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On the efficiency of quantum error correction for quantum image transmission algorithm

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Abstract. Quantum imaging algorithm is modified to reduce the complexity of the quantum circuit. Simplification was done by replacing nonlinear optical elements by linear elements which allow one to obtain conventional quantum entanglement operator. The obtained results show the expected efficiency of data transmission. Quantum error correction is used to improve the quality of image transmission. Modelling of image transmission with classic quantum computer interpreter is suggested.

Keywords. Quantum imaging; quantum communication; image restoration; quantum error correction.

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1. Introduction

A working quantum computer has already become a real possibility due to recent development in the field of nanotechnology but there is still a long way to go [1]. The analogous situation takes place in quantum communication. Optical channels are preferable in quantum communications (see, e.g., [2–10]). The idea of quantum signal transportation appeared at the very beginning of quantum algorithm researches or even earlier. The Abbe Rayleigh diffraction limit constrains the spatial resolution for classical imaging methods. Quantum imaging exploits correlations between photons to reproduce structures with higher resolution. Quantum-correlated N-photon states were shown to potentially surpass the classical limit by a factor of $1/N$, corresponding to the Heisenberg limit, using the method known as optical centroid measurement [11–13]. Quantum imaging has many applications in communications, material investigation, biology, etc. [14–17].

In 1998, Steven Weinberg turned his attention to the problem of measurement which does not give one the ability to use full information contained in quantum entangled system. Due to this reason, researchers try to avoid unnecessary measurements while constructing quantum algorithms including quantum information transfer algorithms. It leads to using uncontrollably large number of elements in transmission system. Another problem is the overwhelming difficulty in maintaining exclusive measurement system. Measuring high-frequency waves is more difficult than measuring low-frequency waves. Lemos *et al* [18] suggested a quantum imaging concept based on induced coherence without induced emission. They used two photons of different wavelengths. The photons that pass through the imaged object are never detected, while they obtained images exclusively with the signal photons, which do not interact with the object. This enables the probe wavelength to be chosen in a range for which suitable detectors are not available. However, this scheme is not simple for implementation due to the nonlinear elements used. In the present work, we suggest to replace all nonlinear components by a linear quantum entanglement operator.

2. Quantum imaging

2.1 *Classic quantum computer interpreter*

Modified quantum imaging algorithm is implemented using a classic quantum computer interpreter. The main elements of quantum algorithm are unitary operators. Operator A is called unitary operator if $AA^+ = A^+A = E$ where E is the identity operator and A^+ is the adjoint operator for operator A. Each unitary operator corresponds to some quantum operation (quantum gate) applied to quantum system. A sequence of quantum gates forms a quantum algorithm.

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Qubit is the main object for quantum computing. Physical qubit is a quantum system which can be a superposition of two states. In quantum informatics, any n-qubit system has 2^n linearly independent states. Each qubit can be represented as the complex 2-vector

$$|\phi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad \alpha, \beta \in \mathbb{C}.$$

If one performs a measurement, he/she observes the qubit in the corresponding state with the probabilities $|\alpha|^2, |\beta|^2$, respectively. Correspondingly, one has the normalisation condition, $|\alpha|^2 + |\beta|^2 = 1$.

The state space for the multiqubit system is the tensor product of the state spaces for separate qubits. Let A be the operator acting on the first qubit $|\phi\rangle$ and B be the operator acting on the second qubit $|\psi\rangle$. Then, the tensor product of operators $A \otimes B$ (in matrix case, it is the Kronecker product of matrices) acts as follows:

$$(A \otimes B)(|\phi\rangle \otimes |\psi\rangle) = A(|\phi\rangle) \otimes B(|\psi\rangle).$$

Particularly, the tensor product of two vectors $|\phi\rangle, |\psi\rangle$ of sizes $n \times 1$, $m \times 1$ is the vector ξ of size $nm \times 1$ where

$$\xi_{ni+j} = \phi_i \psi_j \;\forall i \in (1 \ldots n), \forall j \in (1 \ldots m).$$

The tensor product $a \otimes B$ of two matrices A, B of sizes $n \times m$, $p \times q$ is the block matrix C (of size $np \times mq$) with the following block entries:

$$C_{ij} = A_{ij} B \;\forall i \in (1 \ldots n), j \in (1 \ldots m).$$

So, for implementing the classic quantum interpreter, the following operations should be implemented:

1. Algebra of the application of matrix to vectors.
2. Algebra of the multiplication between two matrices.
3. Algebra of the tensor products between two vectors.
4. Algebra of the tensor products between two matrices.

2.2 *Quantum imaging algorithm*

The object for imaging acts as a quantum gate T. T is a unitary operator depending on the coordinates (x, y) at the object plane. It can be represented in the following form:

$$T(t)(\alpha|0\rangle + \beta|1\rangle) = \xi|0\rangle + \zeta|1\rangle,$$

where

$$\xi = \alpha t - \beta\sqrt{1-t^2}, \quad \zeta = \beta t + \alpha\sqrt{1-t^2}.$$

Here $t : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$. The matrix representation of operator T is as follows:

$$T(x, y) = \begin{pmatrix} t(x, y) & -\sqrt{1-t^2(x, y)} \\ \sqrt{1-t^2(x, y)} & t(x, y) \end{pmatrix}.$$

This rotation operator is a one-qubit operator. For many-qubit system, one can consider the corresponding tensor product

$$T_{k+1} = T \otimes T_k, \quad T_1 = T.$$

Also one can construct T not for each pixel (or other atomic element) but for a cluster. Using this operator, one comes to the final representation of the quantum algorithm (the corresponding circuit is shown in figure 1):

$$Q = (\mathrm{NL1}_{n+n} \otimes E_n) \circ (E_n \otimes T_n(x, y)) \circ (E_n \otimes \mathrm{NL2}_{n+n}).$$

Here NL1 and NL2 elements are nonlinear crystals which split the laser beam into two monochromatic beams and E_n is the identical operator. It is difficult to implement these elements. We use here linear optical elements (CNOT-gates):

$$Q' = (\mathrm{CNOT}_{n+n} \otimes E_n) \circ (E_n \otimes T_n(x, y)) \circ (E_n \otimes \mathrm{CNOT}_{n+n}).$$

2.3 *Measurements with interference model*

The measurements determine particle numbers which are recorded by the sensors. It is important that the sensors detect low-frequency signal only (high-frequency signal is not detected) which match them before they have an interference. Taking into account the distribution for the modes, one can calculate the interference between them by simple addition:

$$A(x, y) \dot{+} B(x, y) = I(x, y),$$

where $A(x, y)$ is the distribution after entanglement with modified high frequency, $B(x, y)$ is the distribution after entanglement with non-modified high frequency and $\dot{+}$ is the binary operator which sums only projections with right angle. So there are two different interference pictures:

$$I_A(x, y) = A(x, y) + T^+(x, y)B(x, y),$$
$$I_B(x, y) = T^+(x, y)A(x, y) + B(x, y).$$

Let us perform some pre-transformations. Let $P_h(x, y)$ be the initial high-frequency distribution, $P_l(x, y)$ be the initial low-frequency distribution, $Z(x, y)$ be the constant zero distribution, $H(x, y)$ be the high-frequency after processed distribution,

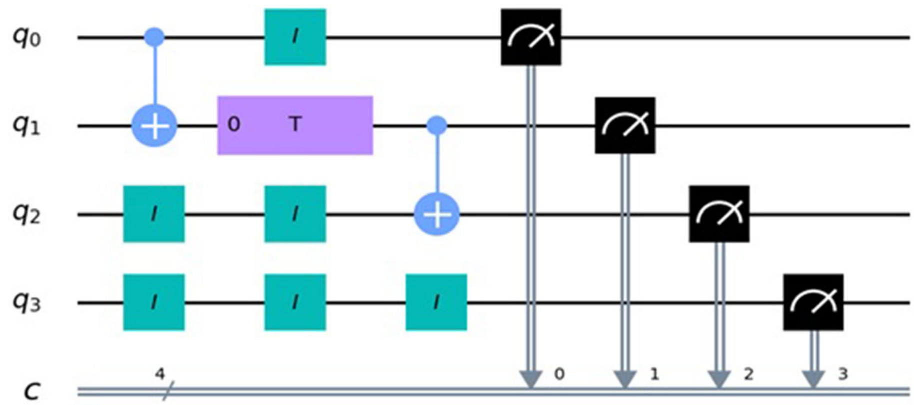

Figure 1. Quantum imaging algorithm. Quantum gate T is related to the object for imaging, q_i marks qubit, C is the classical register for results of measurements, + denotes the CNOT operator controlled by the qubit connected to it, I is the identical gate (operator).

$$Q'(P_h \otimes P_l \otimes P_l) = (A \otimes B \otimes H),$$
$$TI_A = TA + TT^{+}B = TA + B.$$

As A, B are real scalars, their conjugation does not change their inner structure.

$$T^{+}I_A = T^{+}A + B = I_B$$

or

$$TI_B = I_A.$$

In this way, we can get A, B if we know the nature of T. Keeping in mind the known values of A and B, one can obtain the approximate picture.

Let us estimate the error. Mean squared error (ϵ) is a metric for the quality of the transmitted image ($I1, I2$ are the initial and the transmitted signals).

$$\epsilon(I1, I2) = \sqrt{\frac{1}{mn}\sum_{x=1}^{n}\sum_{y=1}^{m}|I1(x, y) - I2(x, y)|^2}.$$

The corresponding results for different number of bits is presented in table 1. One can see that ϵ does not depend essentially on the number of qubits.

2.4 *Algorithm improvements*

The suggested algorithm has non-deterministic nature with non-zero error probability. We improve it by adding Shor's error correction code [19] for the main qubit. The scheme is shown in figure 2. This algorithm incorporates two algorithms intended for correction of bit-flip and sign-flip errors. It should be mentioned that flipped bits are the only type of error in classical computer, but there is another possibility of an error with quantum computers, the sign flip. The 1st, 4th and 7th qubits are for the sign flip code, while the three groups of qubits (q_1, q_2, q_3), (q_4, q_5, q_6) and (q_7, q_8, q_9) are designed for the bit flip code.

As the first step, we should determine the error probability for auxiliary qubits ($q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9$ on the scheme). It is comfortable to have equal error probabilities

$$p_{err}(q_i) = \frac{1}{8}, \quad i \in [2, 9],$$

but it is difficult to achieve this situation. Really, due to the fact that q_1, q_4, q_7 are the most loaded qubits in this quantum algorithm, it would be better if

$$p_{\text{err}}(q_4) \le p_{\text{err}}(q_7) \le p_{\text{err}}(q_j), \quad j = 5, 6, 8, 9.$$

In figure 3, N has close to linear dependence from $p_{\text{err}}(q_1)$ with the same slope angle for different probabilities of error in auxiliary qubits. So, in general

$$p'_{\text{corrected}}(q_1) \approx p_{\text{corrected}}(q_{\text{aux}}),$$

where $p'_{\text{corrected}}(q_1)$ corresponds to new correctness rate. Naturally, in the case of non-zero auxiliary qubit error probability, there is no ideal error correction.

Figure 4 shows the efficiency of corrections, i.e. the per cent of the corrected errors. It strongly depends on probability of error in auxiliary qubits.

3. Conclusion

A quantum image transmission method was suggested by using linear elements in quantum circuit (without nonlinear ones). This scheme is essentially simpler to implement than the corresponding nonlinear analogous ones. It shows an appropriate quality of transmission. But there are two unsolved problems: the image will be blurred and a classic channel (with non-fixed size of data) is to be used. This blurring of images can be par-

Table 1. Mean squared error (ϵ) for different number of qubits.

Size	16	32	64	128	256	512
ϵ	0.5601	0.5559	0.5612	0.5603	0.5600	0.5603

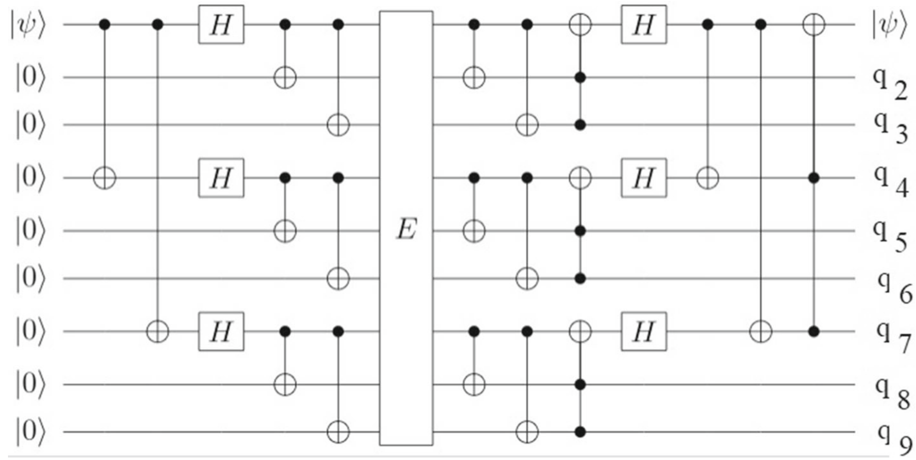

Figure 2. Quantum error correction algorithm. Here H is the Hadamard gate and E is a quantum channel that can arbitrarily corrupt a single qubit. Qubits are enumerated from top to bottom.

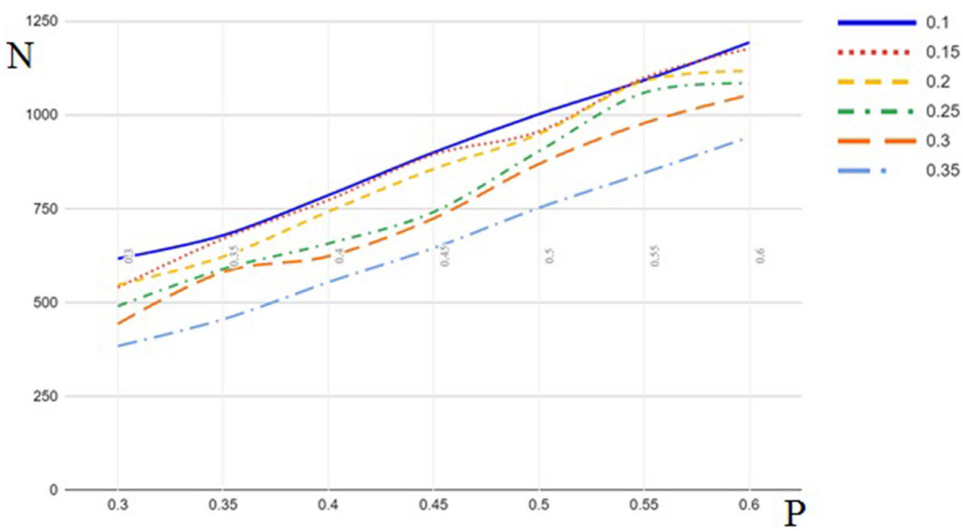

Figure 3. Number N of the corrected errors via the probability $P = p_{\mathrm{err}}(q_1)$ of the error in the main qubit; different curves correspond to different probabilities of error in auxiliary qubits. Number of experiments for each point is 2048.

tially solved by using other non-quantum methods or additional classical data transfer. Need for classic channel, unfortunately, cannot be resolved right now with this algorithm without complicating the scheme. Nevertheless, the simplicity of implementation gives one an essential advantage.

The quantum image transmission could be patched with Shor's error correction algorithm. It leads to some improvement for correctness rate. This rate becomes more stable and less dependent on the main qubit error probability. However, small error rates are required for all the eight auxiliary qubits.

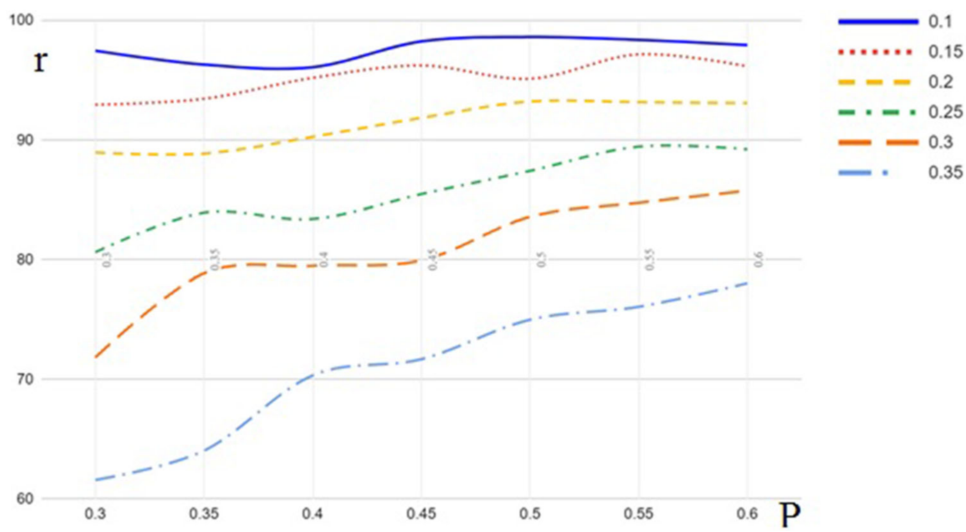

Figure 4. Efficiency r of correction (per cent of corrected errors) via the probability P of the error in the main qubit; different curves correspond to different probabilities of error in auxiliary qubits.

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References

[1] G J Milburn and M J Woolley, *Contemp. Phys.* **49**(**6**), 413 (2008)
[2] A Pathak and A Banerjee, *Optical quantum information and quantum communication* (SPIE Spotlight, NY, 2016)
[3] V O Sheremetev, A S Rudenko and A I Trifanov, *Nanosystems: Phys. Chem. Math.* **9**, 484 (2018)
[4] M P Faleeva and I Y Popov, *Quantum Inform. Process.* **19**, 72 (2020), https://doi.org/10.1007/s11128-019-2569-y
[5] M P Faleeva and I Y Popov, *Nanosyst.: Phys. Chem. Math.* **11**, 651 (2020)
[6] M P Faleeva and I Y Popov, *Indian J. Phys.* **96**, 2501 (2022)
[7] M Bohmann, A A Semenov, J Sperling and W Vogel. *Phys. Rev. A* **94**, 010302(R) (2016)
[8] D Yu Vasylyev, A A Semenov and W Vogel, *Phys. Rev. Lett.* **117**, 090501 (2016)
[9] T Herbst, T Scheidl, M Fink, J Handsteiner, B Wittmann, R Ursin and A Zeilinger, *PNAS* **112**, 14202 (2015)
[10] P A Gilev and I Y Popov, *Nanosyst.: Phys. Chem. Mathem.* **10**, 410 (2019)
[11] T B Pittman, Y H Shih, D V Strekalov and A V Sergienko, *Phys. Rev. A* **52** R3429 (1995)
[12] P A Morris, R S Aspden, J E C Bell, R W Boyd and M J Padgett, *Nature Commun.* **6**, 5913 (2015)
[13] M Unternahrer, B Bessire, L Gasparini, M Perenzoni and A Stefanov, *Optica* **5**, 1150 (2018)
[14] R Tenne *et al*, *Nat. Photon.* **13**, 116 (2019)
[15] Ch Schnell, *Nature Meth.* **16**, 214 (2019)
[16] M Genovese, *J. Opt.* **18**, 073002 (2016)
[17] M Kolobov (Ed.), *Quantum imaging* (Springer, Berlin, 2007)
[18] G B Lemos, V Borish, G D Cole, S Ramelow, R Lapkiewicz and A Zeilinger, *Nature* **512**, 409 (2014)
[19] P W Shor, *Phys. Rev. A* **52**, R2493 (1995)