IV, (b) effect of M on temperature distribution and (c) effect of β on temperature distribution.

Figure 4b shows the effect of ambient temperature. The temperature in the fin increases by increasing the value of θ_a . Case IV attains the highest and Case I attains the lowest temperature.

6. Conclusion

In this paper, a one-dimensional moving fin has been studied. LWCM, LSM and MM are used for the com-

Figure 3. Effect of (a) Pe on temperature distribution, (b) Nr on temperature distribution and (c) ϵ_G on temperature distribution.

putation. An exact solution is obtained to validate the results computed by these methods. From the percentage error computation, we conclude that LWCM has the least error compared to other methods. So the effect of parameters on the temperature in a moving fin is demonstrated by LWCM. We concluded that temperature in the fin increases by increasing thermal conductivity, heat generation, ambient temperature, n and G parameters. The temperature in the fin decreases

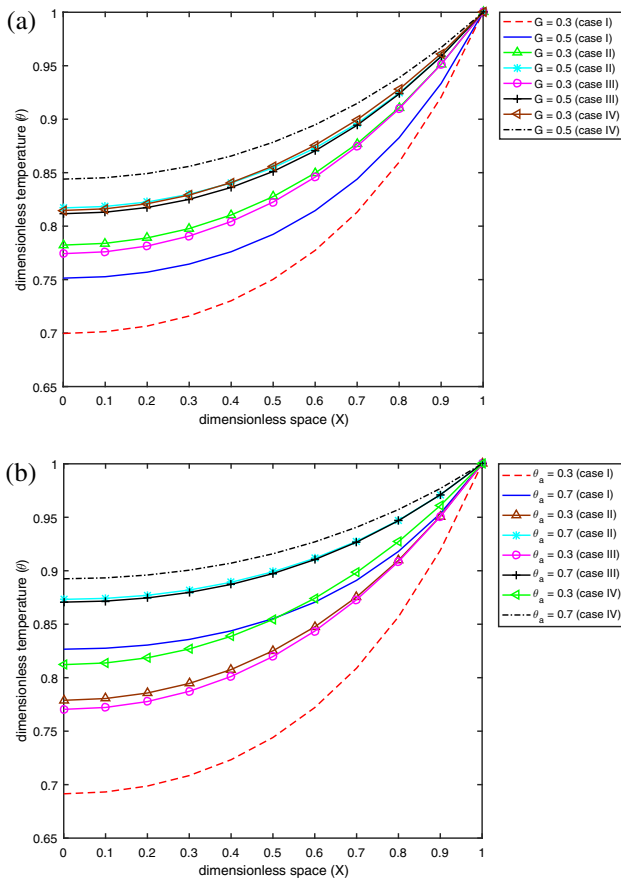


Figure 4. Effect of (a) G on temperature distribution and (b) θ_a on temperature distribution.

by increasing parameters like Peclet number, thermogeometric, radiation–conduction. Meanwhile, the temperature distribution is maximum in Case IV when thermal conductivity is an exponential function of temperature.

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