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Reconstructing Tsallis holographic phantom

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Abstract. The current research considers the Tsallis holographic dark energy (THDE) with the Hubble horizon as an IR cut-off with a non-interacting flat Friedmann–Lemaitre–Robertson–Walker (FLRW) Universe. We investigate the evolutionary behaviour of deceleration and the equation of state parameter for the distinct Tsallis exponent δ and noticed appropriate behaviour in the model. The scalar field of the phantom describes the non-interacting Tsallis holographic dark energy when $\delta > 2$, then we demonstrate the phantomic narration when $\delta > 2$ for the Tsallis holographic dark energy and reconstruct the scalar field potential of the phantom.

Keywords. Tsallis holographic dark energy; Friedmann–Lemaitre–Robertson–Walker Universe; phantom.

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1. Introduction

The demonstration of the canonical ΛCDM scenario provides an excellent match to observations at low and high red-shift, see some examples [1–6]. Despite its enormous success, there are some tensions among the values of cosmological parameters inferred from independent datasets [7–10]. The most famous and persisting one is that related to the value of H_0 (the Hubble constant) which is estimated from Planck cosmic microwave background (CMB) data ($h = 0.6737 \pm 0.0054$) [2] against the value obtained from Cepheid-calibrated local distance ladder measurements ($h = (0.7403 \pm 0.0142$, $R19$) [11]), introduced as the H_0 tension, with $h = H_0/(100\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1})$. This tension now reaches the 4.4σ level. For explaining H_0, two main avenues are shown: The first one is based on the possibility that Planck and/or the local distance ladder measurement of H_0 suffer from unaccounted systematics. The second more intriguing possibility is that the H_0 tension might be the first sign for physics beyond the concordance ΛCDM model. The most economical possibilities in this direction involve some kind of dark radiation (to boost N_{eff} beyond its canonical value of 3.046) or phantom dark energy (DE) (i.e. a component of dark energy (DE)) with the equation of state (EoS) $\omega < -1$) [12–14].

The dynamical DE scenario is also an alternative approach for DE. We can suitably understand the problem of the cosmological constant by offering a DE element with a dynamic variable EoS and considering that some unexplained mechanism eliminated the vacuum energy specifically to zero. Some scalar field mechanism recommended the dynamical DE and advised that when a scalar field evolves under a fitting potential, it gives an energy model with negative pressure. Hence, many researchers have investigated a variety of scalar field models, including quintom [15,16], phantom [17], k-essence [18], quintessence [19], ghost condensate [20], tachyon [21], and so on. Here, one should observe that to explain the underlying philosophy of DE, the scalar field DE models are as useful. Besides, other suggestions on DE holds Chaplygin gas models [22], brane-world models [23], interacting DE models [24], etc. One should realise, nevertheless, that these models are almost settled at the phenomenological level, lacking theoretical root. Modern physics mentions that for the evolution of the Universe, the origin of the prevailing acceleration is a puzzle [25,26]. To resolve the expanded acceleration of late-time, two proposals are suggested: modified gravity or DE theory which are examined extensively in various researches. For a survey on cosmic acceleration of late-time in the hypothesis of modified gravity and DE, see [27,28] and the references therein. For explain-

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ing the issues of DE, the holographic dark energy (HDE) theory can be used [29–34]. Besides, the original HDE hypothesis depends on the Hubble horizon and Bekenstein entropy as the infrared cut-off cannot give a proper explanation for a flat Friedmann–Lemaitre–Robertson–Walker (FLRW) Universe [30–32]. Astrophysicists tried to resolve these issues by examining different cut-offs, interactions among the cosmos sectors and several entropies. They even worked on the sequence of the considered proposals [35,36].

Multiple formalisms of the generalised entropy have been applied to analyse gravitational and cosmological happenings because of the darkness of the unexplored cosmos of time-space and the long-range phase of gravity [37,38]. The conclusions of such research show that the possibilities of the power-law arrangements (generalised entropy formalisms) give a satisfactory settlement with gravity and its associated concerns. Recently, some new HDE models have been introduced [39–42] assigning several generalised entropies to the horizon of the FLRW Universe. The backbone of these attempts comes from the fact that the Bekenstein entropy can be obtained by applying the Tsallis statistics to the system horizon [43–46]. The concerned models themselves exhibit competent resistance [39,40]. Tavayef *et al* [47] proposed a new HDE model named Tsallis holographic dark energy (THDE) using the general model of the Tsallis's entropy expression [48], and taking the holographic hypothesis into account to describe the late-time accelerating Universe. They examined the evolution of a flat FLRW Universe filled with DM and THDE without interaction. Tavayef *et al* [47] have shown that the Tsallis holographic dark energy model in the flat Universe lies in the quintessence regime, while Zadeh *et al* [49] have shown that one can generate phantom behaviour from THDE model in a non-flat Universe.

Setare [50] investigated the correspondence of phantom-like HDE with interacting FLRW cosmos in non-flat Universe and reconstructed the scalar field potential of the phantom. The researchers presented that depending on the values of δ (the Tsallis parameter), THDE acts as phantom-like DE. The EoS of DE crosses the border $\omega_D = -1$ of the cosmological constant depending on the standards of δ in the time of the expansion [51,52]. This research proposes conformity among the framework of phantom DE and THDE. A flat FLRW Universe demonstrates the explanation of phantom-like THDE without interaction with the Tsallis parameter $\delta > 2$, and also reconstructed the scalar field potential of the phantom.

The objective of the manuscript is as follows: In §2, THDE is explained. In §3 and 4, we proceed to explore the cosmological model and the resulting cosmological behaviour. In §5, the scalar potential behaviour of the scalar field of the phantom and the evolution pattern of the Tsallis holographic phantom have been explored and reconstructed. Finally, in §6, the conclusions are given.

2. Tsallis holographic dark energy

Let us recall the derivation and the interpretation of standard holographic energy density ($\rho_D = 3c^2m_p^2/L^2$), which is based on the entropy–area relation $S \sim A \sim L^2$ of black holes. Here, $A = 4\pi L^2$ depicts the area of the horizon [29] though the quantum reflection can modify the description of HDE [35,36]. Tsallis and Cirto displayed the transformation of the horizon entropy of a black hole [48] as

$$S_\delta = \gamma A^\delta. \tag{1}$$

Here, the non-additivity parameter is represented as δ and γ is an unknown constant [48]. Noticeably, the Bekenstein entropy recovers the appropriate limit of $\delta = 1$ and $\gamma = 1/4G$ (in the unit where $h = k_B = c = 1$). Particularly, the power-law distribution of probability becomes ineffective at this point, and the common probability allocation describes the system [48]. The quantum gravity [53] approves this relation and points to interesting outcomes in the holographical and cosmological structures [54].

Based on the holographic principle (HP), which states that the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume and it should be constrained by an infrared cut-off, Cohen *et al* [29] proposed a relation between the IR (L) and the system entropy (S), UV (Λ) cut-off as

$$L^3\Lambda^3 \leq (S)^{3/4}, \tag{2}$$

which after combining with eq. (1) leads to [29]

$$\Lambda^4 \leq L^{2\delta-4}(\gamma(4\pi)^\delta), \tag{3}$$

where Λ^4 indicates the density of vacuum energy and ρ_D is the DE energy density in the theory of HDE [32,39]. We can introduce THDE by applying the above inequality as

$$\rho_D = BL^{2\delta-4}, \tag{4}$$

where B represents an unknown parameter [32,39]. Let us examine a flat FLRW Universe with a proper candidate that is the Hubble horizon as the IR cut-off, and no interaction is there between the DE candidate and other parts of the Universe. The energy density and law of conservation epistolize to THDE in this practice ($L = H^{-1}$) is obtained as

$$\rho_D = BH^{-2\delta+4}. \tag{5}$$

$$\dot{\rho}_D + 3\rho_D H(1+\omega_D) = 0. \tag{6}$$

Here, the EoS parameter is $\omega_D = p_D/\rho_D$ and p_D is the THDE pressure.

3. The cosmological model

We consider an isotropic, spatially flat and homogeneous FLRW metric in the form

$$ds^2 = -dt^2 + a^2(t)\delta_{\mu\nu}dx^\mu dx^\nu, \tag{7}$$

where $a(t)$ is the scale factor. Additionally, we believe that pressureless dark matter (ρ_m) and THDE (ρ_D) fill the Universe. Then the first Friedmann equation is obtained as

$$H^2 = \frac{1}{3M_P^2}\left(\rho_D + \rho_M\right), \tag{8}$$

or similarly [55],

$$\frac{H(z)}{H} \equiv E(z) = \left(\frac{\Omega_{m0}(z+1)^3}{1-\Omega_D}\right)^{\frac{1}{2}}. \tag{9}$$

Here, the Universe red-shift and energy density of the pressureless matter are $z = -1+\frac{1}{a}$ and ρ_m, respectively. Tracing the dimensionless density parameter and critical energy density as $\Omega_i = \rho_i/\rho_c$ ($\rho_c = 3\,M_P^2H^2$ where $M_P \equiv 1/\sqrt{8\pi G}$) [32], we get

$$\Omega_D = \frac{\rho_D}{3M_P^2H^2} = \frac{B}{3M_P^2}H^{-2\delta+2},$$
$$\Omega_m = \frac{\rho_m}{3M_P^2H^2}. \tag{10}$$

By using it we can rewrite eq. (8) as

$$\Omega_m + \Omega_D = \Omega_D(1+u) = 1, \tag{11}$$

where

$$u = \frac{\rho_m}{\rho_D} = \frac{\Omega_m}{\Omega_D}.$$

As there is no interaction among DM and THDE, the conservation equation of matter is

$$\dot{\rho}_m + 3H\rho_m = 0. \tag{12}$$

Applying eqs (12) and (6) and with the time derivative of eq. (8), and combining the result with eq. (10), one can conclude that

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}(1+\omega_D+u)\Omega_D. \tag{13}$$

On the other hand, placing eq. (5) in eq. (6), we obtain

$$\frac{\dot{H}}{H^2} = (1+\omega_D)\frac{3}{2\delta-4}, \tag{14}$$

comparing with eq. (13) to reach

$$\omega_D = -1 - \frac{u\Omega_D(\delta-2)}{1+(\delta-2)\Omega_D}, \tag{15}$$

by eq. (11) in mind, this result takes finally the form

$$\omega_D = \frac{1-\delta}{1+(\delta-2)\Omega_D}. \tag{16}$$

As shown in [47], we have $2-\delta > 1$ for $\delta < 1$ which signifies separation in the nature of Ω_D at the red-shift for which $\omega_D = \frac{1}{2-\delta}$. Hence, the $\delta < 1$ case is not suitable for our structure. We can just conclude from eq. (10)

$$\Omega_D' = \frac{d\Omega_D}{d(\ln\, a)} = (-2\delta+2)\Omega_D\frac{\dot{H}}{H^2}. \tag{17}$$

Using eqs (14) and (16) together we obtain

$$\Omega_D' = 3(\delta-1)\Omega_D\left(\frac{1-\Omega_D}{1-(2-\delta)\Omega_D}\right). \tag{18}$$

As $0 \le \Omega_D \le 1$, it can be noticed easily that for $\Omega_D \to 0\,(\Omega_D \to 1)$, we have $\omega_D \to 1-\delta\,(\omega_D \to -1)$. Also, $\omega_D \to -1$ for $\delta = 2$, a result independent of the value of Ω_D. This case is similar to the DE model of the cosmological constant. One can observe that THDE model is having a vital part in δ. The deceleration parameter (DP) q is shown as

$$q = -1 - \frac{\dot{H}}{H^2}. \tag{19}$$

On working out with eqs (16) and (14), we obtain

$$q = \left(\frac{1}{2}\right)\left[\frac{1+(1-2\delta)\Omega_D}{1-(2-\delta)\Omega_D}\right]. \tag{20}$$

4. Cosmological evolution

Here, we review the cosmological expansion of the EoS parameter and DP in the Universe thoroughly, where THDE is the DE frame. Significantly, eqs (16) and (20) present the expressions of EoS parameters and DP applying Ω_D, the density parameter of THDE. Equation (18) shows a differential equation, which explains the Tsallis energy density parameter Ω_D as a function $x = \ln a$. Unfortunately, when $\delta = 1$, these expressions can be analytically solved in a certain way [31], while for $\delta \neq 1$, it acquires an explicit x-dependence that does not allow for an analytical solution [51]. Hence, to illustrate the expansion of the cosmos, the researchers should utilise the numerical exposition of Ω_D. Consequently, by exaggerating eq. (18) numerically and imposing $\Omega_{D0} = 0.70$ and $\Omega_{m0} = 0.30$ as expected by the survey [2], we analysed the cosmic expansion of the Universe.

Let us precisely examine the EoS parameter and DP of the THDE. Undoubtedly, the form of DP contributes blueprints, and incredible facts concerning the cosmos at a provided span, regardless of whether the Universe is in the acceleration or deceleration stage. Primarily, we shall focus on q. Henceforth, one can distinguish three cases, diverging the physical term of viable q_0:

(1) $q_0 > 0$ indicates the evolution of the cosmos and sustains the deceleration phase. The given form is for both matter-dominated cosmos and pressureless barotropic fluid. $q > 0$ does not get any favour from the observation in the current era as it works for the past Universe, while DE did not dominate the matter.

(2) $-1 < q_0 < 0$ signifies the evolution of the presently speeding up Universe. In actuality, it portrays the condition of the Universe. The researchers assumed that the Universe is surpassed by somewhat antigravitational fluid [56]. In turn, cosmography confirms such characteristics, without postulating any particular form of dark energy evolution [57,58].

(3) $q_0 = -1$ indicates that all the whole cosmological energy budget is dominated by a de Sitter fluid, i.e. a cosmic component with constant energy density which does not evolve as the Universe expands. This is the case of inflation at the very early Universe. However, at the present time,this value is ruled out by observations [56–58].

We draft the evolutionary nature of the DP for distinct values of Tsallis exponent $\delta > 2$. Figure 1 depicts the expansion of the DP (q) of the THDE model with redshift (z). The framework of this graph can dynamically describe the shift from decelerating state to accelerating state as demanded by the findings.

Moreover, we concentrate on the dynamic nature of the EoS parameter of the THDE model and especially review with the Tsallis exponent δ, which quantifies the variation from the general framework. In figure 2, we portray the EoS parameter vs. redshift for diverse values of Tsallis exponent. Usual research is that for extending δ, the complete expansion of ω_D tends to obtain lower values. δ is vital in this research. If $\delta < 2$ then $\omega_D > -1$, i.e. the EoS is greater than -1 (quintessence phase). For $\delta > 2$ the value of ω_D remains in the phantom region. This was assumed because eq. (16) supports phantom values, which exposes its skills and also a general interest in the situation at hand. And, for $\delta = 2$, we get $\omega_D = -1$, a conclusion free of the value of Ω_D. As the Universe expands, its behaviour evolves more like a cosmological constant. We can work out not only on the features of DE but also the future of the Universe that relies on the choices of the Tsallis exponent, which shows the importance of δ for the THDE model. Thus,

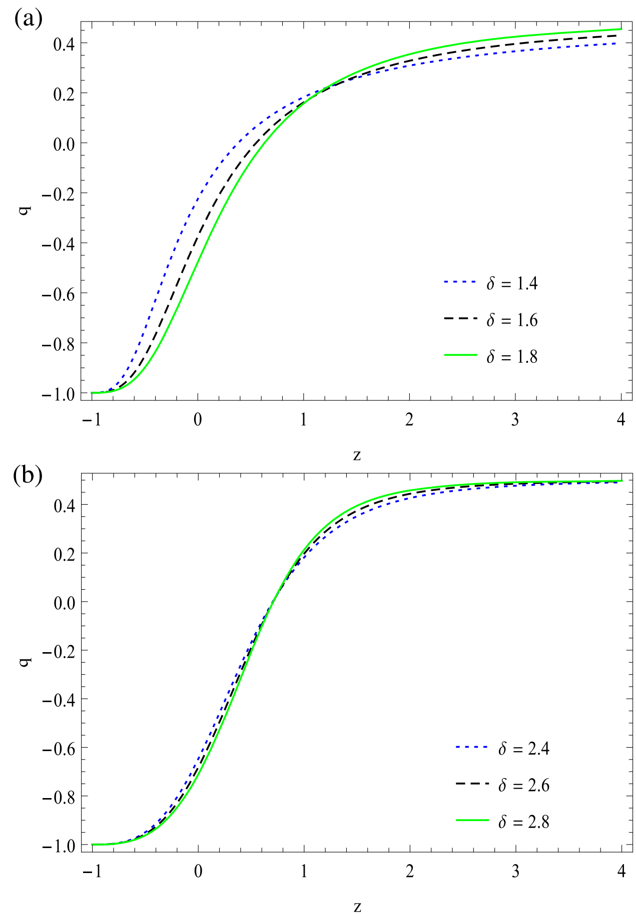

Figure 1. Evolutionary plot for DP (q) vs. red-shift parameter (z) of THDE for (**a**) $\delta < 2$ and (**b**) $\delta > 2$. Here, we have taken $\Omega_{D0} = 0.70$.

we see that by the values of the Tsallis exponent, THDE can lie in the quintessence or phantom region.

From a theoretical perspective, THDE with $\delta < 2$, in precise $\delta = 2$ appears reasonable [47]. Altogether, the preferences of $\delta > 2$ are advantageous for research, and so one cannot overlook the prospect of $\delta > 2$ entirely. Correspondence between scalar fields of the phantom and THDE can be set by $\delta > 2$, and THDE can be explained in this state efficiently with the help of phantom. This correspondence is called 'Tsallis holographic phantom'.

5. Tsallis holographic phantom

As discussed above, the dynamic behaviour of DE can be explained by the THDE framework. The density of DE evolves dynamically in a cosmic scenario, using the standard holographic principle while applying the Tsallis entropy rather than the standard Bekenstein–Hawking one. Taking the scalar field model into account,

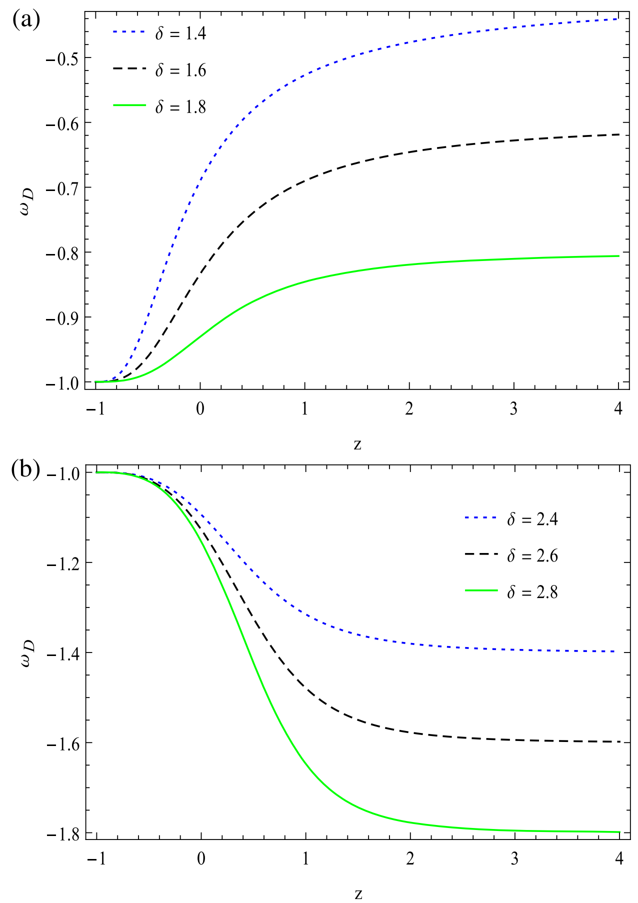

Figure 2. The nature of evolution for the EoS parameter ω_D vs. redshift z of THDE for distinct choices of δ. (**a**) When $\delta < 2$, ω_D behaves as quintessence and (**b**) when $\delta > 2$, ω_D behaves as a phantom. We have considered $\Omega_{D0} = 0.70$.

which is noted individually, provides a fruitful explanation of the fundamental DE hypothesis. It is hard to acquire the underlying hypothesis of DE without including a complete theory of quantum gravity. Yet, using some features of the quantum gravity model, one can achieve this by assuming the fundamental DE hypothesis that provides an uncertain THDE model. Here, interest appears to uncover how the scalar field model is valid effectively by taking THDE as a fundamental DE hypothesis.

In compliance with the Klein–Gordon equation $\ddot{\phi} + 3H\dot{\phi} - \mathrm{d}V/\mathrm{d}\phi$, here ϕ and $V(\phi)$ are phantom scalar and potential fields which arise according to its potential and aims to move towards the lowest of the potential. The slope shows the expansion rate and the potential diminishes due to H, which is accountable for the evolution of the Universe. Hoyle's version first introduced the phantom scalar fields in the steady-state hypothesis. A creation field (C-field) was presented by him through the appearance of new matter in the voids generated at the evolution of the Universe in compliance with the ideal principle of cosmology to adjust the model with the homogeneous density of the Universe [59]. Moreover, Hoyle and Narlikar assembled and refined it into its hypothesis called Hoyle and Narlikar gravitation hypothesis [60]. One can show the scalar field of phantom action coupled minimally towards gravitation as [61]

$$S = \int \mathrm{d}^4x \left[- V_\phi + \frac{1}{2}(\Delta\phi)^2 \right] \sqrt{-g}. \tag{21}$$

Therefore, the pressure and energy density are as follows [50,61]:

$$p_\phi = -\frac{1}{2}\dot{\phi}^2 - V_\phi, \quad \rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V_\phi \tag{22}$$

moving towards the EoS parameter $\omega_\phi = p_\phi/\rho_\phi$. Later, we get $\omega_\phi < -1$ for $\frac{\dot{\phi}^2}{2} < V_\phi$. However, the phantom scalar field ϕ did not across the divide line of CC ($\omega_\phi = -1$) [17,62,63]. Hence, only for $\delta > 2$, THDE can be expressed as the phantom. In the Universe, inside a finite interval, the curvature of the Universe increases to infinity by a phantom fluid [61]. We can attain ϕ (the scalar field) and $V(\phi)$ (the scalar field potential) as

$$\dot{\phi}^2 = -\left(\omega_\phi + 1\right)\rho_\phi, \tag{23}$$

$$V_\phi = \frac{1 - \omega_\phi}{2}\rho_\phi. \tag{24}$$

According to the forms of phantom pressure and energy density, we can acquie kinetic energy and the scalar potential expressions as:

$$\frac{1}{\rho_{c0}}\dot{\phi}^2 = -\left(1 + \omega_\phi\right)\Omega_\phi E^2, \tag{25}$$

$$\frac{1}{\rho_{c0}}V_\phi = \frac{1}{2}E^2\Omega_\phi\left(1 - \omega_\phi\right), \tag{26}$$

where $\rho_{c0} = 3M_P^2H_0^2$ represents the recent critical density of the Universe. To suggest a resemblance among the phantom scalar field and THDE, the energy density ρ_ϕ identifies with p_ϕ. Equations (16), (18) and (9) state ω_ϕ, Ω_ϕ and E. Furthermore, the derivative of the scalar field (ϕ) concerning red-shift (z) is found to be

$$\frac{\mathrm{d}\phi/\mathrm{d}z}{M_p} = \pm\frac{\sqrt{-3\left(\omega_\phi + 1\right)\Omega_\phi}}{1 + z}. \tag{27}$$

However, we can find the scalar field evolutionary structure as

$$\phi(z) = \int_0^z \frac{\mathrm{d}\phi}{\mathrm{d}z}\mathrm{d}z. \tag{28}$$

For the THDE model investigated in this work, the potential $V(\phi)$ can be obtained using eqs (26), (25) and (27). It is difficult to obtain the analytical structure of

$V(\phi)$ (the potential) because of the intricacy (due to the Tsallis parameter δ, for $\delta \neq 1$, it does not permit an analytical explanation) of those equations, while THDE phantom potential can be solved numerically. Therefore, we mathematically simulate the phantom field expansion with the implicit form of the potential ($V(\phi)$). Also, from some specific forms of the EoS, one can also rebuild the phantom potential ($V(\phi)$) [64,65].

Various cosmologists have modelled the number of phantom potentials [66–69]. Caldwell *et al* [70] investigated the scalar field of the phantom and noticed that a big-rip of the Universe could be generated when $\omega_D < -1$. The phantom region having zero kinetic energy directs to the maximum of the potential, and the domain determines the top of the potential all the time, mocking the de-Sitter-like nature ($\omega_D = -1$) due to its distinct features [71]. Figure 3 reveals that using eqs (27) and (28), we should reconstruct $\phi(z)$ and figure 4 shows the plot of phantom potential (V_ϕ). We plotted the arcs by selecting $\delta = 2.4$, 2.6 and 2.8. Figures 4 and 3 altogether portray the dynamics of the potential and the scalar field. The nature of the Tsallis holographic phantom is depicted in figure 5 as the ω_D–$\omega_{D'}$ plane. This figure shows that the EoS parameter evolves from $\omega_D < -1$, $\omega_{D'} > 0$ to $\omega_D \to -1$, $\omega_{D'} \to 0$ in the ω_D–$\omega_{D'}$ plane.

This research relates the non-interacting THDE with the phantom scalar field in a flat cosmos. We show that phantom can explain THDE when $\delta > 2$ (here, in the general case δ is a completely free parameter, but in the theory of similar probabilities, it is associated with the dimensionality of the system d, especially, $\delta = d/(d-1)$ for $d > 1$ [48]) in a specific manner. Taking $\Omega_{D0} = 0.70$ for the present time, the research formed the conformity among the non-interacting THDE and phantom and also reconstructed the Tsallis holographic phantom potential. Other forms of HDE extensively examined in the literature indicated the phantom. For example, Setare [72] proposed conformity among the interacting HDE framework in flat cosmos and the phantom DE model with potential in the scenario of Brans–Dicke hypothesis. The researchers of [50] presented that phantom can explain the HDE when $c < 1$ (c is a positive constant in the holographic model of DE) in a specific form. Kim *et al* [73] showed that the interacting HDE model cannot adjust a shift from DE with $\omega_D \geq -1$ to the phantom region with $\omega_D < -1$. Lately, Varshney *et al* [74] specified the conformity among the phantom DE and THDE in the modified $f(R,T)$ gravity with Hubble horizon as the IR-cut off. Additionally, Elizalde *et al* [75] and Nojiri and Odintsov [76] suggested the generalised HDE model from which some examples might cross the phantom border line. However, the present work has significance with additional research given by [50,72–74] in diverse forms. Firstly, in several issues of current interest, general relativity

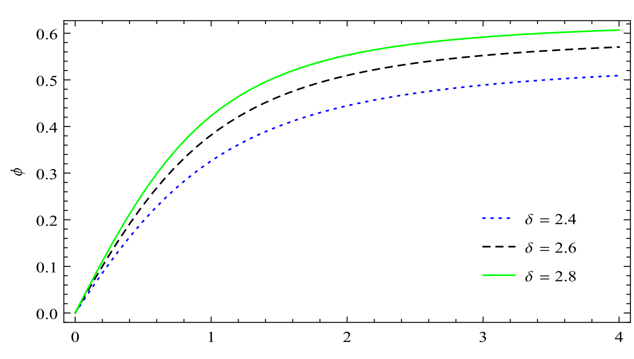

Figure 3. The nature of evolution for ϕ (the scalar field) with with z (red-shift) for the Tsallis holographic phantom having $\delta > 2$. Here, ϕ is in form of M_p^2. We have considered $\Omega_{D0} = 0.70$.

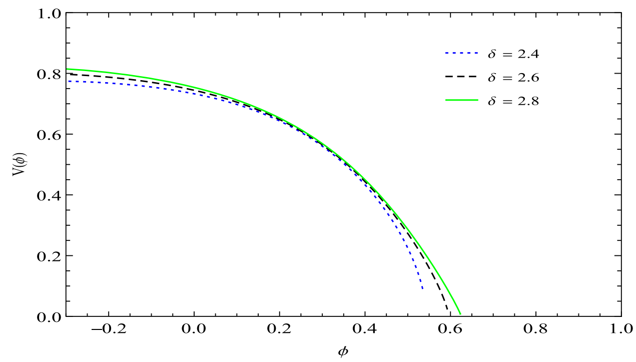

Figure 4. The reconstruction of the potential of Tsallis holographic phantom with ϕ (scalar field) for $\delta > 2$. Here the potential V is in the form of ρ_{c0} and ϕ (the scalar field) in M_p^2. Also, we have considered $\Omega_{D0} = 0.70$.

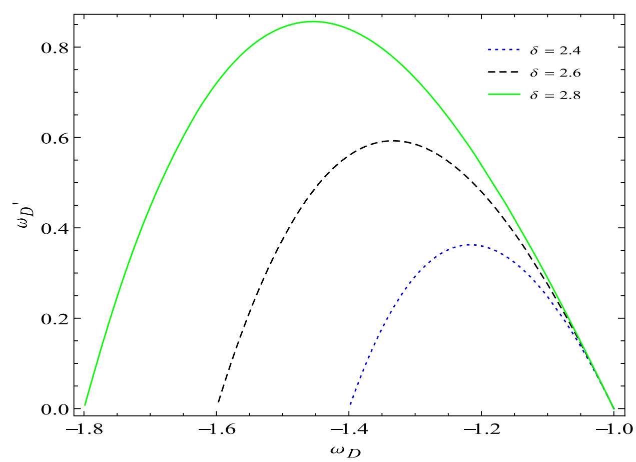

Figure 5. The trajectories of evolution for the Tsallis holographic phantom in ω_D–ω'_D plane for distinct values when $\delta > 2$. Here, we have considered $\Omega_{D0} = 0.70$.

(GR) has some significant distinctions compared to the hypothesis of modified gravity. Secondly, phantom correspondence with THDE has been established without the interaction of THDE and dark matter, which has not been done previously. Thirdly, this research includes the recently suggested THDE model that depends upon non-extensive Tsallis entropy similar to the HDE in [34,50,72,73,77,78]. Very recently, some more versions of HDE based on entropy formalism were proposed as Barrow HDE [40], Kaniadakis HDE [79] and New THDE [80]. All mentioned variants of HDE exhibited the phantom-divide crossing during the expansion of the Universe as IR cut-off with the future event horizon.

6. Conclusions

The Tsallis holographic dark energy based on Tsallis entropy [48] and holographic principle has been proposed and examined recently in literature [47,51,74,81–84]. D'Agostino [85] constrained THDE model considering δ as a free parameter applying a Bayesian analysis at low red-shift cosmological data as matter fluctuations growth rate factor (GRF) data combined byobservational Hubble data (OHD) and type Ia Supernovae (SN) data. D'Agostino showed that the EoS of the THDE models lies in the quintessence regime for $\delta < 2$. Sadri [86] combined the data such as BAO, CMB and SNIa and gamma-ray burst to constrain the Tsallis free parameter δ of the THDE model. Although in modern observation it is assumed that $\delta > 2$ is compatible, yet, future investigations are required to compile this model more accurately. However, for $\delta > 2$, THDE will act like a DE phantom model. Consequently, we understand that for the future of the Universe and THDE characteristics resolving the value of δ is essential. Different groups provided various ranges for the Tsallis parameter δ in modern research. Similarly, Sadri [86] applied the data evolving from the BAO, CMB and SNIa to constrain THDE and obtained the suitable results $\delta = 2.121^{+0.150}_{-0.229}$.

In summary, we see that the scenario can very well describe the shift from decelerating phase to accelerating phase as demanded by observations. Besides, the EoS parameter extends to the phantom domain ($\delta > 2$) and resembles -1 in the future. Further, the conformity among the scalar field of the phantom and THDE is shown in the research. After assuming that the scalar field models of the DE are effective theories of an underlying theory of DE. Although the fundamental theory holds some characteristics of a gravitational quantum theory, it is not attained presently that can be investigated by practicing the holographic principle and generalised entropy formalism. Therefore, while remarking the Tsallis entropy and the principle of holographic, the vacuum energy obtains the dynamical property. However, accessible observations indicate that THDE resembles the phantom and does not rule out the probability of $\omega_B > -1$. If we notice the model of a scalar field (like a phantom) as an efficient explanation of such a hypothesis (THDE), then we need to be proficient in rebuilding these models of scalar field according to the evolutionary nature of THDE and practicing the model of the scalar field to imitate the evolving nature of the dynamical vacuum energy. We note that for $\delta > 2$, the phantom region can explain THDE completely in a specific approach. We established the conformity among the phantom and THDE and reconstructed the dynamics of the field and holographic phantom potential.

In the future, by assuming the IR cut-off as a combination of particle and future horizon, one can hope to acquire the phantom phase. Furthermore, the phantom scalar field can explain THDE with the choice of L as the combination of particle and future horizon in GR and also in modified gravity theories.

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