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Spontaneous symmetry breaking and massive photons from a Fresnel-type potential

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Abstract. We discuss spontaneous symmetry breaking in the presence of a new type of symmetric potential based on Fresnel integrals which give an infinite number of minima. Several interesting points were raised, in particular the emergence of massive Goldstone boson and an enhancement of the photon mass. The new theory depends on discrete numbers $n, N \in \mathrm{Z}$ and hence a large family of massive particles may be obtained filling the gap between the electroweak scale and the Planck scale in the standard model.

Keywords. Higgs boson; massive Goldstone boson; photon mass enhancement; standard model.

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As it stands today, the standard model (SM) of particle physics is the most accurately verified theory by several experiments done at CERN through its phenomenological properties and particle contents. Although the puzzle piece of SM has been found after the discovery of the Higgs boson based on ATLAS and CMS collaborations at LHC, we still believe that it is not the complete picture and that a new physics beyond the standard model and around the TeV scale is required, the Higgs mass being quadratic sensitive to the Planck scale [1–3]. A tricky problem that arises with the discovery of the Higgs boson is the well-known hierarchy problem which concerns the big gap or desert between the electroweak scale of about 125 GeV and the Planck scale which is about 2.4×10^{18} GeV. Although the new physics would give rise to physical corrections to the Higgs mass, e.g. quadratically divergent mass corrections and to its quartic Mexican potential (QMP), fine tune cancellations are strongly required [4]. There are several arguments in favour of the presence of new physics including dark matter and massive neutrinos. Whatever the case is, the central question is how we construct new physics? Particle physics is a huge information quantum ground and to decipher the new physics is a tricky problem. In this letter, we would like to attack the problem from a different simple perspective.

The Higgs is a typical example where its scalar field naturally is governed by the QMP $V(\phi) = -\frac{1}{2}\mu^2\phi\phi^* + \frac{1}{4}\lambda\phi^2\phi^{*2}$. Here ϕ is the complex scalar field and ϕ^* its complex conjugate, $\lambda > 0$ is the Higgs field self-coupling constant and $\mu^2 < 0$ has unit GeV^2. Although the Higgs boson is based on QMP, its self-couplings are inadequately constrained and hence leave the nature of the Higgs boson mysterious. However, there are several physical scenarios where scalar fields are dominated by periodic potentials, e.g. periodic Higgs potential, periodic axion potential and periodic inflationary potential [5]. In general, the standard model of particle physics is dominated by QMP or Higgs potential which is used to generate mass for the electroweak gauge bosons W and Z below a certain critical temperature. However, several theoretical arguments suggest that new terms must be added to the Higgs potential if we need to connect the Higgs potential to the inflationary paradigm which gives rise to local minima. It is noteworthy that at high energies, quantum gravitational corrections are important

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since the Higgs potential barrier is required to be dependent on the effective Higgs mass at high energy limits [6]. These extended potentials must be stable and their constancy or stability has been explored using the renormalisation group technique. However, this method has proved that oscillatory potentials are motivating since an oscillatory Higgs potential is characterised by infinitely many degenerate minima although it requires a careful renormalisation group analysis. This has important consequences on spin models in quantum field theory and on the physics of the early Universe, dark matter and dark energy [7–11].

One motivation of this paper is to see if an oscillatory non-quartic potential with infinite number of minima can help in filling the desert between the electroweak and the Planck's scales. To do this, we introduce the following new class of extended symmetric potential:

$$V(\phi) = \frac{\mu^4}{2\lambda}\phi \sinh\left(C\left(-\sqrt{-\frac{\lambda}{\mu^2}\phi}\right)\right), \tag{1}$$

where $\phi \to \sqrt{\phi^*\phi}$ is a complex scalar field, ϕ^* is its complex conjugate and

$$C(\phi) = \int_0^{\phi} \cos x^2 \mathrm{d}x = \sum_{n=0}^{\infty} \frac{(-1)^n \phi^{4n+1}}{(2n)!(4n+1)} \tag{2}$$

is the Fresnel integral largely used in optics. This potential has a maximum at $\phi = 0$ and first minima around a circle. Hence, the ground state, i.e. the vacuum state $|0\rangle$ breaks the symmetry which gives rise to degenerate vacuum state. It is notable that

$$\lim_{x\to\pm\infty} x \sinh(C(-x)) = -\infty,$$
$$\lim_{x\to 0} x \exp(C(-x)) = 0,$$
$$x \sinh(C(-x)) \approx -x^2 - \frac{1}{6}x^4 + \frac{3\pi^2 - 1}{120}x^6 + \frac{63\pi^2 - 1}{5040}x^8 - \frac{105\pi^4 - 378\pi^2 + 1}{362880}x^{10} + \mathrm{O}(x^{12}),$$

and the potential is characterised by the first derivative:

$$\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} = -\sqrt{-\frac{\lambda}{\mu^2}\phi} \cos\left(\frac{\pi}{2}\frac{\lambda}{\mu^2}\phi^2\right) \cosh\left(C\left(\sqrt{-\frac{\lambda}{\mu^2}\phi}\right)\right) - \sinh\left(C\left(\sqrt{-\frac{\lambda}{\mu^2}\phi}\right)\right). \tag{3}$$

These types of symmetric potential are motivating since they give rise to infinite discrete minima and enhanced weighty Higgs boson, i.e. Higgs cascade growth. Besides, potential (1) differs from various types of potentials introduced in literature, mainly the Nambu–Goldstone potential, the Coleman–Weinberg potential and the tadpole-induced potential [12]. It is notable that the employed potential, when expanded around certain minimum, contains terms in the 6th, 8th,..., powers in the field. It is therefore manifestly not renormalisable. In fact, dynamics of non-renormalisable potentials in electroweak symmetry breaking have been reported in [13] mainly in phenomenological models where the Higgs boson is composite and emerges as a light pseudo-Goldstone boson of a strongly interacting sector. Besides, such an approach has been explored in the supersymmetric Landau gauge extension to the standard model where all components are quantum supermultiplets [14]. Moreover, in general, the vacuum stability problem is of particular importance since at large scales, the effective scalar potential can get second minimum besides the one at the Fermi scale depending on the values and the ratio of the Higgs and top mass [15]. If the 2nd minimum of the effective potential is the lowest energy state and if the inverse decay rate from the false electroweak vacuum to the true vacuum state surpasses the lifetime of the Universe, then metastability occurs [16]. Yet, the potential is unstable if it is unbounded below or for huge decay rates. In general, the lower bound of the Higgs mass may be obtained from the renormalisable group approach and is unswervingly connected to the vacuum stability problem of the effective Higgs potential [17–22]. Undoubtedly, the topology of the effective potential depends on the numerical estimates of various parameters, including the Higgs mass and the value of the strong coupling constants. Since at high energy scales, the measured Higgs mass is not in agreement with the lower bound, a careful understanding of the lower Higgs mass bound and dynamics of non-renormalisable effective potential within the standard model are compulsory. It should be stressed that higher-order corrections to Higgs boson decays and Higgs sectors have been studied in literature through various phenomenological theories, e.g. minimal supersymmetric models (MSSM) [23] and non-renormalisable supersymmetric models [14,24]. In general, higher-order multifield potentials have motivating consequences in cosmology and Higgs-octic inflation paradigm [21,22,25–28]. These higher-order terms may be generated by strong dynamics at the TeV scale or by the presence of additional singlet scalar field which gives rise to the emergence of heavy particles [29]. These may have important consequences on electroweak baryogenesis problem and pseudo-Goldstone

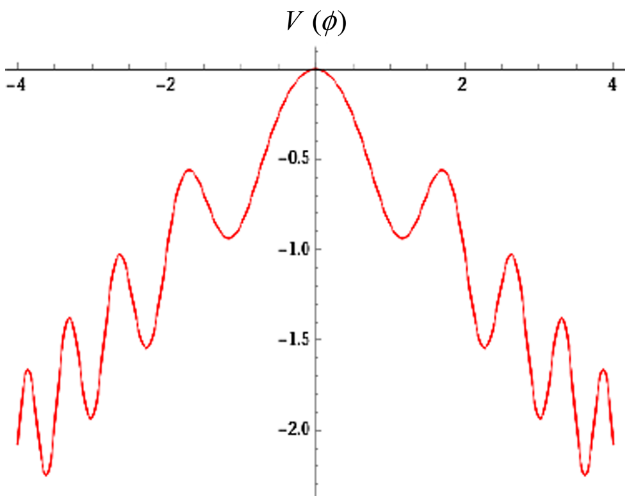

Figure 1. 1D plot of $V(\phi)$ for $\mu^2 < 0, \lambda > 0$.

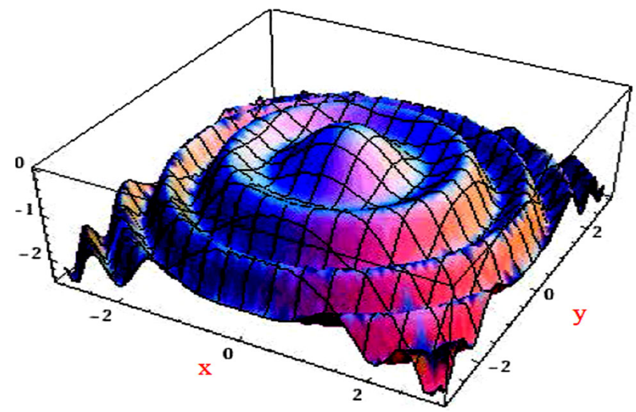

Figure 2. 3D plot of $V(\phi = \sqrt{x^2 + y^2})$ for $\mu^2 < 0, \lambda > 0$.

Higgs model [30,31]. Let us add that the number of coupling constants is limited in the standard model with one Higgs doublet for which the Higgs potential is bounded from below. However, in supersymmetric or Grand Unification theories, several scalar doublets emerge which give rise to several multiplets of scalar particles. Hence, the necessary and sufficient conditions under which the potential is bounded from below, becomes non-trivial [32]. At the end, it is noteworthy that the exact shape of the Higgs potential can have profound theoretical consequences on our thoughtful fundamental interactions not only at the electroweak scale but also at high energy limits. Since $-\lambda/\mu^2 \ll 1$, we can approximate $\sinh(C(\sqrt{-a}x)) \approx \sqrt{-a}x + \mathrm{O}(x^3), a < 0$ and $\cosh(C(\sqrt{-a}x)) \approx 1 + \mathrm{O}(x^2)$. Hence

$$\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} \simeq -\sqrt{-\frac{\lambda}{\mu^2}}\phi\left(\cos\left(\frac{\pi}{2}\frac{\lambda}{\mu^2}\phi^2\right)+1\right) \tag{4}$$

and hence

$$\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} = 0$$

$$\Rightarrow \begin{cases} \phi_0 = 0 \\ \phi_{n1} = \pm\sqrt{\frac{2\mu^2}{\lambda}}\sqrt{2n-1} \equiv \nu_{n1}, \quad n \in \mathrm{Z} \\ \phi_{n2} = \pm\sqrt{\frac{2\mu^2}{\lambda}}\sqrt{2n+1} \equiv \nu_{n2}, \quad n \in \mathrm{Z} \end{cases}. \tag{5}$$

The potential has a local maximum at $\phi = 0$ where $V(\phi) = 0$. Its 1D and 3D plots in normalised units are given in figures 1 and 2 for $\mu^2 < 0, \lambda > 0$ (ϕ^* being the complex conjugate of ϕ) for two different scales (all figures are plotted in normalised units):

These figures illustrate the dissimilarities between our proposed potential and the Higgs potential for different scales. For large scales, potential (1) is characterised by disordered circular ripples due to the numerical values of the coupling parameters as observed in the Taylor series expansion of $x \sinh(C(-x))$. Potential (1) has a minimum along the circles which correspond to $n \in Z$. Hence, along the curvatures of the potential, radial excitations correspond to massive particles, whereas along the circle the potential is flat, thus the massless excitation. It is notable that energy quantisation is not in fact a common property of quantum mechanical systems and the quantisation is at all times a consequence of a particular feature of the potential. Since the scalar potential can obtain a second minimum and more at large scales in addition to the one at the Fermi scale obtained within the standard Higgs model, then one naturally expects that the potential is metastable. In the case of a stable potential, the electroweak minimum is nothing but the ground state of the theory. In general, metastability occurs when the second minimum is the lowest energy state and the inverse decay rate of electroweak vacuum in the standard model is much larger than the age of the Universe [33,34]. It is believed that some mechanism may reserve the behaviour of the potential at very high energies due to non-perturbative physics undetectable to a loop expansion. This already means that the minimum energy is bounded from below, and the generation of at least a second local minimum beyond the barrier may be successful due to quantum correction effects. This is the case of the standard model and effective theory where the problem of an infinitely deep well is absent. But such a state will probably not persist in the presence of gravitational corrections which are expected to become important at large field values and hence rendering the potential unstable. Whatever the case is, if the potential possesses regions with lower energy than at the origin, a minimum beyond the barrier has no major role due to the decay of the vacuum at the origin of the potential. These conclusions are untimely in reality and much work is still required and probably the need of a new physics beyond the standard model [35]. It is notable that Higgs-like potentials with multiple minima have been considered in literature and may also be considered unnatural regarding the sizes of their dimension-full operators. Yet, the problem may be solved by using loop quantum corrections [36–38]. They have important considerations in astrophysics and cosmology. For example, a potential with two-degenerate minima may imply that quantum chromodynamics matter at two separate sets of quark masses is significant for high-energy physics and early-epoch of time [36]. It should be stressed that, although instabilities are not favoured in physics, it has been known that the standard Higgs potential develops an instability at large field values at a scale of the order of 10^{11} GeVand dark matter in the form of primordial black holes seeded by Higgs fluctuations during the inflationary epoch of time may be correlated to this instability [39,40].

To determine the particle spectrum, we study the theory in the region of each minimum $\phi_n(x) = \nu_{nk} + \eta_n(x), n \in Z, k = 1, 2$. We consider the case $k = 2$ with $n \in Z^-$ since its gives the plausible solution. The Lagrangian of the theory $L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$ is therefore replaced after calculation by

$$L = \frac{1}{2}\partial_\mu\eta_n\partial^\mu\eta^n - \frac{1}{2}\mu^2\eta_n^2 \times \left(1 - 2(2n+1) - \frac{3\pi^2 - 1}{16}(2n+1)^2 + \cdots\right) + \text{interaction terms.} \tag{6}$$

This leads to masses

$$m_\eta^2 = \mu^2 \times \left(1 - 2(2n+1) - \frac{1}{16}(3\pi^2 - 1) \times (2n+1)^2 + \cdots\right).$$

For $n = 0$, we find $m_\eta^2 \approx -2.8\mu^2$, for $n = 1$ we find $m_\eta^2 \approx -21\mu^2$, for $n = 2$, we get $m_\eta^2 \approx -53.7\mu^2$ and so on, whereas for $n = -1$, we get $m_\eta^2 \approx -9\mu^2$ and for $n = -2$, we get $m_\eta^2 \approx -33.7\mu^2$. In the Higgs mechanism with global $U(1)$ symmetry, we assume that the scalar field ϕ is complex and we look for a solution of the form

$$\phi_{nN}(x) = \frac{1}{\sqrt{2}}(\nu_{n2} + \eta_n(x) + i\rho_N(x)).$$

Here, $\rho_N(x)$ is a new scalar field known as the Goldstone field where we associate to it the vacuum expectation value ν_{N2}, $N = 1, 2, 3, \ldots$. We find therefore,

$$\begin{aligned} L &= \left(\partial_\mu\phi\right)^*\left(\partial^\mu\phi\right) - \frac{\mu^4}{2\lambda}\left(-\left(-\frac{\lambda}{\mu^2}\right)\phi\phi^* - \frac{1}{6}\left(-\frac{\lambda}{\mu^2}\right)^2(\phi\phi^*)^2 + \frac{3\pi^2-1}{120}\left(-\frac{\lambda}{\mu^2}\right)^3 \times(\phi\phi^*)^3 + \mathrm{O}((\phi\phi^*)^4)\right) \\ &\approx \frac{1}{2}\left(\partial_\mu\rho_N\right)^2 + \frac{1}{2}\left(\partial_\mu\eta_n\right)^2 - \frac{1}{2}\left(1 - 2(2n+1) - \frac{3\pi^2-1}{16}(2n+1)^2 + \cdots\right)\mu^2\eta_n^2 \\ &\quad - \frac{1}{2}\left(1 - \frac{1}{3}(2N+1) - \frac{3\pi^2-1}{40}(2N+1)^2 + \cdots\right)\mu^2\rho_N^2 + \cdots. \end{aligned} \tag{7}$$

Any (local) minimum which is flat in an azimuthal direction should have an associated massless degree-of-freedom. However, in our case for $n = 0$, we find $m_\eta^2 = -\mu^2$ whereas for $N = 1$ the boson associated with the field $\rho_N(x)$ has mass $m_\rho^2 \approx -6.43\mu^2$. For $N = 2$ we find $m_\rho^2 \approx -18.54\mu^2$ and so on. Yet, for higher-order corrections and for $N \in Z^-$, it is easy to verify that the Goldstone boson is therefore massive. In fact, massive Goldstone boson is explored in various aspects of quantum many-body systems [41,42]. Moreover, Goldstone bosons are associated with dark matter production. More precisely, the stability of dark matter particle could be attributed to the residue of Z_2-symmetry emerging from the spontaneous symmetry breaking of the global $U(1)$ symmetry. The emergent Goldstone boson is a good candidate for dark radiation [43]. It was shown in [44,45] that this emergent Goldstone boson plays a leading role in the production of dark matter. Besides, it was revealed in [46] that fermionic superpartner of a weak-scale Goldstone boson can be a plausible WIMP candidate. More probes of Goldstone dark matter are discussed in [47–50].

We can now study the Abelian mechanism in the presence of an electromagnetic field A_μ by requiring local gauge invariance, i.e. $\partial_\mu \to D_\mu = \partial_\mu - igA_\mu$ and the unitary gauge $A_\mu \to A'_\mu = A_\mu - g^{-1}\partial_\mu\rho(x)$ (g being a coupling constant). For suitable $\rho(x)$ (under a certain phase of rotation), we look for a solution such that

$$\phi(x) = \frac{1}{\sqrt{2}}(\nu_n + H(x)),$$

$H(x)$ being the Higgs field for which we associate the vacuum expectation value ν_{n1} whereas for A_μ we associate ν_{N2}, $N \in \mathrm{Z}^{*-}$ and we write the Lagrangian as

$$\begin{aligned}L &= \left(D_\mu\phi\right)^*\left(D^\mu\phi\right) - \frac{\mu^4}{2\lambda}\left(-\left(-\frac{\lambda}{\mu^2}\right)\phi\phi^* \right.\\ &\quad -\frac{1}{6}\left(-\frac{\lambda}{\mu^2}\right)^2(\phi\phi^*)^2 + \frac{3\pi^2-1}{120}\left(-\frac{\lambda}{\mu^2}\right)^3\\ &\quad \left.\times(\phi\phi^*)^3 + \mathrm{O}((\phi\phi^*)^4)\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\ &= \frac{1}{2}\partial_\mu H\partial^\mu H + \frac{\mu^2 g^2}{\lambda}(2N+1)A_\mu A^\mu\\ &\quad -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2H^2A_\mu A^\mu\\ &\quad -\frac{1}{2}\left(\frac{1}{2} - 2n - 1 - \frac{3\pi^2-1}{32}(2n+1)^2\right.\\ &\quad \left.+\cdots\right)\mu^2H^2 + \cdots . \end{aligned} \tag{8}$$

Hence, the photon square mass is

$$m_A^2 = \tfrac{1}{\lambda}(2N+1)\mu^2g^2, \quad \mu^2 < 0, \quad N \in \mathrm{Z}^{*-},$$

the Higgs boson square mass is

$$m_H^2 = \left(\frac{1}{2} + 2n + 1 - \frac{1}{32}(3\pi^2-1)(2n+1)^2 + \cdots\right)\mu^2.$$

The in-vacuum motion of a photon is known to be described by a massless propagator. The appearance of a simple mass term violates a Ward–Takahashi identity for the photon vacuum polarisation. A problem of this type was associated with the original vector meson dominance model of Sakurai. Additional terms in a Lagrangian are necessary to eliminate the defect. For $n = 0$ and $N = -1$,

$$m_H^2 = -0.5\mu^2 \quad \text{and} \quad m_A^2 = -\frac{1}{\lambda}\mu^2g^2$$

whereas for $n = 1$ and $N = -2$,

$$m_H^2 \approx -10.5\mu^2 \quad \text{and} \quad m_A^2 = -\tfrac{3}{\lambda}\mu^2g^2.$$

For $n = -1$, we get $m_H^2 \approx -1.4\mu^2$ and for $n = -2$, we find $m_H^2 \approx -10.54\mu^2$. We have therefore an enhancement in the photon mass with respect to the $U(1)$ gauge case which may have important consequences in cosmology, astrophysics [51–54] and magnetic materials [55].

The Abelian scenario can now be generalised in a clear-cut way to a non-Abelian gauge theory, in particular the Weinberg–Salam electroweak gauge theory, which combines electromagnetic and weak interactions [56,57]. This standard model is an $SU(2)_L \otimes U(1)_Y$ gauge theory holding three $SU(2)$ gauge bosons $W_\mu^i (i = 1, 2, 3)$ and one gauge boson B_μ with a kinetic energy term

$$L = -\frac{1}{4}(W_{\mu\nu}^i W^{\mu\nu i} + B_{\mu\nu}B^{\mu\nu}),$$

where $W_{\mu\nu}^i = \partial_\nu W_\mu^i - \partial_\mu W_\nu^i + g\varepsilon^{ijk}W_\mu^i W_\nu^k$ and $B_{\mu\nu} = \partial_\nu B_\mu - \partial_\mu B_\nu$. Here ε^{ijk} are the $SU(2)_L$ structure constants. We consider a complex scalar field in the spinor representation of $SU(2)_L$ such that

$$\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix},$$

with $U(1)$ charge $Y(\phi) = \frac{1}{2}$. Here the subscripts + and 0 indicate the electric charge Q of the components. The covariant derivative is

$$D_\mu = \partial_\mu + igT^iW_\mu^i + i\frac{1}{2}g'B_\mu,$$

where $T^i = \frac{1}{2}\tau^i$ are the generators of the fundamental representation of the $SU(2)_L$ Lie algebra (τ^i are the Pauli matrices, g and g' are the $SU(2)$ and $U(1)$ coupling constants respectively. By taking the symmetric potential of the form

$$\begin{aligned}V(\phi) &= \frac{\mu^4}{2\lambda}\left(-\left(-\frac{\lambda}{\mu^2}\right)\phi\phi^* - \frac{1}{6}\left(-\frac{\lambda}{\mu^2}\right)^2(\phi\phi^*)^2\right.\\ &\quad \left.+\frac{3\pi^2-1}{120}\left(-\frac{\lambda}{\mu^2}\right)^3(\phi\phi^*)^3 + \mathrm{O}((\phi\phi^*)^4)\right),\end{aligned} \tag{9}$$

and choosing

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \nu_{nk}\end{pmatrix}, \tag{10}$$

will result in $SU(2)_L \otimes U(1)_Y \to U(1)_Q$. Therefore, to generate the masses of W and Z fields, let

$$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \nu_{nk}+h\end{pmatrix}, \tag{11}$$

which gives, after associating with these fields the vacuum ν_{N2}, $N \in \mathrm{Z}^{*-}$,

$$\begin{aligned}&(D^\mu\phi)^\dagger(D_\mu\phi)\\ &= \frac{\nu_n^2}{8}\left(g^2\left(\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2\right) + \left(gW_\mu^3 - g'B_\mu\right)^2\right)\\ &= \frac{\mu^2}{\lambda}\frac{2N+1}{4}\left(g^2\left(\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2\right)\right.\\ &\quad \left.+\left(gW_\mu^3 - g'B_\mu\right)^2\right),\end{aligned} \tag{12}$$

where

$$W_\mu^{\pm} = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2).$$

Therefore, we find

$$m_W^2 = \frac{2N+1}{\lambda} g^2 \mu^2.$$

Besides, the mass associated with $Z_\mu = (g^2 + g'^2)^{-1/2}(g W_\mu^3 - g' B_\mu)$ and $A_\mu = (g^2 + g'^2)^{-1/2}(g' W_\mu^3 + g B_\mu)$ are respectively given by

$$m_Z^2 = \frac{2N+1}{\lambda}(g^2 + g'^2)\mu^2 \quad \text{and} \quad m_A^2 = 0.$$

For $N = -1$, we find

$$m_W^2 = -\frac{1}{\lambda} g^2 \mu^2, \quad m_Z^2 = -\frac{1}{\lambda}(g^2 + g'^2)\mu^2$$

whereas for $N = -2$, we find

$$m_W^2 = -\tfrac{3}{\lambda} g^2 \mu^2 \quad \text{and} \quad m_Z^2 = -\tfrac{3}{\lambda}(g^2 + g'^2)\mu^2.$$

The experimental values of the weak gauge boson masses are given by $m_W = 80.385 \pm 0.015$ GeVand $m_Z = 91.1876 \pm 0.0021$ GeV. This falls within the mass band predicted by the standard model. For $N = -2$, we therefore expects that $m_W \approx 140$ GeV and $m_Z \approx 156$ GeV. For lower values of N, these masses became more weighty and the gaps between the electroweak scale and the Planck's scale become more and more narrow. One open question to be addressed in the future: the model as it is, allows only for one choice of the (metastable) vacuum. The hierarchy problem translates then into the question why the SM is realised for the first non-trivial vacuum and not for the second, or the third, etc.

The QMP is still mysterious and not well understood in elementary particle physics and much work is still required. In this letter, we have introduced a new type of symmetric potential based on Fresnel integral function which extends the prediction of the SM. We have discussed the basic set-ups and we have found that such types of potentials can give rise to a large family of particles whose discrete masses are larger than the ones obtained in the conventional SM, filling therefore the gaps between the electroweak scale and the Planck's scale. Undoubtedly, this type of potential requires much study including its implementations in particle physics and its confrontation with experiments done at LHC. Nevertheless, it is important to carry out the electroweak renormalisation of all autonomous parameters of the present theory and address numerous diverse renormalisation schemes for the scalar mixing angles of the extended Higgs sectors [40,58–67]. It should be stressed that based on very accurate measurements of the Higgs boson mass in addition to the top quark mass, a new minimum may emerge at high energy limits, i.e. Grand Unified Theory and even Planck scales. This may have important consequences on the standard electroweak model since its associated vacuum state will be metastable. In other words, this implies that a tunnel transition to a new vacuum state with unusual physical properties may take place after a period much larger than the predictable age of the Universe [60]. This metastability could have important consequences on primordial black holes, gravitational waves and the nature of quantum gravity. In fact, the quartic Higgs potential becomes unstable at high energy limits, i.e. at large values of the Higgs field. The instability scale is too enormous (about 10^{11} GeV) and it is accordingly inaccessible by Earth colliders. It was revealed in [61] that Higgs instability may be detected during the inflationary period of time mainly from the stochastic background of gravitational waves sourced by Higgs fluctuations. These may have important impacts on dark matter, dark energy, black holes and several cosmological aspects predicted by higher-dimensional quantum gravity theories [40,62–72]. Nevertheless, these effects deserve to be tested experimentally at LHC and future 100 TeV hadron colliders, circular and linear e^+e^- colliders and a futuristic muon colliders [12]. It is also interesting to confront in a future work the Higgs sector predictions of the present model with results from Tevatron, LEP and LHC Higgs searches and to consistently estimate loop quantum corrections in the presence of deformations of the quartic Higgs potential [73]. Work in this direction is under progress.

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