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Temperature field as a codifier of entanglement

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Abstract. Vectorial fields such as the electromagnetic field are commonly employed for codifying both qubits and information. It is considered as a three-qubits Heisenberg chain with multiple interaction in the presence of a scalar field, as it is the temperature field. The corresponding pairwise thermal entanglement is calculated. Our results corroborate with the results of Yang and Huang, *Quant. Inf. Proc.* **16**:281 (2017). To first order of approximation in powers of $\beta = 1/T$, it is shown that a temperature field can codify entanglement. A scalar field substrate can be used to distribute quantum entanglement in communication protocols that require this ingredient. Present results can be extrapolated to other physical systems such as Heisenberg spins (two-site) or superconducting qubits, provided the respective quantum correlation (entanglement or discord) expression depends explicitly on the temperature scalar field.

Keywords. Temperature field; scalar field; entanglement.

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1. Introduction

In the discrete variable regime of quantum information processing (QIP), photons are employed as the carriers of information with qubits encoded in their polarisations. In the continuous variable regime, information is contained in continuous degrees of freedom of the electromagnetic field such as field quadratures. So vectorial fields are commonly employed as codifiers of information [2–6]. On the other hand, an essential physical quantity for QIP is quantum entanglement [7] which plays an important role in both theoretical and experimental branches of physics [8–10]. In particular, the thermal entanglement refers to such a quantity in the presence of a temperature field. This quantity has been studied in many situations such as in one-dimensional Heisenberg model [11], Heisenberg spin chain with multiple interaction [12] and Yang Baxter models [13]. In ref. [12] it was found that entanglement survives at higher temperatures. At this stage, the following question arises: Can a scalar field, as it is the temperature field, codify quantum entanglement? In order to investigate further on this topic, a three-qubits system is considered here with $XXZ + YZY$, XZY and YZX interactions in the presence of a temperature field. Respective pairwise entanglement is calculated and the result corroborates with the results of [1]. Furthermore, it is found that a scalar field, e.g. a temperature field, can codify quantum entanglement. This opens the way for further searches of scalar fields which are different from the temperature field that can codify quantum entanglement. It is worth to mention that the recent findings pave the way for further revolutionary technological applications in long-distance quantum communications. Thus, a scalar field substrate can be used to distribute quantum entanglement in protocols that require this ingredient. Concerning to the question whether the present results can be extrapolated to other physical systems such as Heisenberg spins (two-site) or superconducting qubits, the answer is affirmative. It can be done, provided the respective quantum correlation (entanglement or discord) expression depends explicitly on the temperature scalar field. Present work is organised as follows. In §2, a brief account of the Heisenberg XXZ spin chain with multiple interaction is given. In addition, eigenvalues of the respective Schrödinger equation are analytically obtained. In §3, the temperature field (scalar field) is identified as a codifier of ground thermal entanglement. Finally, the results are summarised.

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2. Heisenberg spin chain with multiple interaction

The respective Hamiltonian is given by

$$H = \sum_{a=1}^{N}\Big[J(\sigma_a^x\sigma_{a+1}^x + \sigma_a^y\sigma_{a+1}^y) + J_z\sigma_a^z\sigma_{a+1}^z + B\sigma_a^z\Big] + \sum_{a=1}^{N} J'\Big[\sigma_a^x\sigma_{a+1}^z\sigma_{a+2}^x + \sigma_a^y\sigma_{a+1}^z\sigma_{a+2}^y\Big] + \sum_{a=1}^{N} J''\Big[\sigma_a^x\sigma_{a+1}^z\sigma_{a+2}^y - \sigma_a^y\sigma_{a+1}^z\sigma_{a+2}^x\Big], \quad (1)$$

where B is the external magnetic field, J' and J'' denote the $XZX + YZY$ and $XZY - YZX$ three-sites interactions, respectively. In what follows, we constrain ouselves to a $N = 3$ qubits ring. The corresponding eigenvalues associated with the Schrödinger equation $H|\Psi\rangle = E|\psi\rangle$ are as follows:

$$\begin{aligned} E_0 &= 3(J_z + B),\\ E_1 &= B - J_z - 2J - 2J' - 2\sqrt{3}J'',\\ E_2 &= B - J_z - 2J - 2J' + 2\sqrt{3}J'',\\ E_3 &= B - Jz + 4J + 4J', \end{aligned} \quad (2)$$

$$\begin{aligned} E_4 &= -B - J_z - 2J + 2J' - 2\sqrt{3}J'',\\ E_5 &= -B - J_z - 2J + 2J' + 2\sqrt{3}J'',\\ E_6 &= -B - J_z + 4J - 4J',\\ E_7 &= 3(J_z - B). \end{aligned} \quad (3)$$

The values of the coupling constants J, J_z, J' and J'' shall be subjected to constraints (see the discussion below).

3. Temperature field and entanglement

The density matrix associated with a thermal equilibrium state is

$$\rho(T) = \frac{1}{Z}\exp\Big(-\frac{H}{T}\Big),$$

where $Z = \sum_{a=0}^{7}\exp(-\beta E_a)$. In what follows, we choose Boltzmann's constant as the unit in such a way that $\beta = 1/T$ where T is the temperature. If pairwise entanglement is considered, then the reduced density matrix $\rho_{12}(T) = \mathrm{tr}_3\rho(T)$ is required which is given by [12]

$$\rho_{12}(T) = \frac{1}{Z}\begin{pmatrix}\alpha & 0 & 0 & 0\\ 0 & \nu & \delta & 0\\ 0 & \delta^* & \nu & 0\\ 0 & 0 & 0 & \mu\end{pmatrix}, \quad (4)$$

where

$$\begin{aligned} \alpha &= \frac{1}{3}\big(3\mathrm{e}^{-\beta E_0} + 2\mathrm{e}^{-\beta E_1} + \mathrm{e}^{-\beta E_3}\big),\\ \nu &= \frac{1}{3}\big(2\mathrm{e}^{-\beta E_1} + \mathrm{e}^{-\beta E_3} + 2\mathrm{e}^{-\beta E_4} + \mathrm{e}^{-\beta E_6}\big),\\ \delta &= \frac{1}{3}\big(-\mathrm{e}^{-\beta E_1} + \mathrm{e}^{-\beta E_3} - \mathrm{e}^{-\beta E_4} + \mathrm{e}^{-\beta E_6}\big),\\ \mu &= \frac{1}{3}\big(3\mathrm{e}^{-\beta E_7} + 2\mathrm{e}^{-\beta E_4} + \mathrm{e}^{-\beta E_6}\big). \end{aligned} \quad (5)$$

The concurrence is given by (due to the invariance under cyclic shifts, the value of the concurrence does not depend on the pairs of the qubits chosen)

$$\mathcal{C} = \max\Big\{\frac{2}{Z}\big(|\delta| - \sqrt{\alpha\mu}\big), 0\Big\}. \quad (6)$$

In the present work, the following expansions is made in powers of β to a first-order of approximation

$$\mathrm{e}^{-\beta E_a} \simeq 1 - \beta E_a + \mathcal{O}\big(\beta^2\big), \qquad a = 0, 1, 2, ..., 7, \quad (7)$$

where the eigenvalues E_a ($a = 0, 1, 2, ..., 7$) are given by eq. (2). To substitute (7) in (5) and (6) and solving for β, the following equation is obtained:

$$\frac{\beta - \beta_0}{\beta_0} \simeq 2\mathcal{C}, \quad (8)$$

where

$$\beta_0 = \Big(J + J_z + J' - \frac{\sqrt{3}}{6}J''\Big)^{-1}. \quad (9)$$

The values of the magnetic fields J, J_z, J' and J'' must be such that $\beta_0 > 0$. Equation (8) shows that a temperature field (scalar field) can codify entanglement.

4. Conclusions

To determine whether a temperature field (scalar field) can codify entanglement, a three-qubits Heisenberg spin chain has been considered in the presence of multiple interaction. The respective Schrödinger equation has been solved and the eigenvalues analytically found in terms of the involved magnetic fields. Without loss of generality, it is traced out over the third qubit the density matrix associated with the thermal equilibrium state. As a result, the two-qubits density matrix is obtained in the presence of a temperature field which is given by eqs (4) and (5). From these equations, the pairwise concurrence given by eqs (5) and (6) is derived. Concurrence is considered as a measure of entanglement. A series expansion in powers of $\beta = 1/T$ is made as indicated by eq. (7) and substituting in eqs (5) and (6), eq. (8) is obtained. It is important to emphasise that the validity of our approximation is for values of the magnetic fields J, J_z, J' and J'' for which $0 < \beta_0 \ll 1$ in eq. (9). From such an equation it is seen that a temperature field (scalar field) can codify quantum entanglement. The above can pave the way for further searches to check whether scalar fields other than temperature fields can codify entanglement.

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References

[1] J Yang and Y Huang, *Quant. Inf. Proc.* **16**, 281 (2017)
[2] C Cabrillo, J I Cirac, P García-Fernández and P Zoller, *Phys. Rev. A* **59**, 1025 (1999)
[3] L-M Duan, M D Lukin, J I Cirac and P Zoller, *Nature* **414**, 413 (2001)
[4] S L Braunstein and H J Kimble, *Phys. Rev. Lett.* **80**, 869 (1998)
[5] H-J Briegel, W Dür, J I Cirac and P Zoller, *Phys. Rev. Lett.* **81**, 5932 (1998)
[6] P van Loock *et al*, *Phys. Rev. Lett.* **96**, 240501 (2006)
[7] J Brody, *Quantum entanglement* (MIT Press, 2020)
[8] G Alber, T Beth, M Horodecki, P Horodecki, R Hodorecki, M Rotteler, H Weinfurter, R Werner and A Zeilinger, *Quantum information: An introduction to basic theoretical concepts and experiments* (Beijing World Publishing Corporation, 2004)
[9] M A Nielsen and I L Chuang, *Quantum computation and quantum information* (Cambridge University Press, 2000)
[10] L Amico, R Fazio, A Osterloh and V Vedral, *Rev. Mod. Phys.* **80**, 51 (2008)
[11] M C Arnesen, S Bose and V Vedral, *Phys. Rev. Lett.* **87**, 017901 (2001)
[12] W W Cheng, C J Shan, Y X Huang, T K Liu and H Li, *Physica E* **43**, 235 (2010)
[13] C Sun, T Hu, G Wang, C Wu and K Xue, *Int. J. Quant. Inf.* **7**, 879 (2009)